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Economic manufacturing quantity model with investment and tolerance

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Abstract

In this article, the paper addresses a modified EMQ model with inspection. For the process parameters problem, one considers that quadratic loss function with process adjustment cost. For the integrated model, one considers the specified process capability index with consumer's risk.

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1. Introduction

Traditional EMQ model considers the perfect product. Previous researchers, e.g., Porteus [1] and Rosenblatt and Lee [2-3] addressed the imperfect process. Chen and Chou [4] considered the inventory decision model with quality loss Chou [5] addressed the quality investment applied in the reducing the product's bias and variability.

The process mean and standard deviation can affect the bias and variability of product. Taguchi [6] proposed the quadratic loss function for evaluating the product. Huang [7] considered that the adjustment cost is concern to process parameters.

In 2010, Shin et al.'s [8] process parameters setting model addressed Taguchi's [6] quality loss and process cost. Their tolerance design model

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considers the trade-off problem for decreasing the probability of defective product and increasing the manufacturing cost.

Quality investment is an important method for quality promotion. Some researchers, e.g., Hong et al. [9], Ganeshan, Kulkarni, and Boone [10] Chen and Tsou [11], and Tsou [12], have adopted the application of quality investment.

In this article, we present the modified EMQ model under the specified C_{pmk} value and consumer's risk for product's quality protection. The quantity, quality investment, tolerance, and parameters of sampling inspection plan will be determined.

2. Modified Economic Manufacturing Quantity Model

2.1 *Process Parameters Setting*

According to Shin et al. [8] and Huang [7], the modified process parameters setting model is as follows:

Minimize

$$\begin{aligned} E[TC(\mu,\ \sigma)] &= \int_{-\infty}^{\infty} k_1 (y-T)^2 f(y)dy + Z_1\mu^2 + Z_2\frac{1}{\sigma^2} \\ &= k_1[(\mu-T)^2+\sigma^2] + Z_1\mu^2 + Z_2\frac{1}{\sigma^2} \end{aligned} \tag{1}$$

Where k_1 : quality loss coefficient; T : target value of process mean; μ : unknown process mean; σ : unknown standard deviation; Z_1 : coefficient of μ^2 in the process cost; Z_2 : coefficient of $1/\sigma^2$ in the process cost; $E[TC(\mu,\sigma)]$: process expected total cost without inspection.

Let $\frac{\partial E[TC(\mu,\ \sigma)]}{\partial \mu} = 0$ and $\frac{\partial E[TC(\mu,\ \sigma)]}{\partial \sigma} = 0.$ We obtain the optimal $\mu_0 = \frac{k_1 T}{k_1+Z_1}$ and $\sigma_0 = \sqrt[4]{\frac{Z_2}{k_1}}$. The specified values of μ_0 and σ_0 are the input values of modified EMQ model.

2.2 *EMQ, Quality Investment, and Tolerance Settings Model*

Consider the EMQ model with lot size of rectifying inspection plan. We inspect all units when the rejected lot occurs. If the inspected unit is the defective, then it will be replaced. Hence, the conditional profit is

$$W_0 = \begin{cases} A_2Q - nI_c - QC\mu_0 - npR_I - (Q-n)LF_0 - nLF_1, & \text{if } X \le d_0 \\ A_1Q - R_IQp - QI_c - QC\mu_0 - QLF_1, & \text{if } X > d_0 \end{cases} \tag{2}$$

where A_1 : selling price per unit when lot rejected; A_2 : selling price per unit when lot accepted; Q : lot size; X : number of non-conforming unit; n : sample size; d_0 : allowance number of defective unit; I_c : inspection unit cost; R_I :the replacement unit cost; p : probability of non-conforming unit; C : processing unit cost; μ_0 initial value of process mean; LF_0 : expected quality loss without inspection; LF_1 : expected quality loss with inspection.

Hence, the expected total profit is

$$W_1 = \{(A_2 - A_1)Q + R_I \cdot Qp + (Q-n)I_c - npR_I + (Q-n)LF_1 - (Q-n)LF_0\} \\ P(X \le d_0) + (A_1 Q - R_I \cdot Qp - QI_c - QC\mu_0 - QLF_1) \tag{3}$$

where $P(X \le d_0)$: accepted probability of inspected lot..

