

Sombor-index-like invariants of some graphs

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Abstract

The Sombor index (SO) is a vertex-degree-based graph invariant, defined as the sum over all pairs of adjacent vertices of $\sqrt{d_i^2 + d_j^2}$, where d_i is the degree of the i -th vertex. It has been conceived using geometric considerations. Recently, a series of new SO-like degree-based graph invariants (denoted by $SO_1, SO_2, \dots, SO_n$) is taken into consideration, when the geometric background of several classical topological indices (Zagreb, Albertson) has considered. In this paper, we compute and study these new indices for some graphs, cactus chains and polymers.

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1. Introduction

Let $G = (V, E)$ be a finite, connected and simple graph. We denote the degree of a vertex v in G by d_v . A topological index of G is a real number related to G . It does not depend on the labeling or pictorial representation of a graph. In the mathematical and chemical literature, several dozens of

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vertex-degree-based (VDB) graph invariants such as the first and second Zagreb index, the Albertson index and the Sombor index have been introduced and extensively studied [11, 12, 13]. Their general formula is

$$TI(G) = \sum_{ij \in E} F(d_i, d_j),$$

where $F(x, y)$ is some function with the property $F(x, y) = F(y, x)$.

The Sombor index, is defined as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_u^2 + d_v^2}.$$

This index soon attracted much attention, and its numerous mathematical [3, 13, 14, 16, 17, 19, 21] and chemical [1, 2, 5, 6, 9, 10, 15, 18, 19, 20] applications have been established. Recently, Gutman [10] intended to point out the great variety of Sombor-index-like VDB graph invariants that can be constructed by means of geometric arguments and so generated a number of new Sombor-index-like VDB invariants, denoted below by $SO_1, SO_2, \dots, SO_6$.

$$SO_1(G) = \frac{1}{2} \sum_{uv \in E(G)} |d_u^2 - d_v^2|.$$

$$SO_2(G) = \sum_{uv \in E(G)} \left| \frac{d_u^2 - d_v^2}{d_u^2 + d_v^2} \right|.$$

$$SO_3(G) = \sum_{uv \in E(G)} \sqrt{2} \frac{d_u^2 + d_v^2}{d_u + d_v} \pi.$$

$$SO_4(G) = \frac{1}{2} \sum_{uv \in E(G)} \left(\frac{d_u^2 + d_v^2}{d_u + d_v} \right)^2 \pi.$$

$$SO_5(G) = \sum_{uv \in E(G)} \frac{2|d_u^2 - d_v^2|}{\sqrt{2} + 2\sqrt{d_u^2 + d_v^2}} \pi.$$

$$SO_6(G) = \sum_{uv \in E(G)} \left[\frac{d_u^2 - d_v^2}{\sqrt{2} + 2\sqrt{d_u^2 + d_v^2}} \right]^2 \pi.$$

Polymer graphs can be decomposed into subgraphs that we call monomer units. Blocks of graphs are particular examples of monomer units, but a monomer unit may consist of several blocks.

Let G be a connected graph constructed from pairwise disjoint connected graphs $G_1, \dots, G_k$ as follows. Select a vertex of G_1 , a vertex of G_2 , and identify these two vertices. Then continue in this manner inductively. Note that the graph G constructed in this way has a tree-like structure, the G_i 's being its building stones (see Figure 1). Usually we say that G is a polymer graph, obtained by point-attaching from $G_1, \dots, G_k$ and that G_i 's are the monomer units of G . A particular case of this construction is the decomposition of a connected graph into blocks (see [4, 7, 8]).

In Section 2, We compute the Sombor-like degree-based indices for certain graphs such as path, cycle, complete bipartite, friendship, ladder, book, Dutch-windmill and cactus chains. In Section 3, we obtain some relations between indices $SO_i(G)$ ($i = 2, 3, 4, 5, 6$) and index $SO_1(G)$.

