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A variable neighborhood search for a new multi-depot green vehicle routing problem with multiple starting points and distinct serving areas for recyclable waste collection

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Abstract

The transportation of recyclable waste collection is significant for reducing the effect of greenhouse gases. In this study, we investigate a multi-depot green vehicle routing problem in which one group of vehicles starts their routes from different points – excluding depots – and the other group of vehicles starts from one of the multiple depots and additional the serving areas of vehicles are predetermined. Although the groups of vehicles serve different areas, the problem is merged together since the depots have capacity. Furthermore, the objective of the problem is to minimize carbon emissions related to traveling distance and vehicle load on the route. We modeled this as a mixed integer programming problem and we developed variable neighborhood search algorithms with two different strategies. The performance of both approaches was evaluated using test instances which have been modified from published literature.

Subject Classification: *(2010) 90B06, 68T20, 90C11, 90C59, 90C90.*

Keywords: *Variable neighborhood search, Green vehicle routing problem, Multi-depot, Multiple starting, Serving area, Recyclable waste collection.*

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1. Introduction

Climate change affects the average temperature of our world and is called global warming. The main reason for global warming is greenhouse gases (carbon dioxide, methane, nitrous oxide, and fluorinated gases) that come from burning fossil fuels, deforestation, and farming etc. Among greenhouse gases, carbon dioxide makes up the biggest percentage with 81% [12]. Powering cars and trucks with fossil fuels causes an increase in the percentage of carbon dioxide in the atmosphere [47]. Thus, the transportation industry plays an important role in reducing these effects, and one of the main issues in transportation is the vehicle routing problem (VRP).

The VRP was introduced by Dantzig and Ramser [8] and the aim was to find the optimal routes that minimize the total cost or distance. The main constraints of the problem are as follows: each vehicle starts and ends at the depot, the capacity of each vehicle is not exceeded, and each customer is served by only one vehicle. By considering new constraints, variants of VRP have been developed in published literature. One of the well-known extensions of VRP is the Multi-Depot Vehicle Routing Problem (MDVRP). In MDVRP, every customer is visited by a vehicle based at one of several depots. For other variants of the VRP, we refer readers to the book by Toth and Vigo [44].

Minimizing the impact of pollution, such as the greenhouse gas emission caused by transportation, is a significant environmental issue other than cost minimization, and in published literature there are a considerable number of studies that consider the environmental factors involved with VRP and its variants where the problem is defined as green VRP (GVRP) (see e.g. [31]). On the other hand, recycling itself has a green aspect as a means of processing used materials to make them reusable instead of going to waste. In managing the recyclable waste systems, the collection of recyclable materials from different sources such as from houses and firms are one of the activities. In the collection process, the materials are collected from sources by a vehicle, and then these materials are brought together at some points such as depots before recycling. The previously mentioned nature of the system makes recyclable waste collection (RWC) a routing problem. Hence the RWC has been studied as a type of VRP with cost minimization in the published literature (see [43]). However, greenhouse gases are naturally produced during the RWC activities, and so the GVRP is also related to this problem along with cost minimization.

Our motivation in this study is RWC problem in a recycling facility. We considered this problem as multi-depot GVRP (MDGVRP) with additional aspects. We dealt with MDGVRP having heterogeneous vehicles by considering that each vehicle does not need to start its route from one of the depots. The vehicles' routes can start from different points excluding depots, e.g., the driver's home and then end at one of the depots. This gives flexibility to drivers and reduces the green gas emission, especially when starting from their homes, since there is no need to get on the depot. In addition, there are areas to be served where the vehicles are predetermined, e.g. one area of vehicles is within urban area and the other area of vehicles is within organized industrial site. Moreover, the depots have capacity and in this case, routing problem of the vehicles in the predetermined areas are merged in one problem since the returning depot is based on the capacity of the depots. Thus, the MDGVRP with multi-starting points and distinct serving areas (MSDSA) is defined as MDGVRP-MSDSA.

The rest of the study is organized as follows. Section 2 provides a literature review. The mathematical model of the MDGVRP-MSDA is presented in Section 3, and the proposed VNSs are explained in Section 4, while the computational results are given in Section 5. Some conclusions are drawn in Section 6.

2. Literature Review

In this section, we review published literature in two subsections: green vehicle routing problems, and vehicle routing problems in waste collection. Then, we emphasize our contribution to the related literature.

2.1 *Green vehicle routing problems*

Recently, the term green (GVRP) has been used for VRPs aimed at environmental issues (see [31]). Several variants of GVRP and algorithms have been studied in published literature. These studies were based on either energy minimizing vehicle routing problem (EMVRP) or cumulative VRP (cumVRP), on fuel cost rating VRP (FCVRP), or on pollution routing problem (PRP). We start with the studies related to EMVRP. Kara et al. [21] proposed the energy minimizing vehicle routing problem (EMVRP) that minimizes the total energy consumption related to the load of each vehicle and the distance travelled. They developed a mathematical model and solved examples of EMVRP using Cplex 8.0. Then, Xiao et. al. [49] extended

EMVRP by taking the impact of load and distance into account with FCVRP, and a simulated annealing algorithm was developed to solve the FCVRP. After that, Gaur et. al. [13] examined four variants of the EMVRP and proposed a constant factor approximation algorithm. Moreover, Li et. al. [27] considered minimizing the fuel consumption, in other words, carbon dioxide emissions per kilometer as the objective of the tractor and semitrailer routing problem with a many-to-many demand, which is an extension of the roll on-roll off VRP. A three-stage heuristic algorithm with a savings algorithm, improvements, and local search phases was developed. To solve cumVRP, Cinar et. al. [7] extended cumVRP for multiple trips and limited duration, and proposed a hybrid methodology that obtained an initial solution with the Clarke-Wright algorithm and used simulated annealing for the construction of routes. Gaur et. al. [14] studied cumVRP with stochastic demands and proposed randomized approximation algorithms. Xiao and Konak [50] proposed a mathematical model with minimal fuel consumption for the case of a heterogeneous VRP and scheduling for time-varying traffic congestion. The hybrid algorithm consisted of a mixed-integer programming optimization and iterative neighborhood search.

