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Inverse Chen reliability function of $P(X < Y < Z)$

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Abstract

In models of reliability of stress-strength, the stress can fall under the influence of two strengths, and to varying degrees, the reliability of this factor can be expressed by the formula $P(X<Y<Z)$. The paper aims to find the formula of reliability for two models: the first is the cascade model (1+1), where it consists of a main component and the second is a standby component, assuming that the component has two strength variables X and Z and is subjected to stress Y, the second is the cascade model (2+1), which consists of two main components and a third is standby component, assuming that both components have two strength variables X and Z, and the first component is subjected to stress Y_1 and the second component to stress Y_2, in both cases, the stress and robustness random variables will follow the inverse Chen distribution.The reliability function is estimated in two different methods maximum likelihood and percentile. Monte Carlo simulation is performed to compare the estimation results using the statistical criteria: meansquare error and mean absolute percentage error the results showed that the best estimator is ML.

Subject Classification: *Primary 68M15, Secondary 62F10.*

Keywords: *Reliability, Probability P(T<X<Z), Stress-strength, Cascade, Estimation.*

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1. Introduction

Reliability is defined as the period of non-stop operation of the component under the influence of Y stress, where the component resists stress with its strength X where $p(Y < X)$ [1]. In another case, the model can work when the pressure Y on the component falls between two specific values of the component strengths X and Z, that is, the stress is within certain limits, and in this case, the reliability can be found as follows: $R = P(X < Y < Z)$[2]. The (1+1) model, which is a special cascade model, consists of a main component $\mathcal{C}_1$ and a component $\mathcal{C}_2$ with an active standby state, and the component $\mathcal{C}_1$ with a failed state is replaced by the component $\mathcal{C}_2$ [3]. Also in the (2+1) of the cascade model, where it consists of two main components $\mathcal{D}_1$ and $\mathcal{D}_2$ and component $\mathcal{D}_3$ it is in standby case, where if any components of $\mathcal{D}_1$or $\mathcal{D}_2$ fail, component $\mathcal{D}_3$ replaces it so that the model continues to work [1-3].

Many researchers have addressed reliability in their research: Khan and Jan [5] fended the reliability of a multicomponent system of Burr distribution. Karam and Khaleel [4] studied a special serial model where the model consists of two basic units and a standby unit that activates if one of the two main components fails. Khaleel [6] discuss the reliability of a single-component model with strength alone and exposed to four stresses. Mutkekar and Munoli [3] find the reliability of a two-component model., the first basic and the second standby that replace the basic component in case of failure. Doloi and Gogoi [7] studied a cascade model consisting of n-components.

The purpose of the paper is to find the formula when $R = P(X < Y < Z)$ and apply it in the Cascade model (1+1) as well as (2+1) when the random variables of the stress-strength with the inverse Chen. Methods (maximum likelihood and percentile) were used to estimate the model's reliability and simulation was conducted to indicate the best method to estimate by comparing simulation results using MSE and MAPE.

2. The mathematical formula

In 2000, Chen suggested the two-parameter distribution with a cumulative failure rate function [8]. The random variables Y is stress while X and Z are strengths independent identically exponential distributed $X \sim ICh(\beta, \alpha)$, $Y \sim ICh(\beta, \delta)$ and $Z \sim ICh(\beta, \lambda)$. The functions of inverse Chen distribution are [9]:

$$F(x) = \exp\left(\alpha\left(1 - e^{x^{-\beta}}\right)\right) \quad x > 0;\ \alpha, \beta > 0 \tag{1}$$

$$f(x) = \alpha\beta x^{-(1+\beta)}\, e^{x^{-\beta}} e^{\alpha\left(1-e^{x^{-\beta}}\right)} \quad x > 0;\ \alpha, \beta > 0 \tag{2}$$

$$R(x) = 1 - \exp\left(\alpha\left(1 - e^{x^{-\beta}}\right)\right) \quad x > 0;\ \alpha, \beta > 0 \tag{3}$$

$$H(x) = \frac{\alpha\beta x^{-(1+\beta)}\, e^{x^{-\beta}} e^{\alpha\left(1-e^{x^{-\beta}}\right)}}{1-\exp\left(\alpha\left(1-e^{x^{-\beta}}\right)\right)} \quad x > 0;\ \alpha, \beta > 0 \tag{4}$$

