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A note on the multi-depot vehicle routing problem

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Abstract

In a paper, Geetha, Vanathi, and Poonthalir [1] suggested a model for the MDVRP. However, we find that authors ignore the relationship between the two types of proposed binary decision variables, and it leads to the wrong integer programming model for the MDVRP. We give an illustrative example to identify the problem and suggest new constraints to overcome. Also, authors should include the distances between depots and customers in the objective function.

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In logistics, companies often use vehicles that operate from more than one distribution center, referred to as depots to distribute their goods. The problem, which is referred to as Multi-Depot Vehicle Routing Problem (MDVRP), is a practical logistics problem where the objective is to minimize the total distribution cost while satisfying the necessary constraints. The problem has not been studied as much in comparison to the corresponding standard vehicle routing problem (Dantzig and Ramser [2]) where the distribution center is one. Several practical variants of the VRP have been studied in the literature (Bhuranda et al. [3]).

Let a graph G=(V,A) where V={1,2,…n+m} is the set of nodes and A is the arcs or edges set connected each pair of nodes. The node set is partitioned into two subsets, the set of n places to visit and the set of m depots, that is, $V_c=\{v_1,v_2, \dots,v_n\}$ and $V_d=\{v_{n+1},v_{n+2}, \dots,v_{n+m}\}$ respectively. A fleet of homogeneous vehicles capacity Q is available at each depot. A cost matrix $C=(c_{ij})$ corresponding to distance or travel is defined on A. Each node $v_i \in V_c$

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has a nonnegative demand q_i. In the classical MDVRP, the objective minimizes the total distance, the vehicles start their routes and end at the same depot without violating their capacity.

According to Geetha, Vanathi, and Poonthalir [1], their mathematical model of the MDVRP requires the definition of two binary decision variables the x_{ijqp} to be equal to 1 if vehicle q in a depot p moves from customer i to customer j and 0 otherwise, and the binary decision variable y_{iqp} to be equal to 1 if vehicle q serves customer i assigned to depot p and 0 otherwise. Also, k_p is the number of vehicles in depot number p, n_j is the number of customers in depot number j, n_{jq} is the number of customers in depot number j that are included in the route of vehicle number q.

According to the reference the model is as follows.

$$\text{minimize } \Sigma_{p=1}^{m}\Sigma_{q=1}^{k_p}\Sigma_{i=1}^{n}\Sigma_{j=1}^{n}c_{ij}x_{ijqp}$$

subject to

$$\Sigma_{i=1}^{n}q_i y_{iqp} \leq Q \tag{2}$$

$$0 \leq n_{jp} \leq n_j \tag{3}$$

$$\Sigma_{q=1}^{k_j}n_{jp} = n_j \;\forall\, j = 1 \; to \; m \tag{4}$$

$$\Sigma_{j=1}^{m}n_j = n \tag{5}$$

$$\Sigma_{p=1}^{m}\Sigma_{q=1}^{k_p}y_{iqp} = 1 \tag{6}$$

$$x_{ijqp} = 1 \text{ or } 0 \tag{7}$$

$$y_{iqp} = 1 \text{ or } 0 \tag{8}$$

In the formulation, the objective function represents total travel cost; Constraint (2) ensures the feasibility of the loads. Constraint (3) guarantees that the number of customers serviced by vehicles is less equal than serviced by depot customers. In (4), all customers assigned to a depot are served by the vehicles of depot. In (5), all customers are served. In (6) each customer is serviced exactly once; the variables x and y of the problem are binary by equation (7) and (8).

Mathematical formulation proposed by Geetha, Vanathi, and Poonthalir [1] in the objective function should also be considered the connections between depots and customers. The corrected objective function is:

$$\text{minimize } \Sigma_{p=1}^{m}\Sigma_{q=1}^{k_p}\Sigma_{i=1}^{n+m}\Sigma_{j=1}^{n+m}c_{ij}x_{ijqp}$$

Also, let a feasible result in agreement with the proposed model for the same cluster, where y_{234}=1, y_{534}=1 and x_{2524}=1. The y variable assigns the customers 2, and 5 of the depot number 4 to the vehicle number 3. On the other hand, the x variable assigns the arc (2,5) of the depot number 4 to the vehicle number 2. So, customer number 2 and customer number 5 are included in the route of vehicle number 2. It cannot be valid according to the previous result that considers that the customers are served by vehicle number 3.

So, we overcome the problem by setting constraints:

$$x_{ijqp} \leq y_{iqp} \qquad i=1,\ 2,\ \ldots n+m,\quad j=1,\ 2,\ \ldots n$$

and

$$x_{ijqp} \leq y_{jqp} \quad i=1,\ 2,\ \ldots n,\quad j=1,\ 2,\ \ldots n+m$$

For our example the inequalities $x_{2524} \leq y_{224}$ and $x_{2524} \leq y_{524}$ don't allow x_{2524}=1 because the y_{224}=0 (y_{234}=1, the customer number 2 is served by vehicle number 3 of depot number 4) and y_{524}=0 (y_{534}=1, the customer number 5 is served by vehicle number 3 of depot number 4) . If x_{ijqp}=1 the y_{iqp}=1 and y_{jqp}=1 that is correct by the definition of the binary decision variables. If x_{ijqp}=0 the y_{iqp}=0 or 1 and y_{jqp}=0 or 1 that is correct. The arc (i,j) is not included in the route of vehicle number q of depot number p (x_{ijqp}=0), but it is correct to consider that the customers i and j are included or no (setting in the binary decision variables y = 1 or 0) in arc connection with other customers in the vehicle number q of the depot number p.

Declaration of Competing Interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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