As for PRP, Bektas and Laporte [3] described the problem based on the trade-off between several parameters: the speed of the vehicle, the cost of the emissions, the cost of the drivers, and the total cost. Mathematical models were developed, and all experiments were conducted using CPLEX 12.1. Demir et. al. [9] proposed a two-phase algorithm that used an adaptive large neighborhood search algorithm as the first phase and a recursive algorithm for the second phase. Koc et. al. [24] extended PRP using the fleet size by adding a new objective: minimizing the sum of vehicle fixed costs and developing a hybrid evolutionary algorithm. Ehmke et. al. [12] assumed that the speed of the vehicle was variable and time-dependent in urban areas due to the speed of the traffic. A Dijkstra-like label-setting algorithm was used to find the shortest path, and then the routing problem was solved by the tabu search algorithm. Suzuki [40] used PRP with multiple objectives: basic fuel consumption related to the distance and extra fuel consumption related to the vehicles load. They developed an algorithm using simulated annealing for approximating a Pareto frontier and tabu search for improving each element of the frontier.

Besides these studies, Poonthalir and Nadarajan [34] used GVRP by considering a dual objective: minimization of the route cost and fuel consumption, and called the problem F-GVRP. Additionally, the speed varied and thus the fuel consumption rate was not constant. Then, particle swarm optimization was developed to solve the F-GVRP. Behnke et. al. [5] studied VRP based on emission-oriented EVRP and multigraph approach. The emissions were computed using the Kirschtein and Meisel [23] model where the speed of the vehicles was taken as one parameter. Then, a column generation approach was developed.

For the multi-depot GVRP (MDGVRP), Jabir et. al. [19] modeled MDGVRP by minimizing emission costs related to distance and the vehicle load. Then, metaheuristics based on ant colony optimization and VNS were developed to solve the MDGVRP. Peng et. al. [32] studied MDGVRP and developed a hybrid evolutionary algorithm that combined VNS and ant colony optimization. Reyes-Rubiano et. al. [37] approached MDGVRP from economic, environmental, and social aspects. Then they proposed a mathematical model and a biased-randomized VNS.

Additionally, in some studies, the environmental issue was concerned with a second objective. Wang et. al. [45] modeled MDGVRP with soft time windows and a dual objective. The first objective was minimizing the total energy consumption and was modeled as an EMVRP. The second objective was minimizing customer dissatisfaction, which was related to arriving late at a customer site. Small test problems were solved by an augmented constraint method and two non-dominated sorting genetic algorithm-II (NSGA-II) based heuristics were developed for large-scale problems. Wang et. al. [46] studied a dual objective MDGVRP with a sharing strategy where a vehicle starts from one depot and returns to the depot nearest to the last customer. The objectives were to minimize carbon emission and operating costs. A hybrid algorithm by Clarke-Wright, sweep, and multi-objective particle swarm optimization algorithms were designed.

2.2 *Vehicle routing problems in waste collection*

A VRP for waste collection can be formulated as an arch routing problem (ARP) and also modeled as a capacitated ARP (CARP) (see e.g. [26]). Recently, Zsigraiova et. al. [52] used VSRP to investigate the operating cost and pollution reduction in the Geographical Information System (GIS) environment. Furthermore, Kim et. al. [22] considered a waste collection VRP with time windows (WCVRPTW) and also incorporated multiple disposal trips and drivers’ lunch breaks. Benjamin and Beasley [5] used a WCVRPTW to consider drivers’ rest periods and proposed two metaheuristics: tabu search and variable neighborhood search. Wy et. al. [48] proposed a roll on–roll off WCVRPTW and developed a large

Table 1
Reviewed literature

Study	Problem	Objective function	Solution method	Waste collection	Multi depot	Multiple starting points
Teixeira et al. [43]	Periodic VRP	Total cost	Heuristic algorithms	+	-	-
Kim, et al. [22]	WCVRPTW	Route compactness and workload balancing	Insertion algorithm and a clustering-based algorithm	+	-	-
Kara et al. [21]	EMVRP	Total energy consumption	Mathematical model	-	-	-
Benjamin and Beasley [5]	WCVRPTW	Total number of vehicles used	Tabu search and VNS	+	-	-
Bektas and Laporte [3]	PRP	Emission, fuel, travel times and their costs	Mathematical models	-	-	-
Xiao et al. [49]	EMVRP	Fixed costs and fuel costs	Simulated annealing algorithm	-	-	-
Demir et al. [9]	PRP	Fuel, emission and driver costs.	Adaptive large neighborhood search	-	-	-
Gaur et al. [13]	EMVRP	Fuel consumption	Constant factor approximation algorithm	-	-	-
Zsigraviova et al. [52]	VSRP	Operating cost and pollution	GIS optimization tool	+	-	-

Contd...

Wy et al. [48]	WCVRPTW	Number of required tractors and their total route time	Large neighborhood search	+	-	-
Hemmelmayr, et al. [20]	Periodic VRP	Total travel cost	Hybrid algorithm	+	+	-
Ramos et al. [36]	VRP	Distance and carbon emissions	Decomposition solution method	+	+	-
Koc et al. [24]	PRP	Total cost	Hybrid evolutionary algorithm	-	-	-
Li et al. [27]	FCVRP	Carbon dioxide emissions	Three-stage heuristic algorithm	-	-	-
Cinar et al. [7]	cumVRP	Total cost	Hybrid algorithm	-	-	-
Gaur et al. [14]	cumVRP	Fuel consumption	Randomized approximation algorithms	-	-	-
Ehmke et al. [10]	PRP	Time-dependent emissions	Dijkstra-like label-setting algorithm and Tabu search	-	-	-
Xiao and Konak [50]	GVRP	Fuel consumption	Hybrid algorithm	-	-	-
Suzuki [40]	PRP	Fuel consumption	Simulated annealing and tabu search	-	-	-
Markov et al. [28]	GVRP	Total cost	MILP and multiple neighborhood search	+	+	-
Rabbani et al. [35]	MDGVRP	Total cost	Hybrid genetic algorithm	+	+	-
Jabir et al. [19]	MDGVRP	Emission costs	Metaheuristics based on ant colony optimization and VNS	-	+	-

Contd...

