Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 47 (2026), No. 1, pp. 87–98
DOI : 10.47974/JIOS-1425

An inspection and production model with inspection errors

Neng-Hui Shih
Department of Business Administration
National Pingtung University
51 Min-Sheng E. Road
Pingtung 900
Taiwan, R.O.C.

W.C. Tsai
Department of Industrial Management
I-Shou University
Kaohsiung City 84001
Taiwan, R.O.C.

Chih-Hsiung Wang *
Department of Business Administration
National Pingtung University
51 Min-Sheng E. Road
Pingtung 900
Taiwan, R.O.C.

Abstract

When product inspection time is non-negligible, a previous paper developed an online inspection and production model in which inspection errors do not exist. It is assumed that the inspection rate is less than the production rate, to have an economic production lot size, product inspection information needs to be used to infer process quality so that the quality of the uninspected products can be inferred. Once the expected total cost per conforming product for an evaluated production quantity starts to increase, then production is ceased. However, since inspection itself is imperfect in practice, type I and type II inspection errors exist in reality. Thus, in this paper we have reformulated the previous production and inspection model and incorporated these two types of inspection error with the aim of determining their effect on the optimal production lot size. Illustrative numerical examples

* *E-mail:* **chwang@mail.nptu.edu.tw**

are provided to demonstrate the effect of inspection errors on optimal production and on inspection policy.

Subject Classification: *(2010) 90B05, 90B25.*

Keywords: *Imperfect inspection, Imperfect process, Lot sizing.*

1. Introduction

Porteus [7] modelled the imperfect process as being in either an IN or an OUT state, and during the production of an item, the process may shift from an IN state to an OUT state at a constant probability. The relationship between a production lot size and the associated number of defective items was then used by Porteus [7], to determine the optimal production lot size by balancing the costs involved in setup, associated with defective items and related to inventory holding. Based on the Porteus [7] lot size model, Wang and Sheu [18] incorporate an inspection policy that screened for defective products, which were then reworked to make them perfect; in this study, only the later products in the production lot are considered to have been inspected. Other models involving a partial inspection policy have been proposed by Yeh and Chen [21], and Djamaludin et al. [2]. Hybrid decision support systems are used to detect defective products and fix them during production to achieve zero defect goals [9,10]. By way of contrast, Raz et al. [13] propose an economic inspection/disposition model that dynamically selected the products to be inspected in order to minimize the costs associated with product inspection, incorrect rejection of products and the acceptance of nonconforming products. Later, Sheu et al. [14] carried an extension of the Raz et al.'s [13] inspection errors model. Another study by Ramzan et al. [12] investigated the use of an offline inspection policy. On the other hand, when there is no product information of the production sequence, Chen and Chou [1] established a quality investment sampling plan to control the quality level of production batches.

When an imperfect process is being considered, another important research area relates to the study of optimal production lot size when there is a maintenance policy that safeguards the quality status of the process during production. For example, Lee and Rosenblatt [4] and Lee and Rosenblatt [5] assumed that the process shift distribution follows an exponential distribution (alternatively see Porteus [8]). Using equal inspection intervals, the optimal production run length and number of inspections were simultaneously determined by minimizing the expected total cost per unit time. Tseng [17], Kim et al. [3], Wang [19] and Wang and Yeh [20] generalized Lee and Rosenblatt's [4] model to consider a situation where the process shift distribution follows an arbitrary distribution. From this they were able to obtain several useful properties that are necessary for determining an optimal solution. As an alternative, Lin et al. [6] assumed that preventive maintenance itself is imperfect, namely that maintenance might cause a process shift from an IN state to an OUT state during an imperfect process; they then investigated the optimum production lot size and optimum maintenance schedule using such a model.

When online inspection and maintenance is not allowed during production, Shih and Wang [16] proposed that product inspection information can be utilized to infer process quality status. This information can then be used to decide whether a production batch should be stopped; Especially in this case, Shih et al. [15] assume that the product inspection time is not negligible. In the present paper, main aim was to stretch Shih et al.'s [15] model to include two types of inspection errors in order to upgrade its usefulness.

The rest of this paper is organized as follows. In Section 2, mathematical formulae describing the production and inspection policy with inspection errors are established. In Section 3, numerical examples are performed to investigate the effect of inspection errors on the optimal production lot size. In Section 4, conclusions are provided.