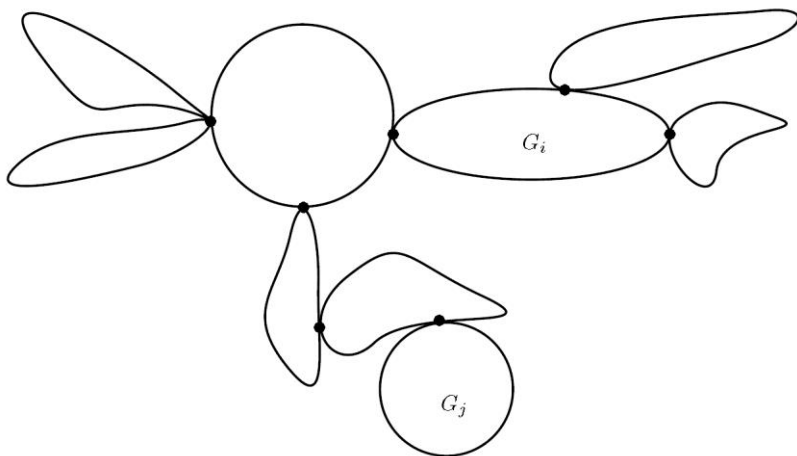


Figure 1
A polymer graph with monomer units $G_1, \dots, G_k$.

2. Sombor-like degree-based indices of certain graphs

In this section, we compute the Sombor-index-like invariants indices for certain graphs, such as paths, friendship graph, grid graphs and cactus chains. By the definitions of $SO_1(G)$, $SO_2(G)$, $SO_5(G)$ and $SO_6(G)$, these indices are 0 for every regular graphs.

2.1 Sombor-like degree-based indices of specific graphs

We begin with the following theorem:

Theorem 2.1:

- (i) For every $n \in \mathbb{N} - \{1, 2\}$, $SO_1(P_n) = 3$.
- (ii) For every $m, n \in \mathbb{N}$, $SO_1(K_{1,n}) = \frac{n(n-1)(n+1)}{2}$ and $SO_1(K_{m,n}) = \frac{mn(m-n)(m+n)}{2}$, if $m \geq n$.
- (iii) For the wheel graph $W_{n+1} = C_n \vee K_1$, $SO_1(W_{n+1}) = \frac{n(n-3)(n+3)}{2}$.
- (iv) For the ladder graph $L_n = P_n \square K_2$, ($n \geq 3$), $SO_1(L_n) = 10$.
- (v) For the friendship graph $F_n = K_1 \vee nK_2$, $SO_1(F_n) = 4n(n-1)(n+1)$.
- (vi) For the book graph $B_n = K_{1,n} \square K_2$, and for $n \geq 3$, $SO_1(B_n) = n(n+3)(n-1)$.
- (vii) For the Dutch windmill graph $D_n^{(m)}$, and for every $n \geq 3$ and $m \geq 2$,

$$SO_1(D_n^{(m)}) = 4m(m-1)(m+1).$$

Proof: The proof of all parts are easy and similar. For instance we state the proof of Part (i).

- (i) There are two edges with endpoints of degree 1 and 2. Also there are $n - 3$ edges with endpoints of degree 2. Therefore we have the result.

Similarly, we can compute $SO_2, \dots, SO_6$ for these graphs. Table 1 and 2 show the exact values for these Sombor-like degree-based indices.

Table 1
The exact values of $SO_i(G)$ ($2 \leq i \leq 4$), for $m \geq n \geq 3$.

G	$SO_2(G)$	$SO_3(G)$	$SO_4(G)$
P_n	$\frac{6}{5}$	$\frac{10\sqrt{2}\pi}{3} + (n-3)\sqrt{2}(2\pi)$	$\frac{25\pi}{9} + (n-3)(2\pi)$
$K_{1,n}$	$\frac{n(n^2-1)}{n^2+1}$	$\frac{\sqrt{2}n\pi(n^2+1)}{n+1}$	$\frac{n\pi}{2} \left(\frac{n^2+1}{n+1} \right)^2$
$K_{m,n}$	$\frac{mn(m^2-n^2)}{m^2+n^2}$	$\frac{\sqrt{2}nm\pi(m^2+n^2)}{m+n}$	$\frac{nm\pi}{2} \left(\frac{n^2+m^2}{n+m} \right)^2$
W_n	$\frac{(n-1)(n^2-2n-8)}{n^2-2n+10}$	$\sqrt{2}\pi \left(\frac{(n-1)(n^2-2n+10)}{n+2} + 3n-3 \right)$	$\frac{\pi}{2} \left((n-1) \left(\frac{n^2-2n+10}{n+2} \right)^2 + 9n-9 \right)$
L_n	$\frac{20}{13}$	$\frac{\sqrt{2}\pi}{5} (45n-48)$	$\frac{\pi}{50} (675n-924)$
F_n	$\frac{2n^3-2n}{n^2+1}$	$\sqrt{2}\pi \left(\frac{4n^3+2n^2+6n}{n+1} \right)$	$2n\pi \left(\frac{2n^4+5n^2+2n+3}{n^2+2n+1} \right)$
B_n	$\frac{2n^3+4n^2-6n}{n^2+2n+5}$	$\sqrt{2}\pi \left(3n+1+2n \left(\frac{n^2+2n+5}{n+3} \right) \right)$	$\frac{\pi}{2} \left(4n+(n+1)^2+2n \left(\frac{n^2+2n+5}{n+3} \right)^2 \right)$
$D_n^{(m)}$	$\frac{2m(m^2-1)}{m^2+1}$	$\sqrt{2}\pi \left(2m(n-1)+4m \left(\frac{m^2+1}{m+1} \right) \right)$	$\frac{\pi}{2} \left(4m(n-1)+8m \left(\frac{m^2+1}{m+1} \right)^2 \right)$

