Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 47 (2026), No. 1, pp. 207–213
DOI : 10.47974/JIOS-1513

Process mean setting and quality investment selection

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Abstract

This work discusses inventory model based on EMQ, process mean setting, and parameters of inspection plan in the first step. Then the optimal quality investment for reducing process variance is determined in the second step.

Subject Classification: *60E05 Probability distributions: general theory.*

Keywords: *EMQ, Inspection plan, Mean, Investment of quality.*

1. Introduction

Traditional EMQ model considers about the perfect product for process. Porteus [1] and Rosenblatt & Lee [2, 3] have addressed the imperfect EMQ model. Chen [4] and Chen & Chou [5] proposed a joint settings about process mean and production quantity for modified EMQ model.

The manufacturing target and variance are two key point parameters affecting the product. Taguchi [6] considered quadratic loss function for evaluating the quality.

The known process variance and unknown process mean are the basic assumptions. Some works have adopted Taguchi's loss function in order to solve mean setting problem, e.g., Chan & Ibrahim [7], Chan et al. [8, 9], and Shin et al. [10].

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Improvement activity is a long-term tool for increasing product quality. Tsou [11], Chen & Tsou [12], Ganeshan et al. [13], and Hong et al. [14] have applied this method.

In this work, we propose a modified model concerning quality investment selection and process mean setting. Firstly, the optimal decision variables are to obtain the EMQ, process mean setting, and parameters of inspection plan. Then the quality investment is determined for reducing process variability and increasing expected total profit.

2. Model

Step 1. One wants to obtain the EMQ, process mean, and parameters of inspection plan for modified inventory model.

Maximize

$$ETP(Q_1, m_0, n_1, d_0) = M_1 \cdot W_{22} - S_t \cdot \frac{M_1}{Q_1} - 0.5 \cdot Q_1 \cdot (1 - \frac{O_1}{I_1}) h_1 \tag{1}$$

subject to
$$\max_{0 \le p \le 1} AOQ = p_L \tag{2}$$

where

$$W_{22} = \{(A_{22} - A_{11}) + RI \cdot p_1 + (1 - \frac{n_1}{Q_1}) IC + (1 - \frac{n_1}{Q_1})(LF1 - LF0)$$
$$- \frac{np_1 R_I}{Q_1}\} P(X_1 \le d_0) + (A_{11} - RI \cdot p_1 - IC - C_1 m_0 - LF1) \tag{3}$$

$$LF0 = k_1 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \Phi\left(\frac{T_1 - m_0}{\sigma_0}\right) - \sigma_0 (m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) \right\}$$
$$+ k_2 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \left[1 - \Phi\left(\frac{T_1 - m_0}{\sigma_0}\right)\right] + \sigma_0 (m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) \right\} \tag{4}$$

$$LF1 = k_1 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \left[\Phi\left(\frac{T_1 - m_0}{\sigma_0}\right) - \Phi\left(\frac{LSL - m_0}{\sigma_0}\right) \right] \right.$$
$$\left. - \sigma_0 \left[(m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) - (m_0 - 2T_1 + LSL) \phi\left(\frac{LSL - m_0}{\sigma_0}\right) \right] \right\}$$
$$+ k_2 \left\{ \left[(m_0 - T_1)^2 + \sigma_0^2 \right] \left[\Phi\left(\frac{USL - m_0}{\sigma_0}\right) - \Phi\left(\frac{T_1 - m_0}{\sigma_0}\right) \right] \right.$$
$$\left. - \sigma_0 \left[-(m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) + (m_0 - 2T_1 + USL) \phi\left(\frac{USL - m_0}{\sigma_0}\right) \right] \right\} \tag{5}$$

$$p_1 = 1 - \Phi\left(\frac{USL - m_0}{\sigma_0}\right) + \Phi\left(\frac{LSL - m_0}{\sigma_0}\right) \tag{6}$$

$$P(X_1 \le d_0) = \sum_{x_1=0}^{d_0} \frac{e^{-n_1 p_1}(n_1 p_1)^{x_1}}{x_1!} \tag{7}$$

$ETP(Q_1, m_0, n_1, d_0)]$ is expected total profit; W_{22} is ETP; A_{11} is sold price with rejected product; A_{22} is sold price for accepted product; Q_1 is production size; X is no. of bad unit; n is sampling no.; d_0 is available no. of bad unit; IC_c is inspection expense; RI is replacement expense; p_1 is bad prob.; C is expense of material; $LF0$ is expected loss for no inspection; $LF1$ is expected loss for inspection; $\phi(\cdot)$ is p.d.f. of SND; $\Phi(\cdot)$ is c.d.f. of SND; LSL is min level; USL is max level; m_0 is process expectation; σ_0 is SD; M_1 is sold product (time); O_1 is sold product (day); I_1 is production product per day; S_t is preparing expense; h is store expense; k_1 & k_2 are coefficients of loss.