2. Mathematical model

The notations for developing the production and inspection model with inspection errors are defined as follows, where most of them are the same as those quoted by Shih et al. [15]:

$\mathbf{Y}_n$: an n- tuples used for recording the quality status of the first n products that have been inspected

$\mathbf{y}_n = (y_1, \cdots, y_\kappa, \cdots, y_n)$: the outcome of $\mathbf{Y}_n$, where $y_\kappa = 0$ or 1, which represents that the kth product has been inspected to be conforming or nonconforming, respectively, for $k = 1,2,\cdots,n$

c_{I}: inspection cost for identifying quality status of a product (\$/product)

c_{R}: unit restoration cost (\$)

c_{m} : manufacturing cost per product

c_{s} : joint cost of process setup and maintenance

λ : market demand for the products per year

c_{h} : inventory holding cost for a product per year

q_j: the actual quality status of the j-th product, where $q_j = 0$ or 1, which represents that the j-th product is conforming and nonconforming, respectively

α: type I error, the probability that, on inspecting, a conforming product is nonconforming

β: type II error, the probability that, on inspecting, a nonconforming product is conforming

r_0: revenue from accepting a conforming product (\$)

c_0: cost of accepting a nonconforming product (\$)

c_1: cost of rejecting a conforming product (\$)

c_2: cost of rejecting a nonconforming product, where $0 \leq c_2 < \min\{c_0, c_1\}$ (\$)

We address Shih et al.'s [15] production and inspection model as follows. For more detailed information, please refer to Shih et al. [15]. The process is setup and maintained to be as good as new and to be in an in-control state with cost c_{s}; however, after producing products, the system may randomly shift from an IN state into an OUT state. Let M be a random variable that represents the number of produced products since the last setup, which is the exact time point that the process shifts from an IN state to an OUT state. The probability that the first j products in a production lot are produced in an IN state is denoted by $\overline{P}_j = \Pr(M > j)$. If the process is trapped in an OUT state, it remains there until the completion of the production batch. In addition, it is assumed that the deteriorating process shows manufacturing variation. Therefore, the probability that a product will be produced as conforming when the process is in an IN state or an OUT state are assumed to be θ_0 and θ_1, respectively, where $\theta_1 < \theta_0$ Product inspection information can be used to infer the process quality status. The products are inspected based on the principle of first produced first inspected. As soon as the n-th product has just been inspected, one has that $1 + \lceil n\delta \rceil$ products have been produced when the ration of the inspection rate to the production rate is considered to be δ, where $1 < \delta < \infty$. The $n+1$-th product will be completely inspected when product $1 + \lceil (n+1)\delta \rceil$ has been completely produced. Therefore, before the $n+1$-th product whose inspection information is available, one uses the inspection information of the first n products to infer the process quality, used to evaluate the quality status of product N_n, where $N_n = n+1,\ n+2, \cdots, 1 + \lceil (n+1)\delta \rceil$. Let $V(N_n) = \frac{c_s + \phi(N_n)}{a(N_n)}$ be the expected total cost per conforming product (ECCP) when the product quantity is N_n, where $\phi(N_n)$ is comprised of the costs of process restoration, manufacturing, related quality control, and inventory, as well as revenue from accepted conforming products, with $a(N_n)$ representing the number of conforming products within N_n, respectively. Once $V(N_n)$ starts to increase with N_n, the process ceases production and a production lot is created. Note that the resulting optimal production quantity (denoted by N^*) is greater or equal to $1 + \lceil n\delta \rceil$.

The following mathematical formulas are mainly obtained by referring to the derivation process of Shih et al. [15], in which we appropriately include two types of inspection errors.

When type I and type II inspection errors are considered, taking into account the different time points when the process shifts from an IN state to an OUT state (i.e., $M = 1,2,\cdots,n,n+1\cdots$), the probability that the k-th product will be inspected to be conforming (i.e., $y_k = 0$) or nonconforming (i.e., $y_k = 1$) is generally expressed by the following equation:

$$\begin{cases} D_k = (1-y_k)(\theta_1(1-\alpha) + (1-\theta_1)\beta) + y_k(\theta_1\alpha + (1-\theta_1)(1-\beta)),\ M = 1 \\ E_k = (1-y_k)(\theta_0(1-\alpha) + (1-\theta_0)\beta) + y_k(\theta_0\alpha + (1-\theta_0)(1-\beta)),\ M = i,\ i = 2,\dots,n, k < i \\ F_k = (1-y_k)(\theta_1(1-\alpha) + (1-\theta_1)\beta) + y_k(\theta_1\alpha + (1-\theta_1)(1-\beta)),\ M = i,\ i = 2,\dots,n, k \ge i \\ G_k = (1-y_k)(\theta_0(1-\alpha) + (1-\theta_0)\beta) + y_k(\theta_0\alpha + (1-\theta_0)(1-\beta)),\ M > n \end{cases}$$