Table 2
The exact values of $SO_5(G)$ and $SO_6(G)$, for $m \geq n \geq 3$.

G	$SO_5(G)$	$SO_6(G)$
P_n	$\frac{12\pi}{\sqrt{2}+2\sqrt{5}}$	$\frac{18\pi}{(\sqrt{2}+2\sqrt{5})^2}$
$K_{1,n}$	$\frac{2n\pi(n^2-1)}{\sqrt{2}+2\sqrt{n^2+1}}$	$n\pi \left(\frac{n^2-1}{\sqrt{2}+2\sqrt{n^2+1}} \right)^2$
$K_{m,n}$	$\frac{2nm\pi(m^2-n^2)}{\sqrt{2}+2\sqrt{n^2+m^2}}$	$nm\pi \left(\frac{m^2-n^2}{\sqrt{2}+2\sqrt{n^2+m^2}} \right)^2$
W_n	$2(n-1)\pi \left(\frac{n^2-2n-8}{\sqrt{2}+2\sqrt{n^2-2n+10}} \right)$	$\pi(n-1) \left(\frac{n^2-2n-8}{\sqrt{2}+2\sqrt{n^2-2n+10}} \right)^2$
L_n	$\frac{40\pi}{\sqrt{2}+2\sqrt{13}}$	$\frac{100\pi}{(\sqrt{2}+2\sqrt{13})^2}$
F_n	$4n\pi \left(\frac{4n^2-4}{\sqrt{2}+2\sqrt{4n^2+4}} \right)$	$2n\pi \left(\frac{4n^2-4}{\sqrt{2}+2\sqrt{4n^2+4}} \right)^2$
B_n	$4n\pi \left(\frac{n^2+2n-3}{\sqrt{2}+2\sqrt{n^2+2n+5}} \right)$	$2n\pi \left(\frac{n^2+2n-3}{\sqrt{2}+2\sqrt{n^2+2n+5}} \right)^2$
$D_n^{(m)}$	$16m\pi \left(\frac{m^2-1}{\sqrt{2}+2\sqrt{4m^2+4}} \right)$	$32m\pi \left(\frac{m^2-1}{\sqrt{2}+2\sqrt{4m^2+4}} \right)^2$

Here, we consider the grid graph $(P_n \square P_m)$, and compute the Sombor-like degree-based indices for it.

Theorem 2.2: Let $P_m \square P_n$ be the grid graph (Figure 2). For every $n \geq 6$ and $m \geq 6$,

$$SO_1(P_m \square P_n) = 14m + 14n - 16,$$

$$\begin{aligned}
SO_2(P_m \square P_n) &= \frac{40}{13} + \frac{14m+14n-56}{25}, \\
SO_3(P_m \square P_n) &= \sqrt{2}\pi \left(\frac{104}{5} + \frac{50n+50m-200}{7} \right), \\
SO_4(P_m \square P_n) &= \frac{\pi}{2} \left(\frac{1352}{25} + \frac{1250m+1250n-5000}{49} \right), \\
SO_5(P_m \square P_n) &= 2\pi \left(\frac{40}{\sqrt{2}+2\sqrt{13}} + \frac{14m+14n-56}{\sqrt{2}+10} \right), \\
SO_6(P_m \square P_n) &= \pi \left(\frac{200}{(\sqrt{2}+2\sqrt{13})^2} + \frac{98m+98n-392}{(\sqrt{2}+10)^2} \right).
\end{aligned}$$

Proof: There are eight edges with endpoints of degree 2 and 3 and there are $m+n-4$ edges with endpoints of degree 3. Also there are $2m+2n-8$ edges with endpoints of degree 3 and 4 and there are $2nm-5n-5m-12$ edges with endpoints of degree 4. Therefore we have the result.