Poonthalir and Nadarajan [34]	GVRP	Route cost and fuel consumption	Particle swarm optimization	-	-	-
Wang et al. [45]	MDGVRPTW	Total energy consumption and customer dissatisfaction	Augmented constraint method and NSGA-II	-	+	-
Molina et al. [30]	WCVRPTW	Pollutant emissions	Variable neighborhood tabu search algorithm	+	-	-
Wang et al. [46]	MDGVRP	Total cost	A hybrid algorithm	-	+	-
Taslimi et al. [42]	CVRP	Transportation risk	Decomposition based heuristic algorithm	+	-	-
Gruler et al. [15]	WCVRPTW	Total cost	Simheuristic	+	-	-
Peng et al. [32]	MDGVRP	Total cost	Hybrid evolutionary algorithm based on VNS and ant colony optimization	-	+	-
Reyes-Rubiano et al. [37]	MDGVRP	Total cost	Mathematical model and a biased-randomized VNS	-	+	-
Behnke et al. [4]	Emission-oriented VRP	Total emission	Column generation approach	-	-	-
Aliahmadi et al. [1]	MDGVRP	Total cost and completion time	Mathematical model and NSGA-II	+	+	-
Our paper	MDGVRP-MSDSA	Total energy	Variable Neighborhood Search	+	+	+

neighborhood search. Molina et. al. [30] considered a waste collection VRP (WCVRP) to reduce pollutant emissions and proposed a variable neighborhood tabu search algorithm. Recently, Taslimi et. al. [42] studied medical waste collection by considering the minimization of the transportation risk depending on the weight of medical waste on the vehicle and the storage risk of medical waste at the health center. Gruler, Pérez-Navarro, Calvet and Juan [15] considered WCVRP with stochastic travel times and developed a simheuristic algorithm based on Monte-Carlo simulation.

Besides waste collection, Teixeria et al. [43] dealt with RWC as a periodic VRP containing a sequence of routes for multiple days and the objective was to minimize the total cost. Ramos et al. [36] investigated RWC by considering distance and carbon emissions. Additionally, the problem had multiple products and multiple depots, and each depot had a single service area. Then, the problem was solved with a decomposition solution method. Markov et. al. [28] studied RWC with intermediate facilities. Also, they investigated a heterogeneous fixed fleet and in addition, they considered the assignment of the destination depots as flexible.

2.3 *Our contribution*

The reviewed literature related to VRP is briefly categorized based on waste collection, multi-depot, and multiple starting points and summarized in Table 1 where "+" and "-" represent the relevant paper contains or do not contain the relevant category, respectively. As can be seen from table, a limited number of contributions currently addresses MDVRP in waste collection ([1], [20], [35]). Furthermore, we introduce a novel MDGVRP, which is inspired by a recycling facility. It is characterized by a heterogeneous vehicle fleet. Unlike the classical MDGVRP, some of the vehicles can start their routes from different points excluding depots. To best of our knowledge, this study is the first that formulated the multiple starting points on the MDGVRP context. Additionally, a mixed integer programming (MIP) model is proposed, and the performance of MIP is tested on instances which are small-sized, and thus MIP can obtain optimal solutions of the instances or is able to give upper/lower bounds of them. VNSs with various initial solution strategies: random generation and nearest neighborhood-based generation (VNS-R, VNS-NN) are proposed, and a comparison is made to see the effect of initial solutions in terms of random and greedy performance. New test problems for the proposed

MDGVRP-MSDSA are generated by modifying test problems from published literature. Computational experiments on test instances derived from the literature yield high quality solutions.

3. Definition and Formulation of the Problem

The MDGVRP proposed in this study is motivated by the RWC from a recycling facility. The depots in this problem are where the recyclable wastes are collected. The collecting points are located in two districts, and each district is served by specific vehicles. The first and second districts cover recyclable wastes collected from the houses in the city center and industrial zones, respectively. The routes of vehicles serving the first district start from different points excluding depots (e.g., drivers' home) and they end at one of the depots. The starting and ending point for each route serving the second district is the depot. These two routing problems are discussed together in this study since each depot has a limited capacity. Thus, the decision in choosing the end depot is related to the capacity of the depots and the objective function, which is to minimize carbon emissions related to traveling distance and vehicle load on the route.

To understand MDGVRP-MSDSA better, a small problem with 15 customers, 2 depots and 4 vehicles is explained in detail. The representative solutions are depicted in Figure 1. Red triangles, yellow stars, blue dots,

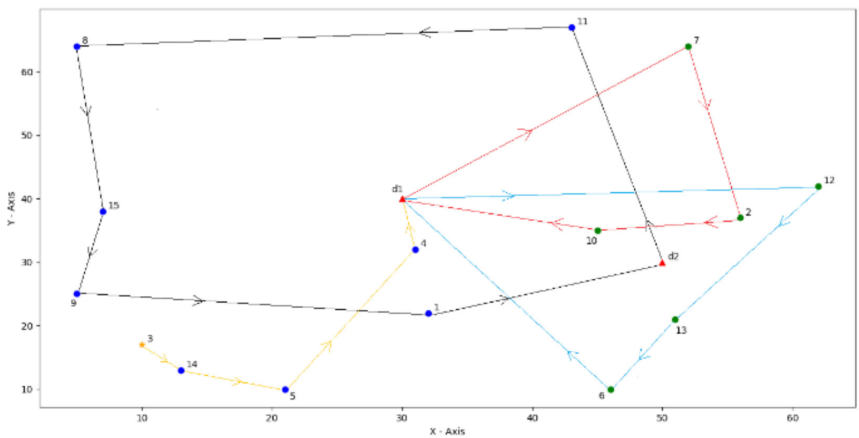


Figure 1

The representative solutions for a small problem with 15 customers, red lines, blue lines, yellow lines and black lines are belong to vehicles 1, 2, 3, 4, respectively.

