

Numerical investigations of flow characteristics around square vs inclined cylinders

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Abstract

This study investigates the flow patterns around square as well as inclined cylinders, a critical area in civil engineering due to their complex flow interactions. The research objectives are to conduct numerical analyses in two dimensions to understand the flow behavior around inclined square cylinders and those enclosed by parallel surfaces, focusing on vortex shedding and flow dynamics at varying Reynolds numbers (Re) from 40 to 200. Key findings reveal that vortex shedding frequencies and Strouhal numbers (St) were calculated for different Re values, showing increased turbulence with higher Re . The inclination of the square cylinder caused significant changes in flow characteristics, including abrupt shifts in separation points and variations in drag and lift forces. Quantitative results included detailed streamlines, flow velocities, vortex shedding frequencies, and Strouhal numbers values across a range of inlet velocities and Reynolds numbers. The practical implications of these research findings are significant because velocity distributions show that asymmetric vortex flows occur around inclined cylinders at Reynolds numbers of 100 and 200. When building slanted structures to withstand approaching winds, civil engineers must consider these facts.

Subject Classification: 76M20, 76M25, 76-10.

Keywords: Flow around square cylinders, Two-dimensional numerical investigations, Reynolds number, Strouhal number.

1. Introduction

The examination of fluid dynamics around blunt bodies (objects with a non-aerodynamic shape) holds significant importance in various applications within the field of engineering, encompassing disciplines such as civil engineering (for the evaluation of wind pressures on buildings and bridges), mechanical engineering (for the design of vehicles and infrastructure), as well as environmental engineering (for the analysis of flow behaviors in rivers and around obstacles). The formation of vortices (resulting from the existence of shear forces and velocity gradients in the fluid) could lead to periodic loading forces on the structure, consequently inducing structural vibrations and noise caused by the flow [1].

In civil engineering, flow conditions greatly influence structure functionality and stability. An understanding of flow patterns around structures, like slanted square cylinders, is vital for efficient design and overcoming challenges. Fluid separation and vortex shedding around a cylinder can compromise structural stability, posing ongoing challenges. Understanding flow dynamics around tall square cylinders is crucial across engineering domains. Insight into flow around inclined square cylinders aids in designing streamlined, resilient objects. Fluid behavior

around such objects can exhibit intricate features, influencing design effectiveness [2].

Factors like inclination angle, flow velocity, and Reynolds number influence flow parameters around inclined cylinders. Empirical studies show displacement of separation points affects attributes like Strouhal number, drag, and lift forces, depending on the angle of attack. Compared to vertical or horizontal cylinders, inclined ones exhibit more [3] intricate three-dimensional flow, with the angle impacting wake vortex configurations and aerodynamic interference [4-5]. A research study [6] used hotwire anemometry to study square cylinder orientation effects at high Reynolds numbers. The angle of attack determines flow separation points, affecting flow patterns and characteristics like Strouhal number [3]. Famous research explored the impact of cylinder aspect ratio on pressure forces and vortex shedding frequencies [7].

Research has explored the impact of nearby plane walls on circular cylinders, analyzing vorticity dynamics and vortex-induced instability theory [8], a similar theory initially proposed by [9]. Instabilities are influenced by gap ratios and cylinder surface characteristics, shedding light on vortex-induced instability mechanisms. Numerical simulations are crucial for understanding flow dynamics and mitigating vibrations, aiding in enhancing aerodynamic stability [5, 10-12]. Two-dimensional numerical investigations into flow characteristics around inclined square cylinders between parallel walls reveal intense vortex shedding and flow oscillations at Reynolds number 50. However, consensus on square cylinder instabilities, especially in transitioning Reynolds number regimes, remains lacking.

Okajima [13] experimentally characterized vortex shedding from square and rectangular cylinders in a controlled water tank over $Re = 70-2 \times 10^4$, identifying distinct Strouhal-Reynolds number trends and noting a breakdown of this correlation beyond $Re \approx 10^4$. Subsequent work on four square-arranged cylinders at low Reynolds numbers revealed that wake topology and shedding frequencies are highly sensitive to cylinder spacing and Reynolds number variations [14].