Using the inspection results, $\mathbf{Y}_n = \mathbf{y}_n$, to infer the reliability of the process for producing the j-th ($j > n$) product gives $\Pr(M > j | \mathbf{Y}_n = \mathbf{y}_n)$. More precisely (also see Shih et al. [15]) for a similar derivation),

$$x_j^{\mathbf{y}_n} = \frac{\overline{P}_j}{\Pr(\mathbf{Y}_n = \mathbf{y}_n)} \prod_{k=1}^{n} G_k, \tag{1}$$

where

$$\Pr(\mathbf{Y}_n = \mathbf{y}_n) = \left(1 - \overline{P}_1\right) \prod_{k=1}^{n} D_k + \sum_{i=2}^{n} \left(\overline{P}_{i-1} - \overline{P}_i\right) \left\{ \prod_{k=1}^{i-1} E_k \prod_{k=i}^{n} F_k \right\} + \overline{P}_n \prod_{k=1}^{n} G_k .$$

Furthermore, the probability that the process will shift from an IN state to an OUT state while producing the j-th product is given by $\tau_j^{\mathbf{y}_n} = x_{j-1}^{\mathbf{y}_n} - x_j^{\mathbf{y}_n}$ for $j > n$ and

$$\mu_j^{\mathbf{y}_n} = \frac{\overline{P}_{j-1} - \overline{P}_j}{\Pr(\mathbf{Y}_n = \mathbf{y}_n)} \prod_{k=1}^{j-1} E_k \prod_{k=j}^{n} F_k$$

for $j \leq n$, respectively (see Shih et al. [15] for a similar derivation).

The probability that whether the j-th product whose actual quality status (i.e., q_j) agrees with the inspection result (y_j), i.e., the probabilities of $\left(y_j, q_j\right) =$ (0,0), (0,1), (1,0) or (1,1) for $j = 1,2,\cdots,n$, can be generally expressed as follows under different shift point M.

$$\begin{cases} d_j = \left(1 - y_j\right)\left(\theta_1(1-\alpha)\left(1-q_j\right) + (1-\theta_1)\beta q_j\right) \\ \quad +y_j\left(\theta_1\alpha\left(1-q_j\right) + (1-\theta_1)(1-\beta)q_j\right), \ M = 1 \\ e_j = \left(1 - y_j\right)\left(\theta_0(1-\alpha)\left(1-q_j\right) + (1-\theta_0)\beta q_j\right) \\ \quad +y_j\left(\theta_0\alpha\left(1-q_j\right) + (1-\theta_0)(1-\beta)q_j\right), \ M = i, \ i = 2,\ldots,n, j < i \\ f_j = \left(1 - y_j\right)\left(\theta_1(1-\alpha)\left(1-q_j\right) + (1-\theta_1)\beta q_j\right) \\ \quad +y_j\left(\theta_1\alpha\left(1-q_j\right) + (1-\theta_1)(1-\beta)q_j\right), \ M = i, \ i = 2,\ldots,n, j \geq i \\ g_j = \left(1 - y_j\right)\left(\theta_0(1-\alpha)\left(1-q_j\right) + (1-\theta_0)\beta q_j\right) \\ \quad +y_j\left(\theta_0\alpha\left(1-q_j\right) + (1-\theta_0)(1-\beta)q_j\right), \ M > n. \end{cases}$$

The inferred probability that the j-th ($j \leq n$) product is conforming (i.e., $q_j = 0$) or nonconforming (i.e., $q_j = 1$) with the given of the inspection information, $\mathbf{y}_n$, can be generally obtained as

$$\Pr\left(q_j \middle| \mathbf{Y}_n = \mathbf{y}_n\right) = \frac{\Pr\left(q_j, \mathbf{Y}_n = \mathbf{y}_n\right)}{\Pr(\mathbf{Y}_n = \mathbf{y}_n)},$$ where

$$\Pr\left(q_j, \mathbf{Y}_n = \mathbf{y}_n\right) = \left(1 - \overline{P}_1\right) d_j \prod_{k=1,k\neq j}^{n} D_k + \overline{P}_n g_j \prod_{k=1,k\neq j}^{n} G_k + \sum_{i=2}^{n} \left(\overline{P}_{i-1} - \overline{P}_i\right) \left\{ e_j \prod_{k=1,k\neq j}^{i-1} E_k \prod_{k=i}^{n} F_k \right\} \text{ for } j < i$$