Table 2

The solution in detail for a small-scale problem

Vehicle No	Serving Area of Each Vehicle	Starting point or Depot	Route
1	Area 1	d1	d1-7-2-10-d1
2	Area 1	d1	d1-12-13-6-d1
3	Area 2	3	3-14-5-4-d1
4	Area 2	d2	d2-11-8-15-9-1-d2

and green dots indicate depots, starting points – e.g. the drivers' homes – the customers in the first region, and the customers in the second region, respectively. The arrows show the routes of the vehicles.

The routes of vehicles are given in Table 2. As can be shown from this table, the routes of vehicles 1, 2 and 4 started from depots and ended at the same depots. In contrast, vehicle 3 started from node 3, which was a starting point, and finished at a depot (d1).

The fundamental assumptions of the mathematical model are as follows:

- Each vehicle serves its own service area.
- Transport between depots is not allowed.
- Each vehicle is used for one route.
- Each customer is served by one vehicle.
- If the route starts from a depot, then the route ends at the same depot.
- If a route starts from multiple starting points (e.g. driver's home), then the route finishes at one of the depots.
- Each depot has a limited capacity for collecting items.

The MDGVRP is modelled on a complete directed graph $G=(N,A)$, where $N=\{1,2\ldots p\}$ is the set of nodes. The depots are the subset $N_d \subset N$, and each depot $d,m \in N_d$ has a capacity Q_d^w. $N_m \subset N$ is the subset with customers in the first districts, $N_h \subset N$ the subset showing the starting points excluding depots in the first district, and $N_s \subset N$ the subset with customers in the second district. Each customer $i \in N_m \cup N_s$ has a fixed non-negative demand p_i. $A=\{(i,j) : i,\ j \in N,\ i \neq j\}$ is the set of arcs. For every arc (i,j), dis_{ij} represents the distance from node i to node j. A limited fleet of heterogeneous vehicles is located at multiple distinct points: depots and driver's home, and the set of vehicles is denoted by $V = \{1,2\ldots n\}$. The vehicle set V is partitioned into two subsets $V_m = \{1,2...l\}$ and $V_s = \{l+1...n\}$, respectively, i.e. the set of vehicles serving the first district and the set of vehicles serving the second district. The set of vehicles where the starting points were the driver's home are denoted by $V_h \in V_m$. Each vehicle k has a curb weight W_k and a capacity Q_k^v. M is an enough large number. The decision variables of problem are given as follows:

Decision variables:

x_{dijk} :	a binary variable which is equal to 1 if there is an arc from node *i* to node *j* using vehicle *k* from depot *d*.
c_{ik} :	the load of vehicle *k* after visiting node *i*
f_{dijk} :	the load of vehicle *k* after traveling between node *i* and *j* from depot *d*

Objective function:

$$\min z = \sum_m \sum_i \sum_j \sum_k dis_{ij} * f_{dijk} \tag{1}$$

In the above formulation, objective function (1) refers to minimized carbon emissions related to traveling distance and vehicle load on the route.

Constraints:

$$\sum_{d \in N_d} \sum_{j \in N_m} x_{dijk} = 1 \qquad \forall i,k, \qquad k \in V_m, k \notin V_h, i \in N_d \tag{2}$$

$$\sum_{d \in N_d} \sum_{j \in N_s} x_{dijk} = 1 \qquad \forall i,k, \qquad k \in V_s, k \notin V_h, i \in N_d \tag{3}$$

$$\sum_{d \in N_d} \sum_{j \in N_m} x_{dijk} = 1 \qquad \forall i,k, \qquad k \in V_h, i \in N_h \tag{4}$$

$$\sum_{d \in N_d} \sum_{i \in N_m} x_{dijk} = 1 \qquad \forall j,k, \qquad k \in V_m, j \in N_d \tag{5}$$

$$\sum_{d \in N_d} \sum_{i \in N_s} x_{dijk} = 1 \qquad \forall j,k, \qquad k \in V_s, j \in N_d \tag{6}$$

$$\sum_{d\in N_d}\sum_{j\in N_m}\sum_{k\in V_m} x_{dijk} = 1 \qquad \forall i, \qquad i\in N_m \tag{7}$$

$$\sum_{d\in N_d}\sum_{j\in N_s}\sum_{k\in V_s} x_{dijk} = 1 \qquad \forall i, \qquad i\in N_s \tag{8}$$

$$\sum_{d\in N_d}\sum_{j\in N_m}\sum_{k\in V_m} x_{dijk} = 1 \qquad \forall j, \qquad j\in N_m \tag{9}$$

$$\sum_{d\in N_d}\sum_{i\in N_s}\sum_{k\in V_s} x_{dijk} = 1 \qquad \forall j, \qquad j\in N_s \tag{10}$$

$$\sum_{i\in N\setminus N_d} x_{mijk} = \sum_{i\in N\setminus N_d} x_{mjik} \qquad \forall j,d, \qquad d\in N_d, j\in N\setminus N_d \tag{11}$$

$$\sum_{i\in N_d} c_{ik} \ge W_k - M\left(1-\sum_{d\in N_d}\sum_{i\in N_d} x_{dijk}\right) \qquad \forall j,k, \quad j\in N\setminus N_d, k\in V \tag{12}$$

$$c_{ik} \ge W_k + p_i - M\left(1-\sum_{d\in N_d} x_{dijk}\right) \qquad \forall i,j,k \quad i\in N_h \tag{13}$$

