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New algorithms for solving fuzzy linear fractional programming problems by ranking functions

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Abstract

The problem of fully fuzzy programming was one of the problems that many treatments have appeared for due to ambiguity and inaccuracy of its ambiguous coefficients. In this study, finding the optimal numerical solution for fully fuzzy linear fractional programming problems (FFLFPP) using the ranking functions was obtained. The proposed pentagonal ranking function (PPRF) multiplied by γ^5 has been derived, as well as the proposed heptagonal ranking function (PHRF) multiplied by γ^5 has been derived too. Two laws were obtained the first one was the novel ranking membership pentagonal and the second one was the heptagonal function then utilized to solve FFLFPP after converting the problems into FFLPP by utilizing the complementary method, then converting them to crisp linear programming problems and finding the optimal numerical solution for problems by using the proposed ranking functions. The paper included numerical example to illustrate the applied technique and the accuracy of the solution, and a comparison with available techniques were add the novelty of this study.

Subject Classification: *Primary 93A30, Secondary 49K15.*

Keywords: *Fractional linear programming (FLP), Fuzzy numbers (FN), Heptagonal ranking functions (HRF), Pentagonal ranking function (PRF).*

1. Introduction

Linear fractional programming is important in various real-life areas such as financial planning, university, and hospital planning also healthcare etc. [1] In such problems, objective transactions and restrictions may be inaccurate values due to errors in measurement or changes in the market or in some problems that cannot be controlled, such as customers in this case decision-makers hesitate to make decisions and cannot be at level of ambition for the problem [2]. In this case of uncertainty and hesitation, fuzzy linear fractional programming can be

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used in these situations and find the required solutions. Multi-linear fractional programming problems have been solved in several methods by researchers [1] using branch and bound to find the integer solution and [3] solving the problem by using triangular fuzzy numbers. Some researchers have enhanced methods for solving (FLPP) [4-6]. Sahoo and Pati [7] in 2022 and Vandana, Namrata, and Deshmukh, [8] were solved multiple objective fractional problems utilizing pentagonal fuzzy numbers. Others have used ranking functions to solve FFLPP [9] and [10]. The applications have also been employed to solve fuzzy problems [11-13]. This paper introduces various ranking functions to solve FFLFPP. The technique involves converting FFLFPP into FFLPP and applying the proposed ranking functions. The research comprises five sections. The second and third sections detail two types of ranking functions: the proposed pentagonal ranking function PPRF with derivation and the proposed heptagonal ranking function PHRF with derivation. The fourth section provides the solution algorithm with an illustrative example along with a comparison of results, while the final section covers the research conclusions.

2. Pentagonal Fuzzy Function (PFF) [12]

A fuzzy function for pentagonal FN $\tilde{A}_{pf} = (pf_1, pf_2, pf_3, pf_4, pf_5 : \lambda)$ where pf_1, pf_2, pf_3, pf_4 *and* pf_5 are real numbers and $\lambda \in [0,1]$ was defined below

$$\mathcal{M}_{A_{pf}}(x) = \begin{cases} \lambda\left(\frac{x-pf_1}{pf_2-pf_1}\right) & pf_1 \le x < pf_2 \\ \lambda + (1-\lambda)\left(\frac{x-pf_2}{pf_3-pf_2}\right) & pf_2 \le x < pf_3 \\ 1 & x = pf_3 \\ \lambda + (1-\lambda)\left(\frac{pf_4-x}{pf_4-pf_3}\right) & pf_3 \le x < pf_4 \\ \lambda\left(\frac{pf_5-x}{pf_5-p_4}\right) & pf_4 \le x < pf_5 \\ 0 & o.w \end{cases} \tag{1}$$

2.1 *The Proposed Pentagonal Ranking Function (PPRF) Method*:

Now by using $\gamma -$ cut and $0 \le \gamma \le 1$ for Eq.1 where $\inf \widetilde{A}(\gamma) = \{x \in \tilde{A} \setminus \mathcal{M}(x) \ge \gamma\}$ is the infimum of $\tilde{A}$, and $\sup \widetilde{A}(\gamma) = \{x \in \tilde{A} \setminus \mathcal{M}(x) \le \gamma\}$ is the suprimum of $\tilde{A}$.