and

$$\Pr\left(q_j, \mathbf{Y}_n = \mathbf{y}_n\right) = \left(1 - \overline{P}_1\right) d_j \prod_{k=1,k\neq j}^{n} D_k + \overline{P}_n g_j \prod_{k=1,k\neq j}^{n} G_k + \sum_{i=2}^{n} \left(\overline{P}_{i-1} - \overline{P}_i\right) \left(f_j \prod_{k=1}^{i-1} E_k \prod_{k=i,k\neq j}^{n} F_k \right) \text{ for } j \geq i.$$

Recall that $\Pr\left(q_j = 0 \middle| \mathbf{Y}_n = \mathbf{y}_n\right)$ represents the probability that product j whose actual quality is conforming with $\mathbf{y}_n$ is given. Since product j would be accepted only when its inspection result is conforming, i.e., $y_j = 0$. Therefore, the expected number of conforming products that will be accepted by inspection within N_n products can be obtained by the following:

$$a(N_n) = \sum_{j=1}^{n}(1 - y_j)\Pr(q_j = 0|\mathbf{Y}_n = \mathbf{y}_n) + \sum_{j=n+1}^{N_n}[\theta_0 x_j^{\mathbf{y}_n} + \theta_1(1 - x_j^{\mathbf{y}_n})]\,(1 - \alpha), \tag{2}$$

where the first term and the second term represent the expected number of conforming products that would be accepted by inspection within the first n products and the last $N_n - n$ products, respectively.

Next, the quality cost and revenue resulting from inspection errors are analyzed as follows. Since the acceptance or rejection of a product depends on its inspection result, the expected total cost for the former is

$$\sum_{j=1}^{n}\{c_0(1 - y_j)q_j\Pr(q_j|\mathbf{Y}_n = \mathbf{y}_n) + c_1 y_j\,(1 - q_j)\Pr(q_j|\mathbf{Y}_n = \mathbf{y}_n) + c_2 y_j q_j \Pr(q_j|\mathbf{Y}_n = \mathbf{y}_n) - r_0(1 - y_j)(1 - q_j)\Pr(q_j|\mathbf{Y}_n = \mathbf{y}_n)\}. \tag{3}$$

Since q_j is either 0 or 1, Eq.(3) can also be expressed as

$$\sum_{j=1}^{n}\{c_0(1 - y_j)\Pr(q_j = 1|\mathbf{Y}_n = \mathbf{y}_n) + c_1 y_j \Pr(q_j = 0|\mathbf{Y}_n = \mathbf{y}_n) + c_2 y_j \Pr(q_j = 1|\mathbf{Y}_n = \mathbf{y}_n) - r_0(1 - y_j)\Pr(q_j = 0|\mathbf{Y}_n = \mathbf{y}_n)\}. \tag{4}$$

On the other hand, for products n to $N_n - n$, when $\mathbf{y}_n$ is given, their inferred probability of being conforming can be obtained by $\theta_0 x_{n+j}^{\mathbf{y}_n} + \theta_1(1 - x_{n+j}^{\mathbf{y}_n})$ for $j = 1,2,\cdots,N_n - n$. Noting that type I inspection errors affect the quality cost and revenue of a conforming product, and they are given by αc_1 and $(1 - \alpha)r_0$, respectively. On the other hand, type II inspection errors affect the quality cost of a nonconforming product, which is given by $\beta c_0 + (1-\beta)c_2$. Therefore, the expected cost for product j can obtained as

$$[\theta_0 x_{n+j}^{\mathbf{y}_n} + \theta_1(1 - x_{n+j}^{\mathbf{y}_n})][\alpha c_1 - (1 - \alpha)r_0] + [1 - \theta_0 x_{n+j}^{\mathbf{y}_n} - \theta_1(1 - x_{n+j}^{\mathbf{y}_n})][\beta c_0 + (1 - \beta)c_2], \tag{5}$$

for $j = 1,2,\cdots,N_n - n$

As a result, $\phi(N_n)$ can be obtained as follows, where the summation of the quality control related cost and revenue is equal to the summation of $N_n c_{\mathrm{I}}$, Eq.(4) and Eq.(5).