$$c_{ik} \ge c_{jk} + p_i - M\left(1-\sum_{d\in N_d} x_{djik}\right) \qquad \forall i,j,k \quad i\in N\setminus N_d, j\in N\setminus N_d, k\in V \tag{14}$$

$$\sum_{d\in N_d}\sum_{i\in N_d} f_{dijk} \ge W_k - M\left(1-\sum_{d\in N_d}\sum_{i\in N_d} x_{dijk}\right) \qquad \forall j,k, \quad j\in N\setminus N_d, k\in V \tag{15}$$

$$\sum_{d\in N_d} f_{dijk} \ge c_{ik} - M\left(1-\sum_{d\in N_d} x_{dijk}\right) \qquad \forall i,j,k, \quad i\in N\setminus N_d,\ j\in N, k\in V \tag{16}$$

$$\sum_{d}\sum_{i}\sum_{j} x_{dijk} * p_i \le Q_k^v \qquad \forall k, \qquad k\in V \tag{17}$$

$$\sum_{i}\sum_{j}\sum_{k} x_{dijk} * p_i \le Q_d^w \qquad \forall d,\ d\in N_d \tag{18}$$

$$c_{ik}\in R^+ \qquad \forall i,k, \qquad i\in N, k\in V \tag{19}$$

$$f_{dijk}\in R^+ \qquad \forall i,j,d,k, \quad i,j\in N, \quad d\in N_d, k\in V \tag{20}$$

$$x_{dijk}\in\{0,1\} \qquad \forall i,j,k,d, \quad i,j\in N, \quad k\in V, d\in N_d \tag{21}$$

Equations (2) and (3) guarantee that each vehicle starts its route from the depot. Equation (4) is formulated for vehicles that start their route from the driver's home. Equations (5) and (6) ensure that each vehicle returns to the departing depot. Equations (7) to (10) show that each vehicle arrives at and leaves from the customer location at once. Flow constraint is declared in Equation (11): if vehicle k arrives at customer i, it should leave it. Constraints (12) and (13) are formulated for the vehicle load, which departs from the depot and driver's home, respectively. In constraints (14) to (16) (modified from [11]), the vehicle loads on arc (i, j) are calculated. Constraint (17) ensures that the total demand of each route does not exceed the vehicle capacity. Constraint (18) states the capacity restrictions of the depots. Finally, constraints (19) and (20) ensure that the decision variables c_{ik} and f_{dijk} are non-negative where constraint (21) demonstrates the decision variable x_{dijk} as binary variable.

4. The Variable Neighborhood Search Algorithm for MDGVRP-MSD-SA

The variable neighborhood search (VNS) was originally proposed by Mladenovic and Hassen [29] (see also [16-17]). There have been studies on the VRP (see [2]) along with its variants, e.g. MDVRP (see [25]), MDVRP with time windows (see [33]), MDVRP with a heterogeneous fleet (see [38]), MDVRP with a heterogeneous fleet and time windows (see [51]), VRP with a heterogeneous fleet (see [18]), and multi-objective open VRP (see [39]).

The VNS starts with a single initial solution and then the initial solution is improved by systematic change within neighborhood structures to escape from local optima (diversification) and local searches, to find better solutions (intensification).

In this section, we explain the VNSs developed to solve MDGVRP-MSDSA. Firstly, we define the ways for generating initial solutions: random generation and nearest neighborhood-based generation, called VNS-R and VNS-NN. At each run, we use one of these initial solutions to see the performance – in terms of both the solution quality and solution time – of the algorithm via the means of the different initial solutions. Then, we apply two operators for neighborhood procedure: the 1-1 swap-inter-route (N_1) and the 1-insertion-inter-route (N_2) and the 1-insertion-inter-route. Finally, we describe five operators for local search procedure: the 1-1 swap-inter-route (L_1), the 1-1 swap-intra-route (L_2), the 1-insertion-intra-route (L_3), the 1-insertion-inter-route (L_4), and the inversion-intra-route (L_5). These structures are explained in the next subsections.

The flow chart of the overall algorithm is represented in Figure 2. The algorithm starts with defining the parameters as operators of the neighborhood procedure (N), operators of local search procedure (L) and the total number of operators (num_K, num_B) of N and L. The counter of the neighborhood procedure (n) is set to 1. Then, an initial solution is generated according to random search-VNS-R or nearest neighborhood search-VNS-N and global best solution is assigned as the current best solution. After that, the neighborhood and local search procedures continue until num_K and num_B are reached. In the neighborhood search procedure, the operator $N_n \in \{N_1, N_2\}$ is applied, then the counter of the local search procedure (m) is set to 1 and local search procedure is started. In local search procedure, the operator is selected $L_m \in \{L_1, L_2, L_3, L_4, L_5\}$. If a better solution is obtained after local search procedure, then the best