A research study numerically investigates flow characteristics around four square-arranged cylinders at low Reynolds numbers ($Re = 40-160$), revealing distinct wake flow patterns influenced by spacing ratio and Reynolds number [15]. The present research endeavors to simulate fluid flow approximately vertical and inclined cylinders at 0° and 45° , and Reynolds numbers ranging from 40 to 200. Another key objective of this study is to investigate the wake dynamics and to quantitatively assess

crucial parameters such as the Strouhal numbers and vortex shedding for cylinders oriented at different angles.

2. Methodology

The present study employs a numerical model based on an incompressible 2D Navier-Stokes solver implemented in MATLAB, utilizing straightforward numerical methods. The solver discretizes the equations on a collocated uniform grid distribution, ensuring precise calculations across the entire flow field. Derivatives are approximated using cell-centered schemes, which are effective in maintaining consistency and reducing numerical diffusion. Additionally, the top and bottom boundaries of the domain are treated as free-slip [16-17], allowing the fluid to move without friction along these surfaces, mimicking open flow conditions typically found in natural environments. A Runge-Kutta third-order scheme is used for time marching, offering high accuracy and stability in simulating transient flows [18].

Figure 1 shows the of configuration of dimensions of the long square cylinder (left figure) and inclined long square cylinder (right figure).

The cylinder within the flow is modeled using the immersed-boundary volume of fluid (IB-VOF) method. This approach accurately represents the fluid-structure interaction by embedding the cylinder into the fluid grid without the need for a conforming mesh, which simplifies the computational process while maintaining accuracy. The specific scenario investigated involves a square cylinder, placed in an incoming stream with a velocity of $V = 5 \text{ cm/s}$ and a fluid with a kinematic viscosity of $\nu = 10^{-4} \text{ m}^2/\text{s}$. The inlet boundary condition is a constant approaching flow, reflecting real-world scenarios where structures encounter steady wind or water currents. The outlet boundary condition follows the Orlansky convective velocity condition, which effectively minimizes reflections and disturbances from the computational domain, ensuring the simulation's realism.

This modeling approach validates the real-world scenario by accurately simulating the fluid dynamics around and in the wake of the inclined square cylinder. The use of the IB-VOF method ensures that the interaction between the fluid and the cylinder is captured realistically, providing insights into the flow patterns and forces experienced by structures in similar conditions. By incorporating practical boundary conditions and precise numerical methods, the study offers credible and applicable results for civil engineers dealing with inclined structures subjected to fluid flow.

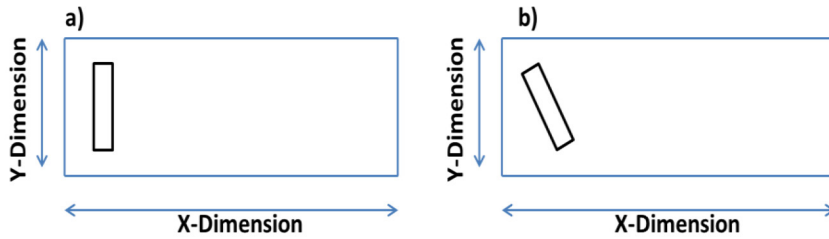


Figure 1

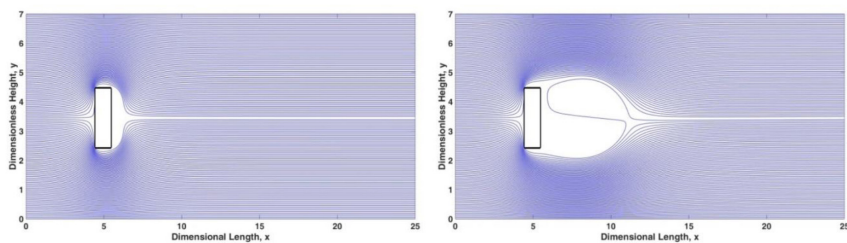
Configuration of dimensions a) Long square cylinder and b) Inclined long square cylinder