We can define the $\gamma -$ cut of pentagonal function as:

$$\tilde{A}_{pf}(\gamma) = \begin{cases} pf_1 + \left(\frac{pf_2-pf_1}{\lambda}\right)\gamma = inf_1\left(\tilde{A}_{pf}(\gamma)\right) \\ pf_2 + \left(\frac{pf_3-pf_2}{1-\lambda}\right)(\gamma-\lambda) = inf_2\left(\tilde{A}_{pf}(\gamma)\right) \\ pf_4 - \left(\frac{pf_4-pf_3}{1-\lambda}\right)(\gamma-\lambda) = sup_1\left(\tilde{A}_{pf}(\gamma)\right) \\ pf_5 - \left(\frac{pf_5-pf_4}{\lambda}\right)\gamma = sup_2\left(\tilde{A}_{pf}(\gamma)\right) \end{cases} \tag{2}$$

Where ≤ 1 , $\gamma \in [0,1]$

Now by applying ranking function via equation Eq.2 obtained

$$R_\gamma(\tilde{A}_{pf}) = \frac{\frac{1}{4}\int_0^1[\gamma(inf_1\tilde{A}_{pf}(\gamma)+inf_2\tilde{A}_{pf}(\gamma))+(1-\gamma)(sup_1\tilde{A}_{pf}(\gamma)+sup_2\tilde{A}_{pf}(\gamma))]\gamma^5 d\gamma}{\int_0^1 \gamma^5 d\gamma},$$

$$R_\gamma(\tilde{A}_{pf}) = \frac{\frac{1}{4}\int_0^1[\gamma^6(pf_1+pf_2+pf_4+pf_5)+(\gamma^5-\gamma^6)(pf_2+2pf_3+pf_4)]d\gamma}{\int_0^1 \gamma^5 d\gamma},$$

$$R_\gamma(\tilde{A}_{pf}) = \frac{\frac{1}{4}[\frac{\gamma^7}{7}(pf_1+pf_2+pf_4+pf_5)+\left(\frac{\gamma^6}{6}-\frac{\gamma^7}{7}\right)(pf_2+2pf_3+pf_4)]\Big|_0^1}{\frac{\gamma^6}{6}\Big|_0^1},$$

$$R_\gamma(\tilde{A}_{pf}) = \frac{1}{4}\left[\frac{\frac{1}{7}(pf_1+pf_2+pf_4+pf_5)+\frac{1}{42}(pf_2+2pf_3+pf_4)}{\frac{1}{6}}\right],$$

We can simplified and deduced the following formula

$$R_\gamma(\tilde{A}_{pf}) = \frac{1}{28} * [6pf_1 + 7pf_2 + 2pf_3 + 7pf_4 + 6pf_5] \tag{3}$$

3. Heptagonal Fuzzy Function (HFF) [14]

A fuzzy function for Heptagonal fuzzy numbers

$$\tilde{A}_{hf} = (hf_1, hf_2, hf_3, hf_4, hf_5, hf_6, hf_7; \lambda)$$

where $hf_1, hf_2, hf_3, hf_4, hf_5, hf_6$ and hf_7 are real numbers and $\lambda \in [0,1]$ weight function, definding heptagonal membership as follows

$$\mathcal{M}_{\tilde{A}_{hf}}(x) = \begin{cases} \lambda\left(\frac{x-hf_1}{hf_2-hf_1}\right) & hf_1 \leq x < hf_2 \\ \lambda & hf_2 \leq x < hf_3 \\ \lambda + (1-\lambda)\left(\frac{x-hf_3}{hf_4-hf_3}\right) & hf_3 \leq x < hf_4 \\ \lambda + (1-\lambda)\left(\frac{hf_5-x}{hf_5-hf_4}\right) & hf_4 \leq x < hf_5 \\ \lambda & hf_5 \leq x < hf_6 \\ \lambda\left(\frac{hf_7-x}{hf_7-hf_6}\right) & hf_6 \leq x < hf_7 \\ 0 & \text{other wise} \end{cases} \tag{4}$$

3.1 *The Proposed Heptagonal Ranking Function (PHRF) Method*:

By using $\gamma -$ cut and $0 \leq \gamma \leq 1$ for Eq.4 then,

$$\tilde{A}_{hf}(\gamma) = \begin{cases} hf_1 + \left(\frac{hf_2-hf_1}{\lambda}\right)\gamma = inf_1\left(\tilde{A}_{hf}(\gamma)\right) \\ hf_3 + \left(\frac{hf_4-hf_3}{1-\lambda}\right)(\gamma-\lambda) = inf_2\left(\tilde{A}_{hf}(\gamma)\right) \\ hf_5 + \left(\frac{hf_4-hf_5}{1-\lambda}\right)(\gamma-\lambda) = sup_1\left(\tilde{A}_{hf}(\gamma)\right) \\ hf_7 + \left(\frac{hf_6-hf_7}{\lambda}\right)\gamma = sup_2\left(\tilde{A}_{hf}(\gamma)\right) \end{cases} \tag{5}$$

where as $0 \leq \lambda \leq 1$, $\gamma \in [0,1]$

Now, using $\gamma -$ cut function for heptagonal fuzzy numbers Eq.5 to find ranking function as follows

$$R_\gamma(\tilde{A}_{hf}) = \frac{\frac{1}{4}\int_0^1[\gamma(inf_1\tilde{A}_{hf}(\gamma)+inf_2\tilde{A}_{hf}(\gamma))+(1-\gamma)(sup_1\tilde{A}_{hf}(\gamma)+sup_2\tilde{A}_{hf}(\gamma))]\gamma^5 d\gamma}{\int_0^1 \gamma^5 d\gamma},$$

By the same technique of pentagonal ranking we can simplified and deduce the following heptagonal fuzzy numbers formula as follows:

$$R_\gamma(\tilde{A}_{hf}) = \frac{1}{28} * [6hf_1 + 6hf_2 + hf_3 + 2hf_4 + hf_5 + 6hf_6 + 6hf_7] \tag{6}$$

4. Solution Algorithem Using Proposed Ranking Function Methods

You can write basically form of FFLFPP as:

$$\text{Max } \tilde{z} = \frac{a\tilde{x}+\vartheta}{b\tilde{x}+\beta}$$

S.to $A\tilde{x} \le B$,

$$\tilde{x} \ge 0$$

where $A(n\times m)$ matrix, $a, b \in \mathcal{R}^n$ and ϑ, β, B are numerical values.

Select a linear fractional programming problem with a solution then transform the problem to FFLFPP by using pentagonal fuzzy numbers with aiding of $\Delta_1 and\, \Delta_2$ after that converts the problem to FFLPP with a complementary method**.** The second step is finding the solution of FFLPP. The end step utilizes PPRF method to find the optimal solution with comparsion. The same technique can be used together with PHRF.

4.1 *Numerical Example*

Here, numerical example will be solved to introduce and explain the proposed ranking function methods PPRF and PHRF.

$$\text{Max z} = \frac{5\tau_1+2\tau_2+\tau_3+1}{3\tau_1+2\tau_2+\tau_3+1}$$

S.to

$$3\tau_1 + 4\tau_2 + \tau_3 \le 4$$
$$\tau_1 + 2\tau_2 + 5\tau_3 \le 6$$
$$\tau_1, \tau_2, \tau_3 \ge 0$$

The crisp sol. is *Max z* = 1.5333, $\tau_1 = 1.3333, \tau_2 = 0, \tau_3 = 0.$

Now convert the example from fractional linear programming to linear programming by using the complementary method *Max z* = $w_1 - w_2$, where w_1 is the numerator and w_2 is the denominator, then the above problem becomes *Max z* = $2\tau_1$ with the same constraints

Change the problem to a pentagonal fuzzy number with aid of Δ_1 and Δ_2 let $\Delta_1 = 0.5$ and $\Delta_2 = 0.5$ then the problem convert to fully fuzzy linear programming and becomes as follows:

Max $\tilde{z}$ = (1, 1.5, 2, 2.5, 3) $\widetilde{\tau_1}$

S.to (2, 2.5, 3, 3.5, 4) $\widetilde{\tau_1}$ + (3, 3.5, 4, 4.5, 5)

$$\widetilde{\tau_2} + (0, 0.5, 1, 1.5, 2)\widetilde{\tau_3} \leq (3, 3.5, 4, 4.5, 5)$$