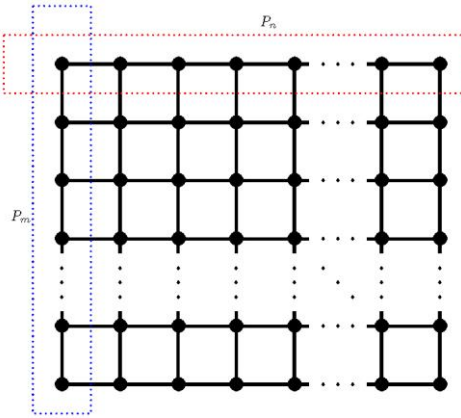


Figure 2
Grid graph $P_m \square P_n$



Figure 3
Chain triangular cactus T_n

2.2 Sombor-like degree-based indices of cactus chains

In this subsection, we consider a class of simple linear polymers called cactus chains. Cactus graphs were first known as Husimi tree, they appeared in the scientific literature some sixty years ago in papers by Husimi and Riddell concerned with cluster integrals in the theory of condensation in statistical mechanics [13, 21, 19]. We refer the reader to papers [9, 17] for some aspects of parameters of cactus graphs.

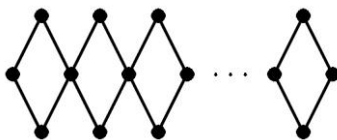


Figure 4
Para-chain square cactus Q_n

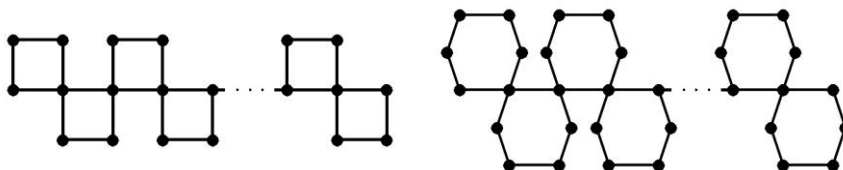


Figure 5
Para-chain square cactus O_n and Ortho-chain graph O_n^h



Figure 6
Para-chain L_n and Meta-chain M_n

Theorem 2.3:

- (i) Let T_n be the chain triangular graph (See Figure 3) of order n . Then for every $n \geq 2$, $SO_1(T_n) = 12n$.
- (ii) Let Q_n be the para-chain square cactus graph (See Figure 4) of order n . Then for every $n \geq 2$, $SO_1(Q_n) = 24n - 24$.
- (iii) Let O_n be the para-chain square cactus (See Figure 5) graph of order n . Then for every $n \geq 2$, $SO_1(O_n) = 12n$.
- (iv) Let O_n^h be the Ortho-chain graph (See Figure 5) of order n . Then for every $n \geq 2$, $SO_1(O_n^h) = 12n$.
- (v) Let L_n be the para-chain hexagonal cactus graph (See Figure 6) of order n . Then for every $n \geq 2$, $SO_1(L_n) = 24n - 24$.
- (vi) Let M_n be the Meta-chain hexagonal cactus graph (See Figure 6) of order n . Then for every $n \geq 2$, $SO_1(M_n) = 24n - 24$.

Proof:

- (i) There are two edges with endpoints of degree 2. Also there are $2n$ edges with endpoints of degree 2 and 4 and there are $n - 2$ edges with endpoints of degree 4. Therefore we have the result.

- (ii) There are four edges with endpoints of degree 2. Also there are $4n - 4$ edges with endpoints of degree 2 and 4. Therefore the result follows.
- (iii) There are $n + 2$ edges with endpoints of degree 2. Also there are $2n$ edges with endpoints of degree 2 and 4 and there are $n - 2$ edges with endpoints of degree 4. Therefore we have the result.
- (iv) There are $3n + 2$ edges with endpoints of degree 2. Also there are $2n$ edges with endpoints of degree 2 and 4 and there are $n - 2$ edges with endpoints of degree 4. Therefore the result follows.
- (v) There are $2n + 4$ edges with endpoints of degree 2. Also there are $4n - 4$ edges with endpoints of degree 2 and 4. Therefore we have the result.
- (vi) There are $2n + 4$ edges with endpoints of degree 2. Also there are $4n - 4$ edges with endpoints of degree 2 and 4. Therefore the result follows.