Where α is shape and β is the scale parameters and since $f(x)$ is pdf then $\int_x f(x)dx = 1$

So that

$$\int_0^{\infty} x^{-(1+\beta)}\, e^{x^{-\beta}} e^{\alpha\left(1-e^{x^{-\beta}}\right)} dx = \frac{1}{\alpha\beta} \tag{5}$$

Now the reliability of three cases will be found in detail:

The first case $p(x < y_1 < z)$: the reliability of the stress-strength model component is defined as the probability that the system can work only when the stress Y on it lies between two certain values of component strengths X and Z. The reliability of this case can be found as follows:

$$R_1 = p(x < y_1 < z) = \int_{y_1} F_x(y_1)(1 - H_z(y_1))g(y_1)dy_1 \tag{6}$$

Using equations 2 and 4 in equation 6 yields:

$$\begin{aligned} R_1 &= \int_{y_1} \exp\left(\alpha\left(1-e^{y_1^{-\beta}}\right)\right)\left(1-\exp\left(\lambda\left(1-e^{y_1^{-\beta}}\right)\right)\right) g(y_1)dy_1 \\ &= \int_{y_1} \exp\left(\alpha\left(1-e^{y_1^{-\beta}}\right)\right) g(y_1)dy_1 \\ &\quad - \int_{y_1}\left(1-\exp\left(\lambda\left(1-e^{y_1^{-\beta}}\right)\right)\right) g(y_1)dy_1 \\ &= \delta\beta \int_{y_1} y_1^{-(1+\beta)} e^{y_1^{-\beta}}\, e^{(\alpha+\delta)(1-e^{y_1^{-\beta}})}\, dy_1 \\ &\quad - \delta\beta \int_{y_1} y_1^{-(1+\beta)} e^{y_1^{-\beta}}\, e^{(\lambda+\delta)(1-e^{y_1^{-\beta}})}\, dy_1 \\ &= \delta\beta \frac{1}{\beta(\alpha+\delta)} - \delta\beta \frac{1}{\beta(\lambda+\delta)} \\ &= \frac{\delta}{(\alpha+\delta)} - \frac{\delta}{(\lambda+\delta)} \end{aligned} \tag{7}$$

Now the reliability of the first case is obtained

$$\mathrm{R}_1 = \frac{\lambda\delta - \alpha\delta}{(\alpha+\delta)(\lambda+\delta)} \tag{8}$$

The second case cascade (1+1) model: the reliability system of the model cascade (1+1) of the two components (one activated and one standby) to be working under the impact of stresses y_1 and y_2 respectively, where are independently and identically distributed, let X and Z be the lower and upper strengths, respectively of each component. To obtain the expression for reliability, consider that:

$$R_{11} = R_1 + R_2 \tag{9}$$

R_1 was found in the first case:

$$R_1 = p(x < y_1 < z) = \int_{y_1} F_x(y_1)(1 - H_z(y_1))g(y_1)dy_1$$

R_2 is found as follows:

$$R_2 = \left(p(x < y_1 < z)\right)^c p(x < y_2 < z) = (1 - R_1)p(x < y_2 < z) \tag{10}$$

In a cascade system after every failure, the stress is k [3] such that

$$y_2 = ky_1,\ y_3 = ky_2 = k^2y_1, \dots, y_i = k^{i-1}y_1\ etc.$$

It is obvious that once the distribution of y_1 specified the distribution of $y_2, y_3, \dots, y_n$ are automatically specified. Then

$$R_2 = (1 - R_1)p(x < ky_1 < z) \tag{11}$$

$$R_2 = (1 - R_1)\int_{y_1} F_x(ky_1)(1 - H_z(ky_1))g(y_1)dy_1$$

For the case that $k = 1$ [3] we get:

$$R_2 = (1 - R_1)R_1 \tag{12}$$

So that

$$R_{11} = R_1 + (1 - R_1) = R_1(1 + 1 - R_1) = R_1(2 - R_1)$$

So the function of reliability for the second case is:

$$\mathrm{R}_{11} = \left[\frac{\lambda\delta - \alpha\delta}{(\alpha+\delta)(\lambda+\delta)}\right]\left[2 - \left(\frac{\lambda\delta - \alpha\delta}{(\alpha+\delta)(\lambda+\delta)}\right)\right] \tag{13}$$