From Eq. (3), the expected total profit is

$$W_2 = \{(A_2 - A_1) + R_I \cdot p + (1 - \tfrac{n}{Q})I_c + (1 - \tfrac{n}{Q})(LF_1 - LF_0) - \tfrac{npR_I}{Q}\}P(X \le d_0) \\ + (A_1 - R_I \cdot p - I_c - C\mu_0 - LF_1) \tag{4}$$

The modified EMQ model is

Maximize

$$ETP(Q,\ Inv,\ \delta,\ n,\ d_0) = M \cdot (W_2 - \tfrac{Inv}{Q}) - S_t \cdot \tfrac{M}{Q} - 0.5 \cdot Q \cdot (1 - \tfrac{O}{I})h \tag{5}$$

subject to

$$W_2 = \{(A_2 - A_1) + R_I \cdot p + (1 - \tfrac{n}{Q})I_c + (1 - \tfrac{n}{Q})(LF_1 - LF_0) - \tfrac{npR_I}{Q}\}P(X \le d_0) \\ + (A_1 - R_I \cdot p - I_c - C\mu_I - LF_1) \tag{6}$$

$$LF_0 = k_1[(\mu_1 - T)^2 + \sigma_1^2] \tag{7}$$

$$LF_1 = \{k_1[(\mu_1 - T)^2][\Phi(\tfrac{USL-\mu_1}{\sigma_1}) - \Phi(\tfrac{LSL-\mu_1}{\sigma_1})] - 2k_1\sigma_1(\mu_1 - T)[\varphi(\tfrac{USL-\mu_1}{\sigma_1}) \\ - \varphi(\tfrac{LSL-\mu_1}{\sigma_1})] - k_1\sigma_1^2[(\tfrac{USL-\mu_1}{\sigma_1})\varphi(\tfrac{USL-\mu_1}{\sigma_1}) - (\tfrac{LSL-\mu_1}{\sigma_1})\varphi(\tfrac{LSL-\mu_1}{\sigma_1}) \\ - \Phi(\tfrac{USL-\mu_1}{\sigma_1}) + \Phi(\tfrac{LSL-\mu_1}{\sigma_1})]\} / [\Phi(\tfrac{USL-\mu_1}{\sigma_1}) - \Phi(\tfrac{LSL-\mu_1}{\sigma_1})] \tag{8}$$

$$LSL = \mu_1 - \delta\sigma_1 \tag{9}$$

$$USL = \mu_1 + \delta\sigma_1 \tag{10}$$

$$f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_1}\exp(-\tfrac{1}{2}(\tfrac{y-\mu_1}{\sigma_1})^2), \quad LSL < y < USL \tag{11}$$

$$\mu_1^2 = T^2 + (\mu_0^2 - T^2)\exp(-\beta_1 Inv) \tag{12}$$

$$\sigma_1^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha_1 Inv) \tag{13}$$

$$p = 1 - \Phi(\frac{USL - \mu_1}{\sigma_1}) + \Phi(\frac{LSL - \mu_1}{\sigma_1}) \tag{14}$$

$$C_{pmk} = (1 - \frac{|\mu_1 - T|}{\frac{USL - LSL}{2}})\frac{USL - LSL}{6\sqrt{(\mu_1 - T)^2 + \sigma_1^2}} = \frac{(\delta\sigma_1 - |\mu_1 - T|)}{3\sqrt{(\mu_1 - T)^2 + \sigma_1^2}} = K_m \tag{15}$$

$$P_a(p = LTPD) = \beta \tag{16}$$

$ETP(Q, Inv,\ \delta, n, d_0)]$: expected total profit per unit time; $\phi(\cdot)$: standard normal

p.d.f.; $\Phi(\cdot)$: standard normal cumulative distribution function; LSL : lower tolerance

limit; USL : upper tolerance limit; μ_0 : initial value of process mean; σ_0 : initial value of standard deviation; Inv : quality investment; μ_1 : the improved mean;

σ_I : improved variability; β_1 : exponential declined coefficient for the process mean; α_1 : exponential declined coefficient for the standard deviation; Y : normal quality characteristic; $f(y, Inv)$: p.d.f. of Y considering quality investment, $f(y, Inv) = \frac{1}{2\sqrt{\pi}\sigma_1}\exp(-\frac{1}{2}(\frac{y-\mu_1}{\sigma_1})^2)$, $-\infty < y < \infty$; σ_T : target value of standard deviation; δ : coefficient of tolerance; C_{pmk} : process capability index; K_m : specified C_{pmk} value; β : customer's risk; $LTPD$:lot tolerance percent defective; $P_a(p = LTPD)$: accepted probability of inspected lot

The combination $(Q, Inv,\ \delta,\ n,\ d_0)$ need to be determined by maximizing $ETP(Q, Inv, \delta,\ n,\ d_0)]$ under the specified C_{pmk} and consumer's risk. We adopt the following method:

Step 1. Set maximum d_0 be $d_{\max}$, maximum Q be $Q_{\max}$ and maximum Inv be $Inv_{\max}$. Set the initial $Inv = 0$, $Q = 1$, and $d_0 = 0$.

Step 2. For a given $LTPD$, β, and d_0, we can obtain the n satisfying Eq. (16).