$(0, 0.5, 1, 1.5, 2)\widetilde{\tau_1}$ + (1, 1.5, 2, 2.5, 3)

$$\widetilde{\tau_2} + (4, 4.5, 5, 5.5, 6)\widetilde{\tau_3} \leq (5, 5.5, 6, 6.5, 7)$$

$$\widetilde{\tau_1}, \widetilde{\tau_2}, \widetilde{\tau_3} \geq 0$$

The optimal sol. of FFLFPP for this example by aiding of pentagonal fuzzy numbers will be as

Max $\tilde{z}$ = (1.5454, 1.5454, 1.5333, 1.5294, 1.5263), and

$\widetilde{\tau_1} = (1.5, 1.5, 1.3333, 1.2857, 1.25)$, $\widetilde{\tau_2} = (0,0,0,0,0)$, $\widetilde{\tau_3} = (0,0,0,0,0)$.

By using Eq.3 of PPRF method concluding the following results

$$R_\gamma(\tilde{z}) = \frac{1}{28} * [6 * 1.5454 + 7 * 1.5454 + 2 * 1.5333 + 7 * 1.5294 + 6 * 1.5263],$$

So $R_\gamma(\tilde{z}) = 1.5364$, $R_\gamma(\widetilde{\tau_1}) = 1.3809$, $R_\gamma(\widetilde{\tau_2}) = 0$, $R_\gamma(\widetilde{\tau_3}) = 0$.

By the same technique we can found the optimal solution of this example by changing the problem in heptagonal fuzzy numbers with aid of $\Delta_1 = 0.5 = \Delta_2$.

The optimal sol. of FFLFPP for above example by heptagonal fuzzy numbers will be

Max $\tilde{z}$ = (1.5555, 1.5454, 1.5454, 1.5333, 1.5294, 1.5263, 1.5238),

$\widetilde{\tau_1} = (1.6667, 1.5, 1.4, 1.3333, 1.2857, 1.25, 1.2222)$, $\widetilde{\tau_2} = (0,0,0,0,0,0,0)$,

$\widetilde{\tau_3} = (0,0,0,0,0,0,0)$

By using Eq.6 for the PHRF method concluding the following results

$$R_\gamma(\tilde{z}) = \frac{1}{28} * [6 * 1.5555 + 6 * 1.5454 + 1.5454 + 2 * 1.5333 + 1.5294 + 6 * 1.5263 + 6 * 1.5238],$$

So $R_\gamma(\tilde{z}) = 1.5371$, $R_\gamma(\widetilde{\tau_1}) = 1.3994$, $R_\gamma(\widetilde{\tau_2}) = 0$, $R_\gamma(\widetilde{\tau_3}) = 0$.

From paper [15] we were used two formulas ranking pentagonal fuzzy numbers, the first one was $R_1 = \frac{(pf_1+pf_2+pf_3+pf_4+pf_5)}{5}$ and the second one was $R_2 = \frac{(pf_1+pf_2+2pf_3+pf_4+pf_5)}{6}$ Moreover, the researcher [16] utilize the ranking triangular fuzzy numbers which $R_3 = \frac{(a+2b+c)}{4}$, where (*a*, *b*, *c*) is triangular fuzzy numbers, to comparison our results.

We can organize our findings in Table 1 to compare them with previous research for different rankings.

Table 1

The numerical results to max$\tilde{z}$ for different rankings are follows

Max z (crisp sol.)	**(PHRF) $Max(\tilde{z})$**	**(PPRF) $Max(\tilde{z})$**	**R_1 Max($\tilde{z}$) [15]**	**R_2 Max($\tilde{z}$) [15]**	**R_3 Max($\tilde{z}$) [16]**
1.5333	1.5371	1.5364	1.5355	1.5359	1.5353

The results $Max(\tilde{z})$ indicate that largest in PHRF method outperformed other ranking methods. Additionally, PPRF method showed greater performance compared to other pentagonal ranking methods (as discussed in [15]). Furthermore, the proposed rankings for heptagonal and pentagonal shapes outperformed the triangular ranking in [16], with all rankings surpassing the crisp solution, as explained in Table 1.

5. Discussion and Conclusion

In the current research, we have developed two novel ranking functions known as PHRF method and PPRF method, each accompanied by its respective set of principles. These functions were subsequently employed to solve FFLFPP, resulting in precise solutions. Our investigation revealed that PHRF demonstrated superior performance compared to PPRF, while PPRF surpassed other ranking functions in diverse research contexts by providing accurate solutions. Comprehensive details are available in Table 1.

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