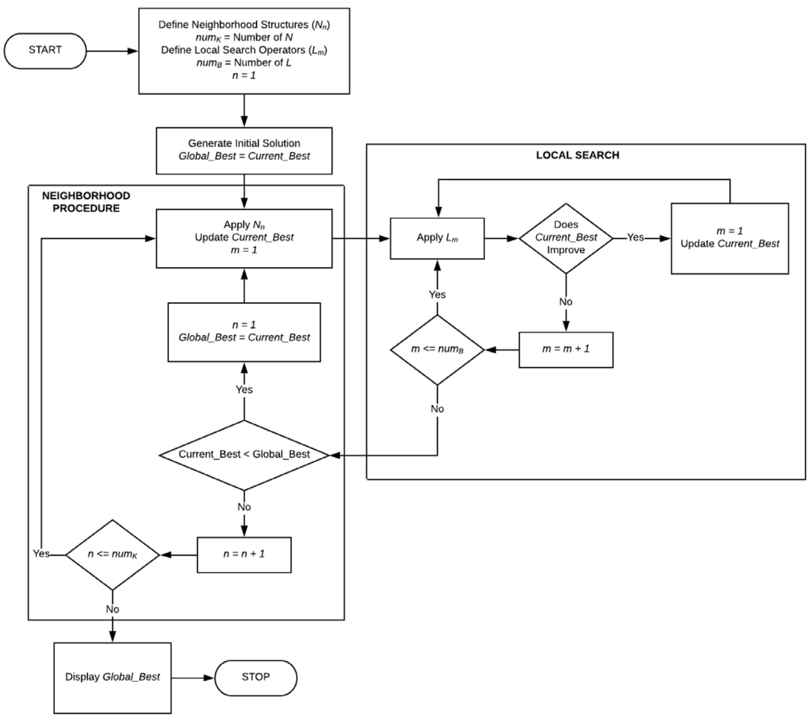


Figure 2

The flow chart of the VNS algorithm for MDGVRP-MSDSA

solution is updated and m is started from 1, otherwise m is increased 1. The local search procedure ends when m is get num_B. After that, if the current best solution is better than the global best solution, then the global solution is updated and n is became 1 again, otherwise n is increased 1. Additional, if n is reached num_K, then the algorithm is terminated.

4.1. *Generating initial solutions for VNS-R and VNS-NN*

The initial solutions were generated by using two different strategies: random generation for VNS-R and nearest neighborhood-based generation for VNS-NN. In general, a random initial solution finds a feasible solution quicker than a greedy initial solution. However, the random initial solution may require a much larger number of iterations than the greedy initial solution. Thus, it may be concluded that there is a trade off [41]. Considering both VNS-R and VNS-NN, the aim was to compare the performance of VNS in terms of the quality of solution and computational time. The greedy algorithm was defined as a nearest neighborhood search, which is easier to implement than other greedy algorithms such as Clark-Wright etc., and they have the capability of finding a feasible solution in a reasonable time.

In the random generation VNS-R, the initial solution is generated randomly under the problem constraints. Firstly, the vehicles started from the depots are assigned to routes, and the routes of the vehicles which are started from the starting point are formed. Then, customers with random selection are designated to the vehicles until the capacities of the vehicles are met. Additional, the returning depots are selected randomly if the capacity of the selected depot is available. Finally the energy consumption of the randomly generated solution is calculated according to Eq. (1).

In the nearest neighborhood-based generation VNS-NN, similar to random generation, the vehicles and the starting depots or the starting points are matched. Different to the random selection, the energy values (e_{ij}) going from current point i to another point j (similar as in Eq. (1), but for only two point) for the relevant vehicle k starting from the depot m are calculated to choose the next customer to assign the route as $e_{ij} = dis_{ij} * f_{mijk}$. The minimum energy value is picked as the nearest neighborhood point and the assignment is made if the capacity is suitable. Similarly, the energy values for returning depots are calculated and the returning depot is determined under the capacity of depot of constraint. Finally the energy consumption of the initial solution is calculated via Eq. (1).

4.2. *Neighborhood procedure*

Two operators are used to define the neighborhood of the VNSs. In the *1-1 swap-inter-route ()*, a route is chosen randomly and the first customer on the route is selected. Then a search for another customer on another route is carried out to find if the two demand points can be swapped under the capacity constraints. If not, the next customer on the route is selected and the procedure is continued until a suitable swap is made. In the *1-insertion-inter-route* (N_2), a route and a customer on this route are selected similarly to N_1. Then the customer is inserted into a first possible route, if that possible route exists. If not, N_2 is applied to the latter customer until an insertion occurs. When applying N_1 and N_2, the constraints of the problem are enforced.

4.3. *Local search procedure*

The following operators for local search are repeated but limited to a certain number of iterations. If a solution having a better objective value is found in the meantime, then the current local search is restarted. Otherwise, the current local search is terminated, and the next local search is launched. The local search strategies apply the constraints of the problem.

The *1-1 swap-inter-route* (L_1) is similar to the neighborhood N_1 with one difference: the customers are chosen randomly. However, the local search L_1 is repeated with a predetermined iteration or until a better solution is found. The *1-1 swap-intra-route* (L_2) starts by choosing a random route and then the swap is made between two random points of demand within the same route. In the *1-insertion-intra-route* (L_3), a random route is selected first, and the local search L_3 tries to put a point of demand on this route into another position of the same route. The *1-insertion-inter-route* (L_4) is similar to the neighborhood N_2. However, both the route and the customer are selected randomly. Then L_4 is repeated in the same way as L_1. In the *inversion-intra-route* (L_5), a route and two points of demand on this route are selected Then the sequence for all points of demand between these two points are inverted.

5. Computational Experiments

The proposed model formulation was coded using the GAMS modeling program and run using the CPLEX solver. The VNS algorithms were coded in Python 3.7. The time limit for proposed methods was stated to be 7200 seconds (2 hours) for each problem**.** All the experiments were conducted on a computer having an Intel® **Core(TM) i7-9750H CPU** dual core 2. 60 GHz 2.59 GHz and 16 GB RAM. The features of the test instances and detailed results of the MIP model and VNSs are presented in the following subsections named as Test problems and Test results.

5.1. *Test problems*

There are no standard benchmark instances for MD-GVRP, because it is tailored to real-life practices. In these situations, it is common to generate random data which is obtained by a company or adopted from literature. In this study, test instances are adopted from Cordeau et al. [7].

For small-scale test problems, the original coordinates of the instances and locations of the depots were selected based on the number of customers and depots. Then, these were grouped into two districts: the nodes for which the value of the x-axis was greater than the value were included in the first district, and the remaining nodes were included in the second district. The value is defined for each problem to obtain the same number of customers in both districts. The nodal starting points were selected randomly. The number of vehicles, and the capacity of each vehicle and each depot were adjusted according to the total demand. The features of the small-scale instances are given in Table 3.