3.0 Results & Discussion

3.1 Flow Patterns around a Cylinder at $Re\ 40$

The streamlines at various time instances are evaluated to comprehend their temporal developments at different Reynolds numbers when the square cylinder is positioned between two parallel walls. The depiction in Figure 2 illustrates the flow (at $t=05$ sec) approaching from the left and establishing a stagnation point at the center of the front surface of the cylinder, where the highest-pressure coefficient is recorded. Moreover, the flow diverges into distinct streams towards the upper and lower edges of the square cylinder, with flow separation manifesting at the forefront of the cylinder. Subsequently, the flow reattaches around 7 and 3.5, denoting the x and y coordinates. A notable density of streamlines signifies escalated velocities at the leading and trailing edge corners of the entity, particularly accentuated at the leading edge.

Figure 3 shows (at $t=100$ seconds) the flow and vortex shedding have fully developed and show a strong oscillation. An increase in velocity with passing time is observed between Figures 2d and 2e, as indicated by the



Figures 2 (left) & 3 (right)

Show streamlines when Re is at 40 at $t=05$ sec & when Re is at 40 at $t=100$ sec

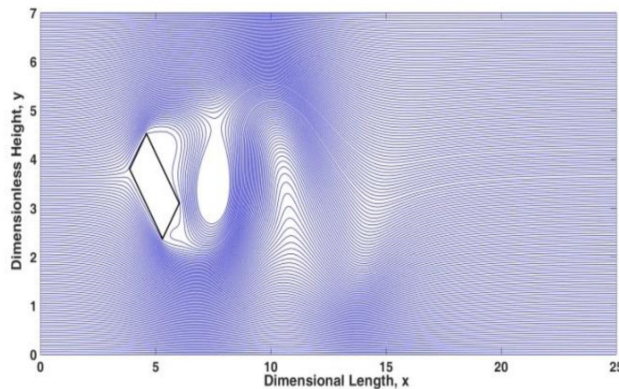


Figure 4

Shows streamlines when Reynold's number (Re) is at 40 at $t=100$ sec in the presence of cylinder at 45° orientation.

reduction in distance between streamlines in the wake of the flow. It is evident that the primary vortices, having opposite circulation, develop in an alternating fashion. The streamlines distinctly reveal the locations of their centres and associated saddle points, consistent with the observations reported in [19].

3.2 Flow patterns around an inclined cylinder at Re 40

The streamlines are evaluated when the long square cylinder is at an inclination angle of around 45° when $t=100$. It is clear from Figure 4 that the vortex is fully formed and oscillates strongly. According to Sohankar et al. [19], vortex shedding was observed even at the lowest Re when the cylinder's orientations fell between 0 and 45° , which implies our findings are consistent with those of previous studies.

3.3 Flow patterns around a cylinder at Re 100

At a Reynolds number of 100, which is relatively low, the flow around a long square cylinder is likely to be in the laminar regime. In laminar flow, the fluid moves smoothly in parallel layers, and the streamlines are well-defined and orderly. However, beyond a certain distance from the cylinder, these streamlines tend to curve due to the influence of the fluid's inertia and the pressure distribution around the cylinder. An attempt is made to know how streamlines look away from the cylinder when the Re is equal to 100.