Step 3. One can compute the δ satisfying Eq. (15) and have $ETP(Q, Inv, \delta,\ n,\ d_0)]$.

Step 4. Let $Inv = Inv + 1$, go to Step 3 until $Inv = Inv_{\max}$.

Step 5. Let $d_0 = d_0 + 1$, go to Steps 2-3 until $d_0 = d_{\max}$.

Step 6. Let $Q = Q + 1$, go to Steps 2-4 until $Q = Q_{\max}$. We have the solution (Q, Inv, δ, n, d_0) with maximum $ETP(Q, Inv, \delta, n, d_0)$].

3. Numerical Example

Assume some parameters: k = 200, T = 50, $Z_1 = 2$, $Z_2 = 6$, $\alpha = 0.5$, $\beta = 0.1$, $\sigma_T = 0.25$, $K_m = 1$, $A_1 = 67.5$, $A_2 = 80$, $R_I = 30.5$, $M = 8000$, $O = 80$, $I = 100$, $h = 10$, $c = 1$ and $I_c = 0.5$.

The optimal solution of Eq. (1) is $\mu_0 = 49.505$ and $\sigma_0 = 0.416$. By solving Eqs. (5)-(16), the optimal solution for sampling inspection is

$$Q = 292, \mu_I = 49.999, \ \sigma_I = 0.25, Inv = 68, \ \delta = 3.002, \ d_0 = 3,$$
$$n = 134, \text{ and } ETP(Q, Inv, \delta, d_0, n) = 137022.1.$$

From Table 1, one has some results:

1. The k, β_1, K_m, σ_T, LTPD, β, I_c, A_1, A_2, M, and R_c occur a significant effect on the quality investment.
2. The σ_T and K_m occur a significant effect on the coefficient of tolerance.
3. The k, β_1, K_m, σ_T, LTPD, β, I_c, A_1, A_2, M, R_c, and h occur a significant effect on the EMQ.
4. The K_m, σ_T, LTPD, β, I_c, A_1, and A_2 occur a significant effect on the inspection.
5. The k, T, σ_T, M, and A_2 occur a significant effect on the profit.

4. Conclusions

The authors have proposed the modified EMQ model under the process parameters setting and inspection. From numerical results, we conclude that the quality loss coefficient, target value of mean and standard deviation, selling price for accepted lot, and demand quantity occur a significant effect on the profit. Further study should consider the modified model with other quality protection.

Table 1

Results for sensitivity analysis

k_1	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
240	106	3.000	266	117277.6	(3, 134)
160	59	3.007	317	156790.6	(3, 134)
T	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
51	68	3.002	293	129022.1	(3, 134)
49	68	3.002	292	145022.1	(3, 134)
α_1	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
0.6	68	3.002	292	137022.1	(3, 134)
0.4	68	3.002	292	137022.1	(3, 134)
β_1	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
0.12	57	3.002	292	137022.1	(3, 134)
0.08	85	3.002	292	137022.1	(3, 134)
σ_T	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
0.30	108	3.000	232	93621.28	(3, 134)
0.20	46	3.026	309	172646.4	(2, 106)
K_m	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
1.2	55	3.608	330	136702.7	(2 106)
0.9	108	2.700	284	138161.1	(9, 284)
$LTPD$	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
0.06	53	3.010	261	137214.5	(2, 89)
0.04	62	3.004	315	136737.9	(3, 167)
β	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
0.05	65	3.003	307	136843.7	(3, 155)
0.01	108	3.000	352	136421.7	(4, 232)
I_c	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
0.6	49	3.015	304	136678.2	(2, 106)
0.4	108	3.000	253	137415.2	(3, 134)
A_1	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
81	108	3.000	219	138508.3	(0, 46)
54	59	3.005	292	136966.0	(3, 134)
A_2	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
96	59	3.005	294	264955.9	(3, 134)
64	108	3.000	220	128520.1	(0, 46)
M	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)

Contd...

9600	65	3.003	321	164732.7	(3, 134)
6400	85	3.000	261	109341.4	(3, 134)
O	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
96	68	3.002	292	137022.1	(3, 134)
64	68	3.002	292	137022.1	(3, 134)
I	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
120	68	3.002	292	137022.1	(3, 134)
80	68	3.002	292	137022.1	(3 134)
S_t	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
24	68	3.002	303	136914.8	(3, 134)
16	74	3.001	281	137133.6	(3, 134)
h	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
12	74	3.001	266	136742.9	(3, 134)
8	64	3.003	327	137331.3	(3, 134)
R_I	Inv	δ	Q	$ETP(Inv, \delta, d_0, n)$	(d_0, n)
36.6	65	3.003	299	136962.7	(3, 134)
24.4	79	3.001	286	137083.1	(3, 134)

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