Table 3

The features of small-scale test instances

Problem no	Number of depots	Number of customers	Number of vehicles	Number of customers in district 1	Number of customers in district 2	Number of starting points excluding depots
S-2-1	2	10	3	6	4	1
S-2-2	2	10	3	5	5	1
S-2-3	2	10	3	6	4	1
S-2-4	2	10	3	6	4	1
S-2-5	2	15	4	9	6	1
S-2-6	2	15	4	9	6	1
S-2-7	2	15	4	9	6	1
S-2-8	2	15	4	6	9	1
S-3-1	3	20	5	12	8	1

Contd...

S-3-2	3	20	5	12	8	1
S-3-3	3	20	5	12	8	1
S-3-4	3	20	5	12	8	1

We adopted large-scale test problems from Cordeau et al. [6]. Only the necessary changes specific to MDGVRP-MSDSA were made the same as the small-scale test problems. The parameters of large-scale test instances are given in Table 4.

Table 4

Parameters of the large-scale test instances

Problem no	Number of depots	Number of customers	Number of vehicles	Number of customers in district 1	Number of customers in district 2	Number of starting points excluding depots
L-2-1	2	80	10	38	40	2
L-2-2	2	100	10	50	45	5
L-2-3	2	100	15	49	48	3
L-3-1	3	100	18	48	48	4
L-4-1	4	48	4	25	21	2
L-4-2	4	50	8	26	22	2
L-4-3	4	50	16	26	22	2
L-4-4	4	96	8	57	35	4
L-4-5	4	100	16	50	45	5
L-5-1	5	75	15	45	24	6
L-6-1	6	72	6	26	43	3

5.2 *Test results*

Firstly, the solution of the MDGVRP-MSDSA using the MIP given in Section 3 was obtained with the GAMS-CPLEX solver. After that, the results obtained using the proposed VNS-R and VNS-NN were compared to the results of the MIP. For both the MIP model and algorithm, a two-hour running time limit was set. The other terminating criteria for the algorithm was the number of iterations of the local search L_1, L_2, L_3, L_4, and L_5, which were determined to be $|N|*0.25$, $|V|*0.3$, $|N|*0.3$,

$|N|*0.25$, and $|V|*0.3$, respectively. Additionally, each problem is solved by VNS-R and VNS-NN under 3 independent runs.

To compare the relative performance of the model and VNSs, we made use of relative percentage deviation (RPD_1, RPD_2 and RPD_3). RPD_1 was measured by the difference between the best solution obtained from VNS-R and the MIP. RPD_2 was the difference between the results of the mathematical model and VNS-NN. RPD_3 was the difference between the best solutions of the VNS-R and VNS-NN for each instance. The formulations for the performance measures are given in equations (22), (23) and (24) as follows:

$$RPD_1 = \frac{Z_{best}^{R} - Z_{GAMS}}{Z_{GAMS}} \tag{22}$$

$$RPD_2 = \frac{Z_{best}^{NN} - Z_{GAMS}}{Z_{GAMS}} \tag{23}$$

$$RPD_3 = \frac{Z_{best}^{R} - Z_{best}^{NN}}{Z_{best}^{VNS-NN}} \tag{24}$$

Table 5 shows the results of test problems which are small-scale and produced by the CPLEX solver in GAMS and those by Python, with the corresponding CPU times in seconds. The best solutions and average value of solutions for VNS-R and VNS-NN are also reported, as well as the percentage improvement (RPD_1 and RPD_2) in solution quality with the proposed algorithms. The last column shows the relative percentage deviation between VNS-R and VNS-NN for each test instance.

There are a few observations that can be made from Table 5. First, the MIP using the GAMS/CPLEX solver gives a feasible solution for almost all small-scale instances including three optimal solutions. RPD_1 and RPD_2 values for one instance (S-3-4) was not computed because the mathematical

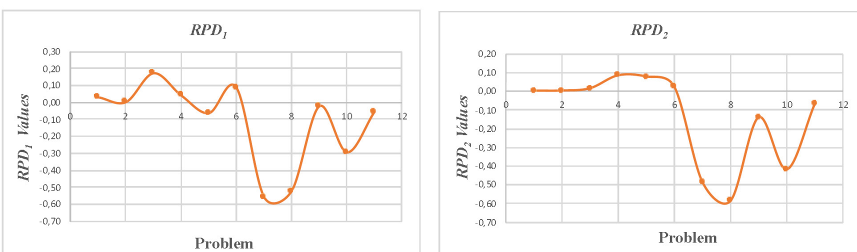


Figure 3

The RPD_1 and RPD_2 values for small-scale test instances

Table 5
Results of small-scale test instances

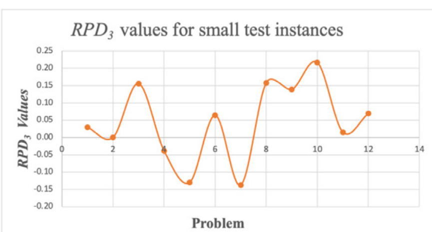
Test No	GAMS		VNS-R					VNS-NN					RPD_3
	Z	**t**	Z^R_{best}	t^R_{best}	$\overline{Z}^R$	$\overline{t}^R$	RPD_1	Z^{NN}_{best}	t^N_{best}	$\overline{Z}^{NN}$	$\overline{t}^{NN}$	RPD_2	
S-2-1	8028.5	103	8286.0	59	9185.0	51	0.03	8052.9	60	8052.9	68	0.00	0.03
S-2-2	10504.2	84	10540.2	60	10578.2	89	0.00	10540.2	88	11104.6	88	0.00	0.00
S-2-3	9673.6	31	11360.0	86	11767.6	109	0.17	9828.7	119	11828.5	106	0.02	0.16
S-2-4	7664.2	7200	7995.9	110	8651.2	105	0.04	8317.5	100	8429.5	99	0.09	–0.04
S-2-5	15758.4	7200	14787.0	350	17051.4	245	–0.06	16995.7	250	17913.4	219	0.08	–0.13
S-2-6	10554.2	7200	11492.6	384	11794.8	319	0.09	10802.9	302	11482.6	299	0.02	0.06
S-2-7	21829.1	7200	9645.5	282	10476.0	200	–0.56	11186.5	93	12615.9	103	–0.49	–0.14
S-2-8	22029.1	7200	10515.1	477	11139.8	270	–0.52	9081.2	502	9957.3	396	–0.59	0.16
S-3-1	13783.6	7200	13489.0	720	15038.6	435	–0.02	11848.8	415	14105.4	412	–0.14	0.14
S-3-2	21646.1	7200	15287.7	269	16943.4	387	–0.29	12565.2	1013	15561.0	712	–0.42	0.22
S-3-3	17047.9	7200	16142.0	868	16907.6	764	–0.05	15901.9	411	17472.1	396	–0.07	0.02
S-3-4	-	7200	15052.3	236	17729.9	284	-	14078.8	478	16394.5	429	-	0.07