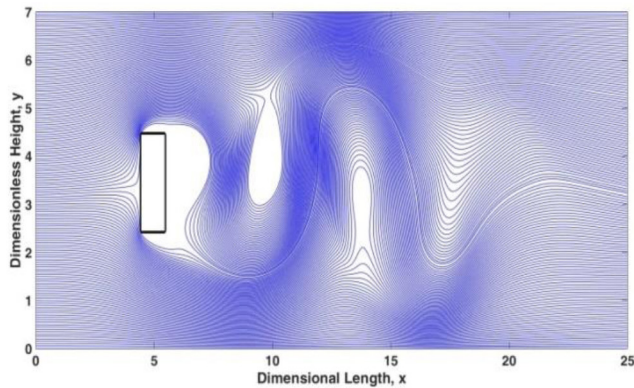


Figure 5

Shows streamlines when the Reynolds' number (Re) is 100 at $t = 100$ sec in the presence of a long square cylinder

Figure 5 shows the streamline contours at $t = 100$ sec. As expected, clear vortices with oscillations are generated behind a long-squared cylinder, as can be witnessed from Figure 5. Further, a pair of counter-rotating vortices could be seen even $t = 40$ sec (not shown here) and their full dominance was witnessed at $t = 100$ sec.

When a fluid flows past an obstacle, such as a long square cylinder, it experiences a pressure drop on the upstream side and a pressure rise on the downstream side. This pressure gradient causes the fluid to accelerate around the corners of the cylinder and creates regions of low pressure in the wake region behind the cylinder. As a result, the streamlines curve and converge in the wake region that, which is what exactly observed in this research. At Reynolds numbers of 100, streamlines can still generate beyond a long square cylinder due to the pressure distribution around the cylinder and the resulting flow patterns.

3.4 Flow Patterns around an inclined cylinder at Re 100

Figure 6 depicts streamline evolution at Re 100 at $t = 100$ sec, and it is obvious that over time, clearer vortex observations emerge. The geometry, orientation, and pressure distribution of the cylinder influence streamline generation. At Re 100, laminar flow prevails, with added complexities due to cylinder inclination [20, 21].

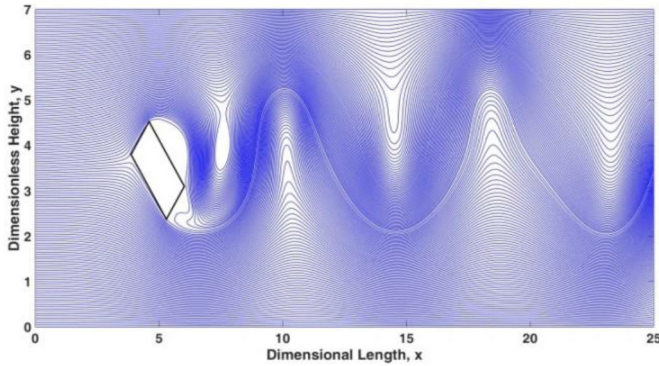


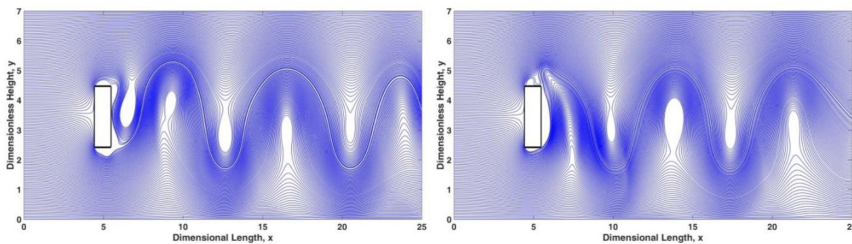
Figure 6

Shows streamlines when Reynolds' number (R_e) is 100 $t=100$ sec in the presence of cylinder at 45° orientation.

3.5 Streamlines in the presence of a cylinder at $Re\ 200$

Figure 7 shows the frequency at 40 seconds, while Figure 8 shows the frequency at 100 sec and it is quite clear that the flow and vortex shedding have fully developed and show a strong oscillation. Further, an increase in velocity with passing time is observed between figures 7 and 8, as indicated by the reduction in distance between streamlines in the wake of the flow. It is also clear that for $R=200$ transition.