The decision variables (Q_1, m_0, n_1, d_0) need to solve under maximizing $ETP(Q_1, m_0, n_1, d_0)]$. Consider following steps:

Step 1. Let maximum Q_1 be $Q_{\max}$ and initial $Q_1 = Q_{\max}$.

Step 2. From Dodge-Romig, we have (n_1, d_0) satisfying Eq. (2).

Step 3. Searching the interval $LSL < m_0 < USL$, one can obtain the corresponding $ETP(Q_1, m_0, n_1, d_0)$ from Eq. (1).

Step 4. Set $Q = Q + 1$ and continue Steps 2-3 with $Q_{\max}$ $Q = Q_{\max}$. We have (Q_1, μ_0, n_1, d_0) with maximum $ETP(Q_1, \mu_0, n_1, d_0)$.

The optimal combination of (Q_1, m_0, n_1, d_0) is the input variables in the next step.

Step 2. One needs to select the quality investment. The model is
Maximize

$$ETP(Inv) = M_1 \cdot \left(W_{22} - \frac{Inv}{Q1}\right) - S_t \cdot \frac{M_1}{Q_1} - 0.5 \cdot Q_1 \cdot \left(1 - \frac{O_1}{I_1}\right) h_1 \tag{8}$$

where

$$W_{22} = \{(A_{22} - A_{11}) + RI \cdot p_1 + \left(1 - \frac{n_1}{Q_1}\right) IC + \left(1 - \frac{n_1}{Q_1}\right)(LF1 - LF0)$$
$$- \frac{n p_1 RI}{Q_1}\Big\} P(X_1 \le d_0) + (A_{11} - RI \cdot p_1 - IC - C_1 m_0 - LF1) \tag{9}$$

$$LF0 = k_1\{[(m_0 - T_1)^2 + \sigma_{\mathrm{I}}^2]\Phi\left(\frac{T_1 - m_0}{\sigma_{\mathrm{I}}}\right) - \sigma_{\mathrm{I}}(m_0 - T_1)\phi\left(\frac{T_1 - m_0}{\sigma_{\mathrm{I}}}\right)\}$$
$$+k_2\left\{[(m_0 - T_1)^2 + \sigma_{\mathrm{I}}^2]\left[1 - \Phi\left(\frac{T_1 - m_0}{\sigma_{\mathrm{I}}}\right)\right] + \sigma_{\mathrm{I}}(m_0 - T_1)\phi\left(\frac{T_1 - m_0}{\sigma_{\mathrm{I}}}\right)\right\} \tag{10}$$

$$LF1 = k_1\left\{[(m_0 - T_1)^2 + \sigma_I^2]\left[\Phi\left(\frac{T_1 - m_0}{\sigma_I}\right) - \Phi\left(\frac{LSL - m_0}{\sigma_I}\right)\right]\right.$$
$$\left.-\sigma_I\left[(m_0 - T_1)\phi\left(\frac{T_1 - m_0}{\sigma_I}\right) - (m_0 - 2T_1 + LSL)\phi\left(\frac{LSL - m_0}{\sigma_I}\right)\right]\right\}$$
$$+k_2\left\{[(m_0 - T_1)^2 + \sigma_I^2]\left[\Phi\left(\frac{USL - m_0}{\sigma_I}\right) - \Phi\left(\frac{T_1 - m_0}{\sigma_I}\right)\right]\right.$$
$$\left.-\sigma_I\left[-(m_0 - T_1)\phi\left(\frac{T_1 - m_0}{\sigma_I}\right) + (m_0 - 2T_1 + USL)\phi\left(\frac{USL - m_0}{\sigma_I}\right)\right]\right\} \tag{11}$$

$$\sigma_I^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha Inv) \tag{12}$$

$$p_1 = 1 - \Phi\left(\frac{USL - m_0}{\sigma_I}\right) + \Phi\left(\frac{LSL - m_0}{\sigma_I}\right) \tag{13}$$

$$P(X_1 \le d_0) = \sum_{x_1=0}^{d_0} \frac{e^{-n_1 p_1}(n_1 p_1)^{x_1}}{x_1!} \tag{14}$$

$ETP(Inv)$ is ETP; Inv is quality investment; σ_I^2 is improved variance; α is declined coefficient.