Similarly, we can compute $SO_2, \dots, SO_6$ for cactus chains. Table 3 shows the exact value for these Sombor-like degree-based indices.

Table 3
The exact values of $SO_i(G)$ ($2 \leq i \leq 6$), for $n \geq 2$.

G	$SO_2(G)$	$SO_3(G)$	$SO_4(G)$	$SO_5(G)$	$SO_6(G)$
T_n	$\frac{6n}{5}$	$3\sqrt{2}\pi \left(\frac{44n-36}{3} \right)$	$\frac{\pi}{2} \left(\frac{776n-1080}{9} \right)$	$\frac{48n\pi}{\sqrt{2}+2\sqrt{20}}$	$\frac{288n\pi}{(\sqrt{2}+2\sqrt{20})^2}$
Q_n	$\frac{12n-12}{5}$	$\sqrt{2}\pi \left(\frac{40n-16}{3} \right)$	$\frac{\pi}{2} \left(\frac{400n-256}{9} \right)$	$\frac{24\pi(4n-4)}{\sqrt{2}+2\sqrt{20}}$	$\frac{144\pi(4n-4)}{(\sqrt{2}+2\sqrt{20})^2}$
O_n	$\frac{6n}{5}$	$\sqrt{2}\pi \left(\frac{38n-12}{3} \right)$	$\frac{\pi}{2} \left(\frac{380n-216}{9} \right)$	$\frac{48n\pi}{\sqrt{2}+2\sqrt{20}}$	$\frac{288n\pi}{(\sqrt{2}+2\sqrt{20})^2}$
O_n^h	$\frac{6n}{5}$	$\sqrt{2}\pi \left(\frac{50n-12}{3} \right)$	$\frac{\pi}{2} \left(\frac{452n-216}{9} \right)$	$\frac{48n\pi}{\sqrt{2}+2\sqrt{20}}$	$\frac{288n\pi}{(\sqrt{2}+2\sqrt{20})^2}$
L_n	$\frac{12n-12}{5}$	$\sqrt{2}\pi \left(\frac{52n-26}{3} \right)$	$\frac{\pi}{2} \left(\frac{472n-256}{9} \right)$	$\frac{24\pi(4n-4)}{\sqrt{2}+2\sqrt{20}}$	$\frac{144\pi(4n-4)}{(\sqrt{2}+2\sqrt{20})^2}$
M_n	$\frac{12n-12}{5}$	$\sqrt{2}\pi \left(\frac{52n-26}{3} \right)$	$\frac{\pi}{2} \left(\frac{472n-256}{9} \right)$	$\frac{24\pi(4n-4)}{\sqrt{2}+2\sqrt{20}}$	$\frac{144\pi(4n-4)}{(\sqrt{2}+2\sqrt{20})^2}$

Corollary 2.4: *Meta-chain hexagonal cactus graphs and para-chain hexagonal cactus graphs of the same order, have the same Sombor-like degree-based indices.*

3. Relations between $SO_i(G)$ ($i = 2, 3, 4, 5, 6$) and $SO_1(G)$

In this section, we obtain bounds for $SO_i(G)$ ($i = 2, 3, 4, 5, 6$) based $SO_1(G)$. We begin with the following proposition which gives a lower and upper bounds for $SO_2(G)$ based $SO_1(G)$.

Proposition 3.1: If G is a graph with minimum degree δ and maximum degree Δ , then

$$\frac{SO_1(G)}{\Delta^2} \leq SO_2(G) \leq \frac{SO_1(G)}{\delta^2}.$$

Proof: First we find the lower bound. We have

$$\begin{aligned} SO_2(G) &= \sum_{uv \in E(G)} \left| \frac{d_u^2 - d_v^2}{d_u^2 + d_v^2} \right| \geq \sum_{uv \in E(G)} \frac{|d_u^2 - d_v^2|}{2\Delta^2} \\ &\geq \frac{1}{\Delta^2} \left(\frac{1}{2} \sum_{uv \in E(G)} |d_u^2 - d_v^2| \right) = \frac{SO_1(G)}{\Delta^2}. \end{aligned}$$

Similarly, we have the result for the upper bound.