$$\phi(N_n) = c_{\mathrm{R}}[\sum_{j=1}^{n}(N_n - j)\mu_j^{\mathbf{y}_n} + \sum_{j=n+1}^{N_n}(N_n - j)\tau_j^{\mathbf{y}_n}] + c_{\mathrm{m}}N_n + N_n c_{\mathrm{I}} + \sum_{j=1}^{n}\{c_0(1 - y_j)\Pr(q_j = 1|\mathbf{Y}_n = \mathbf{y}_n) + c_1 y_j \Pr(q_j = 0|\mathbf{Y}_n = \mathbf{y}_n) + c_2 y_j \Pr(q_j = 1|\mathbf{Y}_n = \mathbf{y}_n) - r_0(1 - y_j)\Pr(q_j = 0|\mathbf{Y}_n = \mathbf{y}_n)\} + \sum_{j=1}^{N_n-n}\{[\theta_0 x_{n+j}^{\mathbf{y}_n} + \theta_1(1 - x_{n+j}^{\mathbf{y}_n})][\alpha c_1 - (1 - \alpha)r_0] + [1 - \theta_0 x_{n+j}^{\mathbf{y}_n} - \theta_1(1 - x_{n+j}^{\mathbf{y}_n})][\beta c_0 + (1 - \beta)c_2]\} + c_{\mathrm{h}}\,[a(N_n)]^2/(2\lambda). \tag{6}$$

The first term and the last term given in Eq.(6) are the parts associated with restoration costs and inventory holding costs, respectively, and their derivation can be seen in Shih et al. [15].

Similar to Shih et al. [15], one can apply Lemma 2.2 of Puri and Singh [11] to determine the optimal production quantity since the following conditions holds: $a(N_n + 1) - a(N_n) > 0$, $\phi(N_n + 1) - \phi(N_n) > 0$ and with $\frac{\phi(N_n+1)-\phi(N_n)}{a(N_n+1)-a(N_n)}$

$= \omega(N_n)$ increasing for N_n (see Appendix for a proof). Consequently, the solution procedure for the optimal production quantity can be addressed as follows (see Shih et al. [15]). Firstly, searching n from 1, 2,…, until $\omega(1 + \lceil\delta(n+1)\rceil)a(1 + \lceil\delta(n+1)\rceil) - \phi(1 + \lceil\delta(n+1)\rceil) > c_s$ is satisfied. Secondly, find $N_0 = \max\{N_n | \omega(N_n)a(N_n) - \phi(N_n) \leq c_s\}$ for $N_n = n+1, n+2, \cdots, 1 + \lceil n\delta\rceil, 2 + \lceil n\delta\rceil, \cdots, \lceil\delta(n+1)\rceil$. Then, determining N_n^* as $\max\{1 + \lceil n\delta\rceil, N_0\}$ or $\max\{1 + \lceil n\delta\rceil, N_0 + 1\}$ by selecting the one which gives a smaller ECCP. Finally, setting N^* as N_n^*.

3. Numerical examples

The following parameter values were used to investigate the effect of the two types of the inspections errors on our proposed inspection and production (IP) policy: $\overline{P}_j = 0.998^{j^{1.003}}$, $\theta_0 = 0.9$, $\theta_1 = 0.1$, $\delta = 1.25$, $c_S = 15$, $c_R = 0.25$, $c_m = 0.5$, $c_I = 1.5$, $r_0 = 5$, $c_0 = 2$, $c_1 = 2.5$, $c_2 = 0$, $c_h = 1$ and $\lambda = 45$.

To determine the effect of inspection errors on the IP model, we first simulated a process shift point (SP) using a random number generated by MATLAB. Based on the obtained SP we further simulate the manufacturing variation and inspection errors using MATLAB random numbers to determine the products' actual quality status and inspection result. The averaged results for 100 simulations are summarized in Table 1, where $\Delta\% = \frac{V(N^*;\alpha=0,\beta=0) - V(N^*;\alpha,\beta)}{|V(N^*;\alpha,\beta)|} \times 100\%$ represents the percentage of the cost that is derived from ignoring inspection errors. From Table 1, one may make the following observations.

When SP is small (see Cases 3 and 15), there would seem to be more nonconforming products produced. For example, in Case 3, one has SP=6. After the 6-th product, these products have more chance of being non-conforming. Nevertheless, the number of non-conforming products will have been underestimated because of Type II errors, which will result in a smaller ECCP than when there is perfect inspection.