The Inv needs to solve with maximizing $ETP(Inv)$. Its solution procedure is

Step 1. Let maximum Inv be $Inv_{\max}$. Let initial $Inv = 0$.

Step 2. Compute the $ETP(Inv)$ from Eq. (8).

Step 3. For $Inv = Inv + 1$, repeat the 2nd Step with $Inv = Inv_{\max}$.

Step 4. The solution is that Inv with maximum $ETP(Inv)$.

3. Numerical Example

Table 1 presents the change of $\pm 20\% \sim \pm 30\%$ for the basic numerical data. From Eqs. (1)-(7), we have $Q_1 = 149$, $m_0 = 49.92$, $n_1 = 18$, $d_0 = 0$ & $ETP(Q_1, m_0, n_1, d_0) = -24963.57$. From Eqs. (8)-(14), we have $Inv = 16$ with $ETP(Inv) = 5933.12$.

Table 1 presents some results:

1. The k_1 and k_2 have a large effect on the mean.
2. The $M_1, O_1, I_1, h_1, S_t, \sigma_0,$ and I_c have a large contribution on the EMQ.
3. The *Inv* has a large contribution on ETP.

4. Conclusions

This work addresses the EMQ one including quality investment and process mean. Numerical results obtain that the Inv can reduce product's variability and improve expected total profit. Further work can include the unknown process parameters with consumer's risk in the model.

Table 1

Some results

C_1	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
0.8	(149, 49.92, 18, 0)	-16976.39	16	13920.32
1.2	(149, 49.92, 18, 0)	-32950.74	16	-2054.08
A_{22}	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
64	(149, 49.92, 18, 0)	-37745.55	16	-6866.88
96	(149, 49.92, 18, 0)	-12181.58	16	18733.12
A_{11}	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
54	(149, 49.92, 18, 0)	-24978.77	16	5933.12
81	(149, 49.92, 18, 0)	-24948.36	16	5933.12
M_1	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
640	(133, 49.92, 18, 0)	-19998.94	16	4709.44
960	(163, 49.92, 18, 0)	-29925.18	16	7160.44
O_1	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
64	(111, 49.92, 18, 0)	-25065.20	15	5800.15
96	(331, 49.92, 18, 0)	-24799.13	18	6150.44
I_1	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
90	(199, 49.92, 18, 0)	-24887.82	17	6032.30
120	(115, 49.92, 18, 0)	-25050.13	15	5819.31
S_t	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
16	(137, 49.92, 18, 0)	-24941.21	16	5947.31
24	(159, 49.92, 18, 0)	-24984.35	16	5918.19
h_1	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
8	(166, 49.92, 18, 0)	-24932.16	16	5974.06

Contd...

12	(136, 49.92, 18, 0)	-24991.96	16	5895.82
RI	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
24.4	(149, 49.92, 18, 0)	-24963.52	16	5933.12
36.6	(149, 49.92, 18, 0)	-24963.62	16	5933.12
k_1	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
160	(149, 49.87, 18, 0)	-19448.26	16	7628.00
240	(148, 49.95, 18, 0)	-29851.17	16	4234.51
k_2	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
240	(149, 49.96, 18, 0)	-20049.65	16	7817.75
360	(148, 49.88, 18, 0)	-29256.96	16	4082.03
α	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
0.4	(149, 49.92, 18, 0)	-24963.57	19	5911.95
0.6	(149, 49.92, 18, 0)	-24963.57	14	5947.25
σ_T	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
0.24	(149, 49.92, 18, 0)	-24963.57	16	12109.23
0.36	(149, 49.92, 18, 0)	-24963.57	15	-1680.63
σ_0	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
0.4	(152, 49.93, 18, 0)	-7432.45	14	6037.99
0.6	(93, 49.90, 18, 0)	-46430.87	16	5532.92
IC	(Q_1, m_0, n_1, d_0)	ETP (Q_1, m_0, n_1, d_0)	*Inv*	*ETP*(*Inv*)
0.4	(144, 49.92, 18, 0)	-24953.62	16	5939.73
0.6	(93, 49.90, 18, 0)	-46430.87	15	5526.43

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Received June, 2023