model did not obtain a feasible solution. Second, looking at the RPD_1 and RPD_2 values, the proposed algorithms provide up to a 59% improvement in the objective function. RPD_1 and RPD_2 values for small-scale tests are illustrated in Figure 3.

Figure 3(a) shows that RPD_1 value is bigger than zero for five test problems (S2-1, S2-2, S2-3, S2-4, S2-6) since the model exhibited better performance. Nevertheless, VNS-R has significantly higher RPD_1 values for the other problems. The widest RPD_1 value, which is equal to –0.56, is recorded for instance S-2-7. The same pattern can be observed in terms of the mathematical model performance in Figure 3(b). For the first six instances, better solutions are obtained by the model. In addition to this, RPD_2 values of these instances is quite small and approximated to zero. RPD_2 values depend on problem size. With the large number of customers and depots for instances which are small-scale, RPD_2 has a peak percentage value (approximately 0.59).

The results of large-scale test problems and corresponding CPU times in minutes are presented in Table 6. This table is similar with Table 5. As can be seen from Table 6, GAMS/CPLEX could not find a feasible solution for all instances. So, RPD_1 and RPD_2 could not be calculated. Other than that, the proposed algorithms provided feasible solutions for all large-scale test problems in a short time.

In Figure 4, the performance of VNS-R and VNS-NN is illustrated based on the results of small- and large-scale test instances. It can be obviously seen that VNS-NN is noticeably more successful than VNS-R regarding solution quality since VNS-NN is better for 8 of 12 small-scale problems (67%) and 9 of 11 large-scale problems (82%). Especially, in large-scale test instances, VNS-R shows better performance for only one instance.



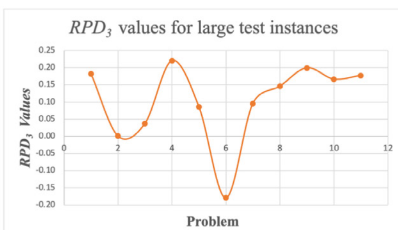


Figure 4

The RPD_3 values for small- and large-scale test instances

Table 6
Results of large-scale test instances

Test No	GAMS		VNS-R				VNS-NN				RPD_3
	Z	$t(s.)$	Z^R_{best}	t^R_{best}	$\overline{Z}^R$	$\overline{t}^R$	Z^{NN}_{best}	t^N_{best}	$\overline{Z}^{NN}$	$\overline{t}^{NN}$	
L-2-1	-	7200	103750.8	27	107424.4	28	87773.6	19	106814.2	15	0.18
L-2-2	-	7200	45375.3	7	50883.6	8	45341.1	11	51164.4	8	0.00
L-2-3	-	7200	56601.8	47	69716.0	39	68981.8	19	72025.5	28	–0.18
L-3-1	-	7200	118999.4	65	122967.8	59	103847.6	39	115325.0	33	0.15
L-4-1	-	7200	210508.6	29	235249.5	28	172485.4	22	175777.2	22	0.22
L-4-2	-	7200	156264.2	70	165432.1	39	142580.3	68	156863.8	54	0.10
L-4-3	-	7200	35260.0	7	38757.5	9	33981.4	5	37185.6	6	0.04
L-4-4	-	7200	73323.1	63	78530.8	55	62842.1	46	71718.5	36	0.17
L-4-5	-	7200	61119.0	35	70455.4	27	56262.1	19	59404.3	22	0.09
L-5-1	-	7200	84741.7	30	89902.7	27	71972.2	35	76731.9	27	0.18
L-6-1	-	7200	74063.1	74	78915.9	49	61727.3	38	66413.4	37	0.20

6. Conclusions

In this study, we proposed an MDGVRP-MSDSA problem. Initially, the mathematical model of the problem was constructed as a mixed integer linear programming (MILP) problem. Because of the Np-hard nature of the problem, MIPL can solve problems up to 20 customers. Then, VNS algorithms were developed to solve large-scale MDGVRP-MSDSA. We considered various generations of the initial solution strategies: random generation and nearest neighborhood-based generation. We adopted 2 neighborhood and 5 local search strategies. After that, small-scale test problems with 10, 15, and 20 customers were generated from published literature to make a comparison between the solution found by MIPL and the VNS. Thereby, the performance of the VNS for each initial solution strategy was demonstrated. Afterward, large-scale problems from published literature were used and solved by both VNS-R and VNS-NN. The computational results showed that the performances of the VNSs were promising. Furthermore, it was concluded that the performance of the VNS-NN was better than VNS-R.

An interesting direction for future research is to handle the MDGVRP-MSDA with multiple objectives considering new objective functions related to reducing the transportation risks of some recycling materials, especially from the areas related to industrial waste. Additionally metaheuristics for multiple objectives could be developed. Furthermore, the MDGVRP-MSDSA could be extended with time windows for collecting items especially for areas where recycling from households is collected.

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