When SP is large (see Cases 5, 8, 10, 12-14 and 16), there would seem to be more conforming products, which results in a larger ECCP than would be the case with the perfect inspection. For example, in Case 8, one has SP=42. Although products 1 to 41 have more chance of being conforming, the number of conforming products is more likely underestimated because of Type I inspection errors. This results in the inferred process reliability being less than is the case with the perfect inspection. As a result, production will cease (on average at 23.6) earlier than when there is a perfection inspection (on average at 24.05). However, a larger SP often results in a larger production quantity even when inspection errors are considered.

When SP is moderate (see Cases 2, 4, 6, 7, 9 and 11), the presence of inspection errors exists results in a larger ECCP, due to the costs associated with the wrongly accepted nonconforming products and the wrong rejection of conforming products.

Finally, ignoring inspection errors is adequate only when SP is small enough (i.e., Cases 3 and 15) since their resulting $\Delta\%$ are tolerable at 1.25% and 4.37%, respectively.

Table 1
The effect of inspection errors on the IP model

				imperfect inspection			perfect inspection			
Case	α	β	SP	n	N^*	$V(N^*)$	n	N^*	$V(N^*)$	$\Delta\%$
1	0	0	62	-	-	-	19.41	25.63	-0.93857	0.00%
2		0.03	21	16.34	21.83	-0.51023	16.33	21.82	-0.57822	-13.33%
3		0.05	6	7.29	10.38	9.780736	7.17	10.25	9.90346	1.25%
4		0.07	19	15.57	20.79	0.11804	15.38	20.55	-0.14093	-219.39%
5	0.04	0	30	20.68	27.23	-0.72033	20.69	27.24	-0.87575	-21.58%
6		0.03	14	12.86	17.56	0.247228	12.24	16.77	0.28539	15.44%
7		0.05	22	18.37	24.43	-0.61374	17.53	23.37	-0.6881	-12.12%
8		0.07	42	17.76	23.6	-0.18191	18.13	24.05	-0.6666	-266.47%
9	0.08	0	12	12.1	16.48	2.108514	11.69	15.73	-0.06463	-103.07%
10		0.03	52	20.46	26.99	-0.25884	21.17	27.77	-0.857	-231.10%
11		0.05	20	17.48	23.05	-0.03749	16.56	21.85	-0.20359	-443.09%
12		0.07	30	17.1	22.77	-0.22383	18.66	24.72	-0.82046	-266.56%
13	0.12	0	30	18.98	25.12	-0.1863	20.54	27.09	-0.33929	-82.12%
14		0.03	52	20.35	26.81	0.135185	23.25	30.35	-1.03263	-863.87%
15		0.05	6	8.02	11.28	11.02123	7.21	10.37	11.50285	4.37%
16		0.07	52	18.15	24.03	0.072228	20.61	27.08	-1.19944	-1760.63%

4. Conclusions

This paper reformulates a previous inspection and production model proposed by Shih et al. [15] so that it considers two types of inspection errors. The numerical examples show that although the optimal number of inspections and the production quantity when there is perfect inspection are close to that of a system with inspection errors, there may still occur a huge percentage deviation in cost. Therefore, it is necessary to carefully evaluate the cost per conforming product when such inspection errors exist.

Acknowledgment

The authors would like to thank the two anonymous referees and the Editor, Professor Khushboo Malik, for their helpful comments.

Appendix

From Eq.(2), the product $N_n + 1$ would be a conforming product and to be accepted by inspection with the expected value of

$$a(N_n+1) - a(N_n) = \left[\theta_0 x_{N_n+1}^{\mathrm{y}_n} + \theta_1\left(1 - x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha) > 0. \quad (7)$$

Note that the last equation is decreasing in N_n since $x_{N_n+1}^{\mathrm{y}_n}$ is decreasing in N_n (please refer to Eq.(1) and also see Shih et al. [15]).