The third case cascade (2+1) model: In this part, we will present the expressions of reliability for cascade (2 + 1) stress-strength of the three components (two activate and one standby) where to be the stresses $Y \sim ICh(\delta_i, \beta);\ i = 1,2$ respectively are independently and identically distributed, we can obtain the reliability formula by:

$$R_{21c} = R_{1c} + R_{2c} + R_{3c} \tag{14}$$

Reliability R_{1c} will be estimated by the probability of the action of the first and second components, as be

$$R_{1c} = p(x < y_1 < z)p(x < y_2 < z) = c_1c_2 \tag{15}$$

Reliability estimate of R_{2c} with the probability of replacing the failure of the first component with the third component and the continuity work of the second component, as being:

$$R_{2c} = (1 - c)\ p(x < ky_1 < z)c_2$$

Since $k = 1$ then

$$R_{2c} = (1 - c_1)c_1c_2 \tag{16}$$

Finally, the reliability estimate R_{3c} with the probability of replacing the failure of the second component with the third component and the continuity work of the first component, as being:

$$\begin{aligned} R_{3c} = &\ (1 - c_2)p(x < ky_3 < z)c_1 \\ &= (1 - c_2)c_1c_2 \end{aligned} \tag{17}$$

For ICh dist. We get

$$c_1 = \frac{\delta_1}{(\alpha+\delta_1)} - \frac{\delta_1}{(\lambda+\delta_1)} = \frac{\lambda\delta_1 - \alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)} \tag{18}$$

$$c_2 = \frac{\delta_2}{(\alpha+\delta_2)} - \frac{\delta_2}{(\lambda+\delta_2)} = \frac{\lambda\delta_2 - \alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)} \tag{19}$$

Now equations 18 and 19 will be used in equations 14, 15, and 16 as follows

$$R_{1c} = \left[\left(\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\right] \tag{20}$$

$$R_{2c} = \left[\left(1-\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\right] \tag{21}$$

$$R_{3c} = \left[\left(1-\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\left(\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\right] \tag{22}$$

Finally, by replacing equations 20, 21, and 22 in equation 14, the function reliability for third case is obtained.

$$\mathrm{R}_{21c} = \left[\left(\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\right] + \left[\left(1-\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\right] + \left[\left(1-\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\left(\frac{\lambda\delta_1-\alpha\delta_1}{(\alpha+\delta_1)(\lambda+\delta_1)}\right)\left(\frac{\lambda\delta_2-\alpha\delta_2}{(\alpha+\delta_2)(\lambda+\delta_2)}\right)\right] \tag{23}$$

3. Estimation

The unknown distribution parameters will be estimated in two different ways, the maximum likelihood method and Percentile method, to find an estimate of the reliability function for (R_1, R_{11}, and R_{21c}).

3.1 *Maximum Likelihood Method*

Suppose $x_1, x_2, \ldots x_n$ form $ICh(\alpha,\beta)$. The function L is given as follows [9]:

$$L = L(\alpha,\beta,x) = \prod_{i=1}^{n} f(x_i) \tag{24}$$

$$= \prod_{i=1}^{n} \alpha\beta\, x_i^{-(1+\beta)}\, e^{x_i^{-\beta}}\, e^{\alpha\left(1-e^{x_i^{-\beta}}\right)}$$

$$= \alpha^n\beta^n \prod_{i=1}^{n} x_i^{-(1+\beta)} \prod_{i=1}^{n} e^{x_i^{-\beta}} \prod_{i=1}^{n} e^{\alpha\left(1-e^{x_i^{-\beta}}\right)}$$