Flow occurs from laminar to turbulent, when a comparison is made between Figures 7 and 8 with those presented in 2, 3, 4, 5, and 6. Further, asymmetric vortex shedding [2] could be seen here and there, which implies that the flow is transitioning from laminar or turbulent regime [22]. In general, the transition from laminar to turbulent flow is triggered by energy amplification away from the wall, facilitated by perturbations



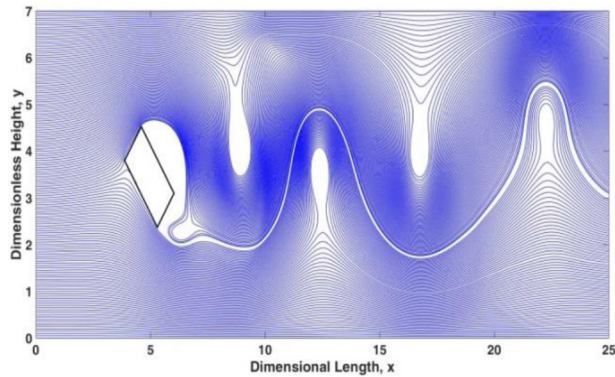
Figures 7 (left) & 8 (right)

Show streamlines when the Reynold's number (R_e) is 200 at $t=40$ sec & $t=100$ sec

near the upper edges, though these features are not predominantly seen in Figures 7 and 8.

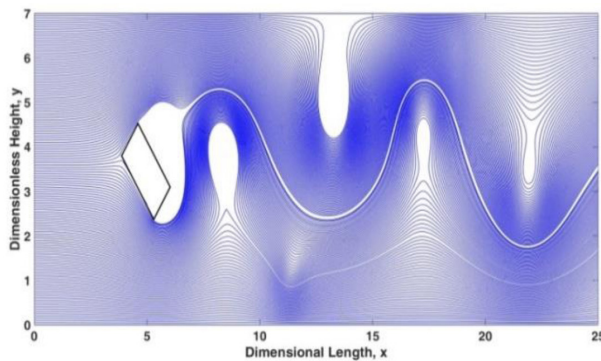
3.6 Streamlines around an Inclined Cylinder at $Re\ 200$

Figure 9 shows the frequency at 40 seconds, while Figure 10 shows the frequency at 100 sec. It is clear that the flow and vortex shedding have further developed and show even stronger oscillations compared to those presented in Figures 7 and 8. The asymmetric vortex shedding has been significantly noticed in both cases in such a way that the vortex shedding has displaced to the upper boundary. This behavior clearly indicates the turbulent nature of the fluid when Re is 200.



Figures 9

Shows streamlines when Reynold's number (Re) is 200 at $t=40$ sec cylinder at 45° orientation.



Figures 10

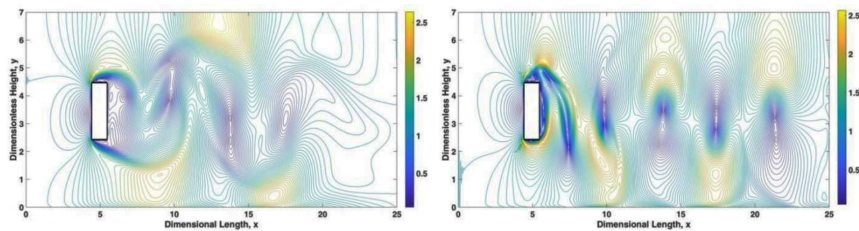
Shows streamlines when Reynold's number (Re) is 200 at $t=100$ sec in the presence of cylinder at 45° orientation.

In a nutshell, Figures 2 to 10 illustrate the transition of shear flows around a cylinder from a laminar state to a turbulent state. The presence of an inclined cylinder introduces additional complexity with regards to vortex shedding, which is influenced by the angle of inclination. As the Reynolds number escalates (from 40 to 200), vortices become unstable and undergo oscillations before eventually detaching periodically. The distinctive swirling wake is recognized as Kármán Vortex Street. It is therefore surmised that the angle of inclination significantly influences the magnitude and distribution of forces acting on the structure. Steeper inclinations may cause the air to directly impact the surface, resulting in elevated pressures and increased drag forces [23].