Next, from Eq.(6), it is easy to obtain

$$\begin{aligned}
\phi(N_n+1) = c_{\mathrm{R}}&\left[\sum_{j=1}^{n}(N_n+1-j)\mu_j^{\mathbf{y}_n} + \sum_{j=n+1}^{N_n+1}(N_n+1-j)\tau_j^{\mathbf{y}_n}\right] \\
&+ (c_{\mathrm{m}} + c_{\mathrm{I}})(N_n+1) \\
&+ \sum_{j=1}^{n}\left\{c_0(1-y_j)\Pr\left(q_j = 1 \middle| \mathbf{Y}_n = \mathbf{y}_n\right)\right. \\
&+ c_1 y_j \Pr\left(q_j = 0 \middle| \mathbf{Y}_n = \mathbf{y}_n\right) + c_2 y_j \Pr\left(q_j = 1 \middle| \mathbf{Y}_n = \mathbf{y}_n\right) \\
&\left.- r_0(1-y_j)\Pr\left(q_j = 0 \middle| \mathbf{Y}_n = \mathbf{y}_n\right)\right\} \\
&+ \sum_{j=1}^{N_n+1-n}\left\{\left[\theta_0 x_{n+j}^{\mathbf{y}_n} + \theta_1\left(1 - x_{n+j}^{\mathbf{y}_n}\right)\right][\alpha c_1 - (1-\alpha)r_0]\right. \\
&\left.+ \left[1 - \theta_0 x_{n+j}^{\mathbf{y}_n} - \theta_1\left(1 - x_{n+j}^{\mathbf{y}_n}\right)\right][\beta c_0 + (1-\beta)c_2]\right\} \\
&+ c_{\mathrm{h}}\left[a(N_n) + \left[\theta_0 x_{N_n+1}^{\mathrm{y}_n} + \theta_1\left(1 - x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha)\right]^2 / (2\lambda).
\end{aligned} \quad (8)$$

Consequently, from Eq.(6) and Eq.(8), one has

$$\begin{aligned}
&\phi(N_n+1) - \phi(N_n) \\
&\quad = c_{\mathrm{R}}\left[\sum_{j=1}^{n}\mu_j^{\mathbf{y}_n} + \sum_{j=n+1}^{N_n}\tau_j^{\mathbf{y}_n}\right] + c_{\mathrm{m}} + c_{\mathrm{I}} + \beta c_0 + (1-\beta)c_2 \\
&\quad + \left[\theta_0 x_{N_n+1}^{\mathrm{y}_n} + \theta_1\left(1 - x_{N_n+1}^{\mathrm{y}_n}\right)\right]\{\alpha c_1 - (1-\alpha)r_0 - \beta c_0 \\
&\quad - (1-\beta)c_2\}
\end{aligned}$$

$$+\frac{c_{\mathrm{h}}}{2\lambda}\left[\theta_0 x_{N_n+1}^{\mathrm{y}_n}+\theta_1\left(1-x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha)\left\{2a(N_n)\right.$$
$$\left.+\left[\theta_0 x_{N_n+1}^{\mathrm{y}_n}+\theta_1\left(1-x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha)\right\}.$$

Then, the ratio $\frac{\phi(N_n+1)-\phi(N_n)}{a(N_n+1)-a(N_n)}$ (denoted by $\omega(N_n)$), can be further shown to be increasing with N_n, as follows. From Eq.(7) and Eq.(8), one has

$$\omega(N_n)=\frac{\left\{c_{\mathrm{R}}\left[\sum_{j=1}^{n}\mu_j^{\mathrm{y}_n}+\sum_{j=n+1}^{N_n}\tau_j^{\mathrm{y}_n}\right]+c_{\mathrm{m}}+c_{\mathrm{I}}\right\}+\beta c_0+(1-\beta)c_2}{\left[\theta_0 x_{N_n+1}^{\mathrm{y}_n}+\theta_1\left(1-x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha)}$$
$$+\{\alpha c_1-(1-\alpha)r_0-\beta c_0-(1-\beta)c_2\}$$
$$+\frac{c_{\mathrm{h}}}{2\lambda}\left\{2a(N_n)+\left[\theta_0 x_{N_n+1}^{\mathrm{y}_n}+\theta_1\left(1-x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha)\right\}.$$

Similar to Shih et al. [15], it is easy to see that both the first term of $\omega(N_n)$ and the c_{h} term are increasing with N_n, where for the c_{h} term one has that

$$\frac{c_{\mathrm{h}}}{2\lambda}\left\{2a(N_n+1)+\left[\theta_0 x_{N_n+2}^{\mathrm{y}_n}+\theta_1\left(1-x_{N_n+2}^{\mathrm{y}_n}\right)\right](1-\alpha)\right\}-\frac{c_{\mathrm{h}}}{2\lambda}\left\{2a(N_n)+\left[\theta_0 x_{N_n+1}^{\mathrm{y}_n}+\theta_1\left(1-x_{N_n+1}^{\mathrm{y}_n}\right)\right](1-\alpha)\right\}=\frac{c_{\mathrm{h}}}{2\lambda}\left\{2\theta_1+(\theta_0-\theta_1)\left(x_{N_n+2}^{\mathrm{y}_n}+x_{N_n+1}^{\mathrm{y}_n}\right)\right\}(1-\alpha)>0.$$

References

[1] C. H. Chen and C. Y. Chou, "The asymmetric specification limit and quality investment settings for rectifying inspection plan," *Journal of Information and Optimization Sciences*, vol. 43, no. 2, pp. 323-330 (2022).