Taking the logarithm of both sides of the above equation:

$$\ln(L) = nln\alpha + nln\beta - (\beta+1)\sum_{i=1}^{n}\ln(x_i) + \sum_{i=1}^{n} x_i^{-\beta} + \sum_{i=1}^{n}\alpha\left(1-e^{x_i^{-\beta}}\right) \tag{25}$$

By deriving the equation 25 for the parameter α:

$$\frac{\partial\ln(L)}{\alpha} = \frac{n}{\alpha} + n - \sum_{i=1}^{n} e^{x_i^{-\beta}}$$

The ML estimator of the parameter α is:

$$\hat{\alpha}_{ML} = \frac{n}{\sum_{i=1}^{n} e^{x_i^{-\beta}} - n} \tag{26}$$

In the same way find The maximum likelihood estimator of the parameters $\delta\delta_1, \delta_{2,}$ and λ are:

$$\widehat{\delta_{J_{ML}}} = \frac{m}{\sum_{j=1}^{m} e^{y_j^{-\beta}} - m},\ \widehat{\delta_{J_1 ML}} = \frac{m_1}{\sum_{j_1=1}^{m_1} e^{y_{j_1}^{-\beta}} - m_1},\ \widehat{\delta_{J_2 ML}} = \frac{m_2}{\sum_{j_2=1}^{m_2} e^{y_{j_2}^{-\beta}} - m_2},$$

$$\hat{\lambda}_{\mathrm{ML}} = \frac{w}{\sum_{h=1}^{w} e^{z_h^{-\beta}} - w} \ldots \tag{27}$$

Where $m, m_1, m_2,$ and w are simple sizes.

3.2 *Percentile Method*

Suppose that $X \sim ICh(\alpha, \beta)$, the estimator of percentile is found by decreasing the square sum among the value and probable value of CDF so as [9]:

$$P = \sum_{i=1}^{n} \left[E\left(F(x_i)\right) - F(x_i)\right]^2 \tag{28}$$

$F(x_i) = e^{\alpha\left(1-e^{x_i^{-\beta}}\right)}$ where $E\left(F(x_i)\right) = P_i$ and $P_i = \frac{i}{n+1}; i = 1,2, \ldots, n$

Then Pr estimator of λ_i is attained as:

$$\ln P = \sum_{i=i}^{n} \left[lnP_i - \ln\left(e^{\alpha\left(1-e^{x_i^{-\beta}}\right)} \right) \right]^2 \tag{29}$$

By deriving the equation 34 for the parameter α:

$$\frac{\partial \ln P}{\partial \alpha} = 2\sum_{i=i}^{n} \left[lnP_i - \alpha\left(1 - e^{x_i^{-\beta}}\right) \right] \left(-\left(1 - e^{x_i^{-\beta}}\right)\right)$$

$$2\sum_{i=i}^{n} \left[lnP_i - \alpha\left(1 - e^{x_i^{-\beta}}\right) \right] \left(-\left(1 - e^{x_i^{-\beta}}\right)\right) = 0$$

Hence the percentile estimator of α is:

$$\hat{\alpha}_{Pr} = \frac{\sum_{i=1}^{n} lnP_i\left(1-e^{x_i^{-\beta}}\right)}{\sum_{i=1}^{n}\left(1-e^{x_i^{-\beta}}\right)^2} \tag{30}$$

Also the estimators of the $\delta, \delta_1, \delta_2,$ and λ are:

$$\widehat{\delta_{\mathrm{J}\,\mathrm{Pr}}} = \frac{\sum_{\mathrm{j}=1}^{\mathrm{m}} \ln \mathrm{P_j}\left(1-\mathrm{e}^{\mathrm{y_j}^{-\beta}}\right)}{\sum_{\mathrm{j}=1}^{\mathrm{m}}\left(1-\mathrm{e}^{\mathrm{y_j}^{-\beta}}\right)^2}, \widehat{\delta_{\mathrm{J_1}\,\mathrm{Pr}}} = \frac{\sum_{\mathrm{i}=1}^{\mathrm{m_1}} \ln \mathrm{P_i}\left(1-\mathrm{e}^{\mathrm{y_{j_1}}^{-\beta}}\right)}{\sum_{\mathrm{j_1}=1}^{\mathrm{m_1}}\left(1-\mathrm{e}^{\mathrm{y_{j_1}}^{-\beta}}\right)^2}, \widehat{\delta_{\mathrm{J_2}\,\mathrm{Pr}}} = \frac{\sum_{\mathrm{j_2}=1}^{\mathrm{m_2}} \ln \mathrm{P_{j_2}}\left(1-\mathrm{e}^{\mathrm{y_{j_2}}^{-\beta}}\right)}{\sum_{\mathrm{j_2}=1}^{\mathrm{m_2}}\left(1-\mathrm{e}^{\mathrm{y_{j_2}}^{-\beta}}\right)^2}$$

$$\hat{\lambda}_{\mathrm{Pr}} = \frac{\sum_{\mathrm{h}=1}^{\mathrm{w}} \ln \mathrm{P_h}\left(1-\mathrm{e}^{\mathrm{z_h}^{-\beta}}\right)}{\sum_{\mathrm{h}=1}^{\mathrm{w}}\left(1-\mathrm{e}^{\mathrm{z_h}^{-\beta}}\right)^2} \tag{31}$$

Where $m, m_1, m_2,$ and w are simple sizes.

4. The Simulation

The simulation was made to compare the outputs of the estimation methods using MSE and MAPE. The simulation study was repeated for a number (10,000) and samples of various small, medium, and large sizes were used independently [10].

4.1 *Algorithm of Simulation*

Simulation algorithms have been written to estimate the reliability of the third cases using the MATLAB software, as in the steps below:

1. Random samples $(x_1, x_2, \dots, x_n)$, $(y_{11}, y_{12}, \dots, y_{1m_1})$, $(y_{21}, y_{22}, \dots, y_{2m_2})$, and $(z_1, z_2, \dots, z_n)$ of sizes (n, m, m_1, m_2, w) = (25, 25, 25, 25, 25) (60, 60, 60, 60, 60), and (100, 100, 100, 100, 100) aregenerated..

2. The parameters values of five experiments $(\alpha, \delta, \delta_1, \delta_2, \lambda)$ in table (1):

Table 1
Values of five experiments

Experiment	α	δ	δ_1	δ_2	λ	β	R_1	R_{11}	R_{21c}
1	0.5	0.9	0.7	0.7	0.7	0.7	0.4629	0.1543	0.0197
2	0.2	0.2	1.1	0.9	1.8	0.9	0.1778	0.6400	0.4636
3	0.2	0.4	0.9	1.9	2.4	0.9	0.7333	0.7732	0.4882
4	0.3	0.5	1.2	1.9	1.1	0.8	0.6061	0.5273	0.1597
5	0.4	0.6	2.2	1.3	1.5	0.9	0.6600	0.5298	0.1850

3. Parameters $(\alpha, \delta, \delta_1, \delta_2, \lambda)$ were estimated (ML and Pr) in equations: (26),(27),(30), and (31) respectively.

4. Calculate the mean by $\text{Mean} = \frac{\sum_{i=1}^{L} \hat{R}_i}{L}$

5. The "mean square error": $\text{MSE}(\hat{R}) = \frac{1}{\text{L}} \sum_{i=1}^{\text{L}} (\hat{R}_i - R)^2$

6. The mean absolute percentage error: $\text{MAPE}(\hat{R}) = \frac{1}{L} \sum_{i=1}^{L} \left| \frac{\hat{R}_i - R}{R} \right|$.

4.2 *Simulation Results*

After applying the previous steps of $\mathcal{R}$ for sample size $(n, m, m_1, m_2, w) = (25, 25, 25, 25, 25) = \text{A}$, $(60, 60, 60, 60, 60) = B$, and $(100,100,100, 100, 100)C$.