Conversely, shallower inclinations can lead to airflow separation or detachment from the surface, creating intricate flow patterns and dynamic loading conditions. When airflow interacts with an inclined square structure, it becomes deflected and perturbed, generating various aerodynamic effects that manifest as forces exerted on the structure. The aerodynamic behavior around a square cylinder differs slightly from that around a cylindrical one, as the separation points in a square geometry are fixed at the leading or trailing edges, contingent upon the Reynolds number. In addition, the region of vortex formation is wider and longer for a square cylinder. The Reynolds number dictates the flow's behavior [24]. At low Re , it is all about symmetry and steadiness. At moderate Re , one might see Karman vortex shedding in the cylinder's wake, due to the fluctuating pressure causing a lift force. When Re is high, turbulence takes over, highlighting some serious vortex shedding, as the present research results have shown.

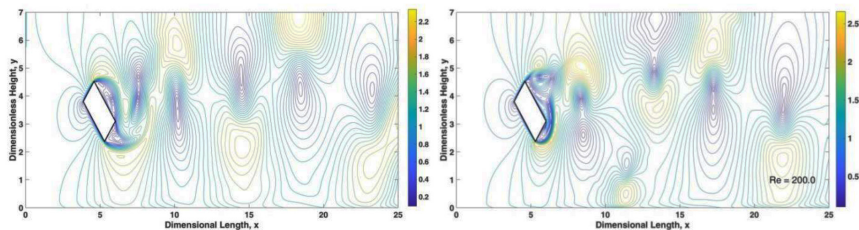
3.7 Velocity Patterns around Square and an Inclined Cylinders at higher Re

With a view to verifying how vortex shedding has occurred for velocities when the Reynolds number is kept at 100 and 200, we present the velocity contours once the vortex shedding has developed fully. Figures 11 and 12 show velocity contours at $t = 100$ sec when the Reynolds number was 100 and 200 in the presence of a long square cylinder, while figures 13 and 14 show velocity contours at $t = 100$ sec when the Reynolds number was 100 and 200 in the presence of an inclined square cylinder. Compared to streamlines, the velocity distributions have shown even better symmetries and steady flow even when the Reynolds numbers were at 100 and 200, respectively. Though the dominance of vortex shedding is



Figures 11 (left) & 12 (right)

Show velocity contours at $t=100$ sec when Re is set at 100 and 200 in the presence of cylinder.



Figures 13 (left) & 14 (right)

Show velocity contours at $t=100$ sec when Re is set at 100 & 200 in the presence of inclined cylinder.

evident in both cases, the vortex shedding frequency is higher for an inclined cylinder compared to a long cylinder.

In addition to the above results, asymmetric vortex flows were associated with inclined cylinders when the Reynolds numbers were 100 and 200, respectively (see figures 13 and 14). Nevertheless, when a cylinder was kept at an inclination angle other than 45° , such asymmetric vortices were found to be observed [2]. However, the size and strength of the vortex associated with the square and inclined square cylinders (when the Reynolds number was at 100 and 200, respectively) are lower than those observed at $t=10$ and $t=20$ sec (not shown here), for both geometries. These results are in great corroboration with the results presented in a research study [2] and the references therein.

Table 1 presents the maximum vortex shedding and Strouhal number at different Re in the presence of square and inclined square cylinders, and these tables also present the inlet velocities. It is clear from this table that most of the values are showing a similarity with earlier studies.

Table 1
Vortex shedding and Strouhal number at different Reynolds numbers

S. No	Inlet velocity	Reynolds Number (Re)	Maximum Vortex Shedding		Strouhal number	
			Square Cylinder	Inclined Square Cylinder	Square Cylinder	Inclined Square Cylinder
1	0.02	40	Not Found	Not Found	Not Found	Not Found
2	0.04	80	0.0510	0.0585	0.3329	0.3332
3	0.05	100	0.2502	0.0710	0.1176	0.1177
4	0.1	200	0.2605	0.0937	0.0526	0.1111

4.0 Conclusion

This research investigates the flow past a long square cylinder and an inclined square cylinder at Reynolds numbers of 40, 100, and 200. The study enhances our understanding of flow characteristics around inclined square cylinders, shedding light on complex flow phenomena and mechanisms. This knowledge advances civil engineering, aiding in the design of more efficient and resilient structures.