[2] I. Djamaludin, D. N. P. Murthy, and R. J. Wilson, "Quality control through lot sizing for items sold with warranty," *International Journal of Production Economics*, vol. 33, pp. 97–107 (1994).

[3] C. H. Kim, Y. Hong, and S. Y. Chang, "Optimal production run length and inspection schedules in a deteriorating production process," *IIE Transactions*, vol. 33, pp. 421–426 (2001).

[4] H. L. Lee and M. J. Rosenblatt, "Simultaneous determination of production cycle and inspection schedules in a production system," *Management Science*, vol. 33, no. 9, pp. 1125–1136 (1987).

[5] H. L. Lee and M. J. Rosenblatt, "A production and maintenance planning model with restoration cost dependent on detection delay," *IIE Transactions*, vol. 4, pp. 368–375 (1989).

[6] Y. H. Lin, J. M. Chen, and Y. C. Chen, "The impact of inspection errors, imperfect maintenance and minimal repairs on an imperfect production system," *Mathematical and Computer Modelling*, vol. 53, no. 9–10, pp. 1680–1691 (2011).

[7] E. L. Porteus, "Optimal lot sizing, process quality improvement and setup cost reduction," *Operations Research,* vol. 34, pp. 137-144 (1986).

[8] E. L. Porteus, "The impact of inspection delay on process and inspecion lot sizing," *Management Science,* vol. 36, no. 8, pp. 999-1007 (1990).

[9] F. Psarommatis and G. Bravos, "A holistic approach for achieving sustainable manufacturing using zero defect manufacturing: a conceptual framework," *Procedia CIRP*, vol. 107, pp. 107-112 (2022).

[10] F. Psarommatis and D. Kiritsis, "A hybrid decision support system for automating decision making in the event of defects in the era of zero defect manufacturing," *Journal of Industrial Information Integration*, vol. 26, p. 100263 (2022).

[11] P. S. Puri and H. Singh, "Optimum replacement of a system subject to shocks: A mathematical lemma," *Operations Research*, vol. 34, no. 5, pp. 782-789 (1986).

[12] M. B. Ramzan, C. W. Kang, and B. Sarkar, "A review paper on offline inspection of finished and semi-finished products and emerging research directions," *Yugoslav Journal of Operations Research*, vol. 26, no. 4, pp. 395-422 (2016).

[13] T. Raz, Y. T. Herer, and A. Grosfeld-Nir, "Economic optimization of off-line inspection," *IIE Transactions,* vol. 32, pp. 205–217 (2000).

[14] S. H. Sheu, Y. C. Chen, W. Y. Wang, and N. H. Shih, "Economic offline inspection with inspection error," *Journal of the Operational Research Society*, vol. 54, pp. 888–895 (2003).

[15] N. H. Shih, Y. S. Liao, and C. H. Wang, "Determining an economic production quantity under a zero defects policy," *Journal of the Operational Research Society*, vol. 69, no. 8, pp. 1309-1317 (2018).

[16] N. H. Shih and C. H. Wang, "An extended production and inspection model with non-rigid demand," *The Scientific World Journal,* p. 623075 (2013).

[17] S. T. Tseng, "Optimal preventive maintenance policy for deteriorating production systems," *IIE Transactions,* vol. 28, pp. 687–694 (1996).

[18] C. H. Wang and S. H. Sheu, "Simultaneous determination of the optimal production-inventory and product inspection policies for a deteriorating production system," *Computers and Operations Research,* vol. 28, pp. 1093–1110 (2001).

[19] C. H. Wang, "Optimal production and maintenance policy for imperfect production systems," *Naval Research Logistics,* vol. 53, no. 2, pp. 151–156 (2006).

[20] C. H. Wang and R. H. Yeh, "Utilizing an approximative solution to obtain the real optimal solution for a production and inspection model," *Applied Mathematical Modelling*, vol. 33, pp. 3769–3775 (2009).

[21] R. H. Yeh and T. Chen, "Optimal lot size and inspection policy for products with warranty," *European Journal of Operational Research,* vol. 174, no. 2, pp. 766–776 (2006).

Received August, 2022

Revised December, 2022