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Integrated model with quality investment and tolerance under the 100% inspection

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Abstract

In this study, we present the integrated design of process parameters and tolerance. For the process parameters setting, one considers the product with quality loss cost and process adjustment cost which is reciprocal to the variance and proportional to the square of process mean. Then, the quality investment and 100% inspection with process capability are considered.

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Keywords: *Process parameters, Tolerance, Quality investment, Process capability index, Quality loss.*

1. Introduction

The output distribution of product can affect the product's bias and variability. One needs to control them in order to have the optimal output of product. One can adopt Taguchi's [1] function of quality loss for measuring the performance of quality. One of Huang's [2] process adjustment cost considers that it is concerned with square of mean and reciprocal to the variability.

In 2010, Shin et al.'s [3] process parameters setting model addresses Taguchi's quality and process costs. The tolerance design considers the trade-off problem for decreasing the probability of non-conforming product and increasing the manufacturing cost. In Shin et al.'s [3] tolerance

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model, the product with 100% inspection considers quality loss, rejection, and tolerance costs. The tolerance cost considers for shortening the specification interval of product. From Case [4] and Jeang [5], the tolerance cost considers the exponential function of product tolerance.

Quality investment can be considered as the method of quality improvement. Tsou [6], Chen and Tsou [7], Ganeshan et al. [8], and Hong et al. [9] addressed it as the function of exponential reduction for mean and variability. The capability index C_{pm} can be used for measuring the optimal value of product. Hence, the setting of specified C_{pm} value can assure the customer's constrained loss.

Chen [10] addressed the different price sold to the markets based on the defective numbers occurred in the inspection. His work is to find the optimal cost of quality investment. Chen and Chou [11] addressed the modified rectifying inspection for obtaining the optimal quality investment and asymmetric product tolerance.

In this work, we address the modified Shin et al.'s [3] parameters and tolerance setting models with quality investment and specified C_{pm} value for 100% product inspection as quality protection.

2. Modified Shin et al.'s Model

2.1 *Notations*

k	quality loss coefficient
μ	unknown process mean
σ	unknown standard deviation
M	manufacturing target of process
C_1	coefficient of μ^2 in the process cost
C_2	coefficient of $1/\sigma^2$ in the process cost
$TC_1(\mu, \sigma)$	process total cost without process inspection
$TC_2(Inv,\ \delta)$	total cost with 100% process inspection
$\phi(\cdot)$	probability density function (p.d.f.) of standard normal distribution
$\Phi(\cdot)$	cumulative distribution function of standard normal distribution
C_R	rejection unit cost
a_2	constant in the tolerance cost

a_3	coefficient of the tolerance range in the tolerance cost
LSL	lower limit of product, $LSL = \mu_0 - \delta\sigma_0$
USL	upper limit of product, $USL = \mu_0 + \delta\sigma_0$
μ_0	known process mean in the phase I
σ_0	known standard deviation in the phase I
Inv	quality investment
μ_1	improved process mean through quality investment
σ_1	improved standard deviation through quality investment
C_{pm}	process capability index of process
K_m	specified value of C_{pm}
β_1	exponential declined coefficient for the process mean through quality investment
α_1	exponential declined coefficient for the standard deviation through quality investment
Y	quality characteristic of product
$f(y)$	p.d.f. of Y without quality investment,

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right), \quad -\infty < y < \infty$$

$f(y, Inv)$	p.d.f. of Y under the quality investment,

$$f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right), \quad -\infty < y < \infty$$

σ_T	target value of standard deviation
δ	coefficient of tolerance
I_c	inspection cost per unit

2.2 *Assumptions*

1. The characteristic of quality is normal distribution.
2. The quality measurement adopts quadratic quality loss function.
3. Consider the quality investment function as the declining exponential reduction for mean and variability.
4. The symmetric specification limits are considered.

5. Consider C_{pm} value as the quality protection when one adopts the 100% inspection in the quality investment and specification limits settings model.

2.3 *Phase I--- Process Parameters Setting*

According to Shin et al. [3] and Huang [2], the modified process parameters setting model is as follows:

Minimize

$$E[TC_1(\mu,\ \sigma)] = \int_{-\infty}^{\infty} k(y-M)^2 f(y)dy + C_1\mu^2 + C_2\frac{1}{\sigma^2}$$
$$= k[(\mu-M)^2+\sigma^2] + C_1\mu^2 + C_2\frac{1}{\sigma^2} \tag{1}$$

Set the first derivative for process mean and standard deviation as zero. We obtain the optimal $\mu_0 = \frac{kM}{k+C1}$ and $\sigma_0 = \sqrt[4]{\frac{C_2}{k}}$. The specified values of μ_0 and σ_0 are the input values of phase II.