The salient observations of this research are: Different inlet velocities established Reynolds numbers for flow around elongated and inclined cylinders. Vortex shedding phenomena, observed across Reynolds numbers, showed increasing turbulence with higher Re. Detailed analyses revealed insights into streamlines and flow velocities over time. Vortex shedding frequencies and Strouhal numbers were calculated for both square and inclined square cylinders across various inlet velocities and Reynolds numbers. The inclination of square cylinders caused abrupt shifts in separation points to different edges. Most importantly, as far as the innovative aspects of this research are concerned, asymmetric vortex flows occur with inclined cylinders at Reynolds numbers of 100 and 200, particularly when the vortices are evaluated in terms of velocity distributions. To our knowledge, such behavior hasn't been reported at higher inclination angles. These findings, therefore, assume great significance when civil engineers are dealing with inclined structures like the leaning tower of Pisa in Italy and the Varanasi Temple in India, which is known as the leaning tower of India.

Acknowledgements

Thanks are due to the Management of GMRIT (Rajam), ANITS (Visakhapatnam), and SVECW (A), Bhimavaram, India for their logistic support.

References

- [1] M. Sarioglu, Y. E. Akansu, and T. Yavuz, "Control of the flow around square cylinders at incidence by using a rod," *AIAA Journal*, vol. 43, no. 7, pp. 1419–1426 (2005), doi: 10.2514/1.9460.
- [2] D.-H. Yoon, K.-S. Yang, and C.-B. Choi, "Flow past a square cylinder with angle of incidence," *Physics of Fluids*, vol. 22 (Apr. 2010), doi: 10.1063/1.3388857.
- [3] T. Igarashi, "Characteristics of the flow around a square prism," *Bulletin of JSME*, vol. 27, no. 231, pp. 1858–1865 (1984), doi: 10.1299/jsme1958.27.1858.
- [4] M. Ali, "Experimental free convection heat transfer from inclined square cylinders," *Heat and Mass Transfer*, vol. 53, pp. 1643–1655 (2017), doi: 10.1007/s00231-016-1881-7.
- [5] Z. Chen, S. Li, L. Zhang, C. Yuan, Z. Zhao, Y. Wu, Y. Xu, and C. Y. Li, "Characteristics of aerodynamic interference and flow phenomenology around inclined square prisms," *Physics of Fluids*, vol. 35, no. 12 (Dec. 2023), Art. no. 125119, doi: 10.1063/5.0182197.
- [6] E. D. Obasaju, "An investigation of the effects of incidence on the flow around a square section cylinder," *Aeronautical Quarterly*, vol. 34, no. 4, pp. 243–259 (1983), doi: 10.1017/S0001925900009768.
- [7] C. Norberg, "Flow around rectangular cylinders: Pressure forces and wake frequencies," *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 49, no. 1, pp. 187–196 (1993), doi: 10.1016/0167-6105(93)90014-F.
- [8] A. Dipankar and T. K. Sengupta, "Flow past a circular cylinder in the vicinity of a plane wall," *Journal of Fluids and Structures*, vol. 20, no. 3, pp. 403–423 (2005), doi: 10.1016/j.jfluidstructs.2005.01.001.
- [9] T. K. Sengupta, S. De, and S. Sarkar, "Vortex-induced instability of an incompressible wall-bounded shear layer," *Journal of Fluid Mechanics*, vol. 493, pp. 277–286 (2003), doi: 10.1017/S0022112003005822.