The following proposition gives a lower and upper bounds for $SO_3(G)$ based $SO_1(G)$, δ and Δ .

Proposition 3.2: Let $G = (V, E)$ be a graph with $|E| = m$ and minimum degree δ and maximum degree Δ . Then

$$\sqrt{2}\pi \left(\frac{SO_1(G) + m\delta^2}{\Delta} \right) \leq SO_3(G) \leq \sqrt{2}\pi \left(\frac{SO_1(G) + m\Delta^2}{\delta} \right).$$

Proof: First we obtain the upper bound. We have

$$\begin{aligned} SO_3(G) &= \sum_{uv \in E(G)} \sqrt{2} \frac{d_u^2 + d_v^2}{d_u + d_v} \pi \\ &\leq \sqrt{2}\pi \left(\sum_{uv \in E(G)} \frac{|d_u^2 - d_v^2| + 2\Delta^2}{2\delta} \right) \\ &\leq \sqrt{2}\pi \left(\frac{1}{\delta} \left(\frac{1}{2} \sum_{uv \in E(G)} |d_u^2 - d_v^2| \right) + \frac{m\Delta^2}{\delta} \right) \\ &= \sqrt{2}\pi \left(\frac{SO_1(G) + m\Delta^2}{\delta} \right). \end{aligned}$$

Similarly, we have the result for the lower bound.

Proposition 3.3: Let $G = (V, E)$ be a graph with $|E| = m$ and minimum degree δ and maximum degree Δ . Then

$$\frac{\pi\delta^2}{2\Delta^2} (m\delta^2 + SO_1(G)) \leq SO_4(G) \leq \frac{\pi\Delta^2}{2\delta^2} (m\Delta^2 + SO_1(G)).$$

Proof: First we obtain the upper bound. We have

$$\begin{aligned} SO_4(G) &= \frac{1}{2} \sum_{uv \in E(G)} \left(\frac{d_u^2 + d_v^2}{d_u + d_v} \right)^2 \pi \\ &= \frac{\pi}{2} \sum_{uv \in E(G)} \left(\frac{(d_u^2 + d_v^2)(d_u^2 + d_v^2)}{(d_u + d_v)^2} \right) \\ &\leq \frac{\pi}{2} \sum_{uv \in E(G)} \left(\frac{(d_u^2 + d_v^2)(2\Delta^2)}{(2\delta)^2} \right) \\ &\leq \frac{\pi\Delta^2}{2\delta^2} \left(\frac{1}{2} \sum_{uv \in E(G)} 2\Delta^2 + |d_u^2 - d_v^2| \right) \\ &= \frac{\pi\Delta^2}{2\delta^2} (m\Delta^2 + SO_1(G)). \end{aligned}$$

Similarly, we have the result for the lower bound.

By similar argument as Proposition 3.1, we have:

Proposition 3.4: Let G be a graph with minimum degree δ and maximum degree Δ . Then

$$\frac{2\sqrt{2}\pi SO_1(G)}{2\Delta+1} \leq SO_5(G) \leq \frac{2\sqrt{2}\pi SO_1(G)}{2\delta+1}.$$

Proposition 3.5: Let $G = (V, E)$ be a graph with $|E| = m$ and minimum degree δ and maximum degree Δ . Then

$$SO_6(G) \leq \frac{2\pi(\Delta^2 - \delta^2)SO_1(G)}{(\sqrt{2} + 2\delta\sqrt{2})^2}.$$

Proof: We have

$$\begin{aligned} SO_6(G) &= \pi \sum_{uv \in E(G)} \left(\frac{d_u^2 - d_v^2}{\sqrt{2} + 2\sqrt{d_u^2 + d_v^2}} \right)^2 \\ &\leq \pi \sum_{uv \in E(G)} \frac{|d_u^2 - d_v^2|(\Delta^2 - \delta^2)}{(\sqrt{2} + 2\delta\sqrt{2})^2} \\ &= \frac{2\pi(\Delta^2 - \delta^2)}{(\sqrt{2} + 2\delta\sqrt{2})^2} \left(\frac{1}{2} \sum_{uv \in E(G)} |d_u^2 - d_v^2| \right) \\ &= \frac{2\pi(\Delta^2 - \delta^2)SO_1(G)}{(\sqrt{2} + 2\delta\sqrt{2})^2}. \end{aligned}$$

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