2.4 *Phase II--- Quality Investment and Specification Limits Settings Model*

The expected total cost of phase II considers some costs, e.g., quality loss, rejection, tolerance, inspection, and quality investment costs. From Chase [4], Jeang [5], and Shin et al. [3], the modified tolerance setting model is to

Minimize

$$\begin{aligned}
&E[TC_2(Inv,\ \delta)]\\
&= \int_{LSL}^{USL} k(y-M)^2 f(y,Inv)dy + C_R[\int_{-\infty}^{LSL} f(y,Inv)dy + \int_{USL}^{\infty} f(y,Inv)dy]\\
&\quad + a_2\exp\left(-\frac{a_3(USL-LSL)}{2}\right) + I_c + Inv\\
&= k[(\mu_1-M)^2]\left[\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right)-\Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right] - 2k\sigma_1(\mu_1-M)\left[\phi\left(\frac{USL-\mu_1}{\sigma_1}\right)-\phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\\
&\quad - k\sigma_1^2\left[\left(\frac{USL-\mu_1}{\sigma_1}\right)\phi\left(\frac{USL-\mu_1}{\sigma_1}\right)-\left(\frac{LSL-\mu_1}{\sigma_1}\right)\phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)-\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right)+\Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\\
&\quad + C_R\left\{1-\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right)+\Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\right\} + a_2\exp(-a_3\delta\sigma_0) + I_c + Inv
\end{aligned} \tag{2}$$

subject to

$$LSL = \mu_0 - \delta\sigma_0 \tag{3}$$

$$USL = \mu_0 + \delta\sigma_0 \tag{4}$$

$$f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_I} \exp\left(-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right), \quad -\infty < y < \infty \tag{5}$$

$$\mu_1^2 = M^2 + (\mu_0^2 - M^2)\exp(-\beta_1 Inv) \tag{6}$$

$$\sigma_1^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha_1 Inv) \tag{7}$$

$$C_{pm} = \frac{USL-LSL}{6\sqrt{(\mu_1-M)^2+\sigma_1^2}} = \frac{\delta\sigma_0}{3\sqrt{(\mu_1-M)^2+\sigma_1^2}} = K_m \tag{8}$$

One needs to obtain the combination (Inv, δ) with minimum value of $E[TC_2(Inv, \delta)]$ under the K_m. There are three steps for solution:

Step 1: Consider the upper value *Inv* as $Inv_{\max}$.

Step 2: Set the initial *Inv* = 0. One can compute the corresponding δ satisfying Eq. (8) and $E[TC_2(Inv, \delta)]$ from Eq. (2).

Step 3: Set *Inv* = *Inv* +0.01, return to Step 2 until $Inv = Inv_{\max}$. We have the optimal combination (Inv, δ) with minimum value of Eq. (2).

3. Numerical Example

Consider the following parameters: k = 200, T =50, $C_1 = 2$, $C_2 = 6$, $a_2 = 2$, $a_3 = 30$, $C_R = 50$, $\alpha = 0.5$, $\beta = 0.1$, $\sigma_T = 0.25$, $K_m = 1.33$, and $I_c = 1$.

The optimal solution of Eq. (1) is $\mu_0 = 49.505$ and $\sigma_0 = 0.416$. From Eqs. (2)-(8), we obtain the optimal decision variables as follows: $\mu_1 = 49.846$, $\sigma_1 = 0.251$, $Inv = 11.67$, $\delta = 2.818$, $LSL = 48.332$, $USL = 50.678$, and $E[TC_2(Inv, \delta)]$ = 29.924. The numerical result of sensitivity analysis is listed. According to Table 1, one has some results as follows:

1. The parameters, k and β, obtain a significant effect on the quality investment.
2. The parameters, β, σ_T, and K_m, obtain a significant effect on the coefficient of tolerance.
3. The parameters, k, β, and σ_T, obtain a significant effect on the expected total cost of product.

4. Conclusions

This paper has proposed about modified Shin et al.'s (2010) work considering quality investment and specified C_{pm} value for quality protection with 100% inspection. Numerical results show that quality loss coefficient, exponential declined coefficient for the process mean, and target value of standard deviation have a significant effect for optimal solution. Further study should consider the modified model with sampling inspection based on the quality investment, specified C_{pm} value, and consumer's risk.

Table 1

Some results in the numerical example

C_1	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
2.4	49.407	0.416	49.845	0.250	13.36	2.823	31.645
1.6	49.603	0.416	49.849	0.252	9.64	2.814	27.866
C_2	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
7.2	49.505	0.436	49.846	0.251	11.67	2.693	29.934
4.8	49.505	0.394	49.845	0.251	11.56	2.988	29.913
k	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
240	49.587	0.398	49.860	0.251	10.83	2.880	31.578
160	49.383	0.440	49.826	0.250	12.63	2.764	28.478
T	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
51	50.495	0.416	50.846	0.251	11.82	2.821	30.113
49	48.515	0.416	48.846	0.251	11.47	2.819	29.732
C_R	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
60	49.505	0.416	49.846	0.251	11.67	2.818	29.929
40	49.505	0.416	49.846	0.251	11.67	2.818	29.919
α	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
0.6	49.505	0.416	49.845	0.250	11.56	2.823	29.878
0.4	49.505	0.416	49.850	0.252	11.88	2.813	30.062
β	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
0.12	49.505	0.416	49.861	0.251	10.58	2.750	27.987

Contd...

0.08	49.505	0.416	49.825	0.250	12.98	2.927	32.605
σ_T	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
0.30	49.505	0.416	49.846	0.300	11.67	3.235	35.385
0.20	49.505	0.416	49.845	0.201	11.56	2.435	25.458
a_2	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
2.4	49.505	0.416	49.846	0.251	11.67	2.818	29.924
1.6	49.505	0.416	49.846	0.251	11.67	2.818	29.924
a_3	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
36	49.505	0.416	49.846	0.251	11.67	2.818	29.924
24	49.505	0.416	49.846	0.251	11.67	2.818	29.924
K_m	μ_0	σ_0	μ_1	σ_1	Inv	δ	$E[TC_2(Inv, \delta)]$
1.6	49.505	0.416	49.845	0.251	11.56	3.401	29.949
1.067	49.505	0.416	49.844	0.251	11.52	2.271	29.889

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