- [10] W. Zhu, K. Luo, Z. Xiao, and S. Fu, "Numerical simulations of the flow dynamics past an oscillating rod-airfoil configuration," in *Proceedings of ICMME 2016* (2016), doi: 10.1007/978-3-319-70031-1_36.
- [11] A. G. da Silva Jr., J. A. Martins, and E. C. Romao, "Numerical simulation of a one-dimensional non-linear wave equation," *Engineering, Technology & Applied Science Research*, vol. 12, no. 3, pp. 8574–8577 (2022), doi: 10.48084/etasr.4920.
- [12] J. U. Rehman, C. N. Nguyen, T. A. Nguyen, T. C. Vo, T. K. Nguyen, and V. Q. Nguyen, "Prediction of the stress wave amplification factor of a spherical blast source using numerical simulations," *Engineering, Technology & Applied Science Research*, vol. 12, no. 5, pp. 9395–9399 (2022), doi: 10.48084/etasr.5233.
- [13] A. Okajima, "Strouhal numbers of rectangular cylinders," *Journal of Fluid Mechanics*, vol. 123, pp. 379–398 (1982).
- [14] Y. Gao, C. Yin, H. Zhang, K. Yang, X. Zhao, and Z. Sun, "Numerical study on flow around four square-arranged cylinders at low Reynolds numbers," *Mathematical Problems in Engineering* (2017), doi: 10.1155/2017/6381256.
- [15] Y.-C. Zhu and S. Zhang, "On the flow characteristics of a four square-cylinder array at a 45° incidence angle and low Re ," *AIP Advances* (2022), doi: 10.1063/5.0091440.
- [16] S. Sen, S. Mittal, and G. Biswas, "Flow past a square cylinder at low Reynolds numbers," *International Journal for Numerical Methods in Fluids*, vol. 67, pp. 1160–1174 (2011), doi: 10.1002/fld.2416.
- [17] W. N. Muyungi, M. H. Mkwizu, and V. G. Masanja, "The effect of Navier slip and skin friction on nanofluid flow in a porous pipe," *Engineering, Technology & Applied Science Research*, vol. 12, no. 2, pp. 8342–8348 (2022), doi: 10.48084/etasr.4763.
- [18] A. E. Perry, M. S. Chong, and T. T. Lim, "The vortex-shedding process behind two-dimensional bluff bodies," *Journal of Fluid Mechanics*, vol. 116, pp. 77–90 (1982), doi: 10.1017/S0022112082000378.
- [19] A. Sohankar, C. Norberg, and L. Davidson, "Low-Reynolds-number flow around a square cylinder at incidence: Study of blockage, onset of vortex shedding and outlet boundary condition," *International Journal for Numerical Methods in Fluids*, vol. 26, pp. 39–56 (1998).
- [20] R. Bourguet and M. S. Triantafyllou, "Vortex-induced vibrations of a flexible cylinder at large inclination angle," *Philosophical Transactions*

- of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 373 (2015), doi: 10.1098/rsta.2014.0108.
- [21] M. Avila, D. Barkley, and B. Hof, "Transition to turbulence in pipe flow," *Annual Review of Fluid Mechanics*, vol. 55, pp. 575–602 (2023), doi: 10.1146/annurev-fluid-120720-025957.
- [22] K. A. Khasawneh and Y. Al-Samamah, "Analytical study of the stagnation point properties and impact forces for non-continuum flow out of planar exit impinging on inclined flat plate surface," *International Review of Aerospace Engineering*, vol. 15, no. 5 (2022), doi: 10.15866/irease.v15i5.22475.
- [23] M. Shademan and A. Naghib-Lahouti, "Effects of aspect ratio and inclination angle on aerodynamic loads of a flat plate," *Advances in Aerodynamics*, vol. 2, pp. 14 (2020), doi: 10.1186/s42774-020-00038-7.
- [24] S. Khurana, K. Suzuki, and E. Rathakrishnan, "Flow field behavior with Reynolds number variance around a spiked body," *Modern Physics Letters B* (2016), doi: 10.1142/S0217984916503620.

Received August, 2025