Table 2
The results of simulation R_1, R_{11} and R_{21c}

E.	s.s.	R_1				R_{11}				R_{21c}			
		ML		Pr		ML		Pr		ML		Pr	
		MSE	MAPE	MSE	MAPE	MSE	MAPE	MSE	MAPE	MSE	MAPE	MSE	MAPE
1	A	0.2931	0.3899	0.1336	0.3513	0.0154	0.0985	0.0219	0.2537	0.0012	0.0241	0.0006	0.0136
	B	0.0934	0.2324	0.1236	0.3443	0.0062	0.0623	0.0133	0.1518	0.0010	0.0157	0.0004	0.0119
	C	0.0560	0.1838	0.1199	0.3415	0.0039	0.0496	0.0103	0.1414	0.0004	0.0126	0.0001	0.0113
2	A	0.0112	0.0728	0.0063	0.0702	0.0055	0.0589	0.0803	0.1667	0.0068	0.0661	0.0610	0.2423
	B	0.0034	0.0440	0.0047	0.0618	0.0024	0.0388	0.0422	0.0587	0.0029	0.0430	0.0569	0.1363
	C	0.0019	0.0333	0.0041	0.0593	0.0014	0.0298	0.0108	0.0463	0.0017	0.0332	0.0457	0.1246
3	A	0.1681	0.2995	0.1702	0.3931	0.0030	0.0423	0.0599	0.1101	0.0063	0.0636	0.0720	0.2644
	B	0.0582	0.1808	0.1521	0.2798	0.0012	0.0271	0.0399	0.0968	0.0028	0.0421	0.0573	0.2577
	C	0.0324	0.1381	0.1355	0.2549	0.0007	0.0209	0.0198	0.0723	0.0016	0.0322	0.0458	0.2253

Contd...

4	A	0.1383	0.2611	0.1427	0.3647	0.0081	0.0707	0.0654	0.1395	0.0042	0.0515	0.0061	0.0709
	B	0.0449	0.1601	0.1298	0.3538	0.0034	0.0462	0.0419	0.1303	0.0018	0.0339	0.0052	0.0682
	C	0.0248	0.1210	0.1246	0.3489	0.0019	0.0349	0.0292	0.1277	0.0010	0.0258	0.0049	0.0677
5	A	0.1648	0.2851	0.1933	0.4301	0.0080	0.0706	0.0484	0.1775	0.0050	0.0565	0.0121	0.1056
	B	0.0541	0.1750	0.1795	0.4189	0.0033	0.0367	0.0201	0.1141	0.0021	0.0367	0.0113	0.1041
	C	0.0295	0.1324	0.1747	0.4151	0.0019	0.0347	0.0114	0.1009	0.0012	0.0280	0.0111	0.1040

4.3 *Discussion of the results*

The results from the three cases of reliability R_1, $R_{11,}$ and $\mathrm{R}_{21\mathrm{c}}$ will be discussed as follows:

About the first case: from Table 1, it was found that R_1 value increases as the parameter δ value increases and decreases as the parameter α value increases also from Table 2, the simulation results showed that the best reliability estimator R_1 is ML estimator in all experiments.

In the second case of reliability: from Table 1 it is found that the value of R_{11} decreases as the values of the (α and δ) parameters increase and increases as the value of the($\delta_1\ and \delta_1$) parameters increases and from Table 2 it was found that the ML estimator is the best to estimate R_{11} when making a comparison within the simulation of the five experiments and different sizes.

Finally, the third case of reliability: from Table 1 it was found that the value of $\mathrm{R}_{21\mathrm{c}}$ increases with increasing parameter values ($\delta_1\ and \delta_1$) and decreases with increasing parameter values (α and δ) also from Table 2 it was found that the ML estimator is the best to estimate $\mathrm{R}_{21\mathrm{c}}$ for all simulation experiments of different sizes and the two criteria: MSE and MAPE.

5. Conclusions

The reliability formula for single-component, (1 + 1), and (2 + 1) models of successive stress strength was derived, by the distribution of, the inverse Chen distribution. A simulation was made to compare the ML and Pr methods in five different experiments for different distribution parameters and at different values for different sample sizes. In general, I found that the best performance was for maximum likelihood estimates.

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