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Assessing effectiveness of specialized private general insurance firms by the application of Malmquist DEA

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Abstract

The Indian insurance industry crossed a significant turning point in FY22, worth $131 billion, and is anticipated to soar to an astonishing $222 billion by FY26. This expansion the country's adoption of insurtech and demonstrates the potential for further progress in the insurance field. While privately held insurance, entities have a greater extent of worldwide knowledge than India's non-life governmental organizations. The Malmquist DEA address for comparing total factor productivity changes, changes in technological advancements, technical efficiency changes and scale efficiency changes in Indian private general insurance organizations. There are 5 private players in India working effectively in the testing period.

Subject Classification: *62P05, 97M40, 90Bxx, 90-XX.*

Keywords: *Data envelopment analysis, Total factor productivity, Efficiency, Technology, Scale efficiency.*

1. Introduction

Indian startups are completely transforming how businesses operate in all sectors. However, firms in the insurance technology area are the most competitive. By integrating data mining, artificial intelligence, and algorithms for prediction to provide clients one-stop insurance solutions,

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insurtech firms businesses are altering the Indian insurance market. An established business intelligence and service company, PGA Labs, recently reported that funding for Indian insurance technology startups has increased at a CAGR of 34% from FY 2017 to FY 20. Specialists forecast that by 2024, internet-based insurance business in India will be value nearly Rs 220 billion. Growth in productivity or technological advancement were linked to the time derivative of the factory's output function, costing function, or profitability function. Using merely observable costs and quantities of outputs and inputs, general production structures can still be empirically implemented. A fundamental paradigm for calculating productive efficiency is presented by Farrell [15], who makes a distinction between technical and allocative efficiency. His contributions established the foundation for Data Envelopment Analysis (DEA) and continue to have a significant impact on productivity and efficiency research.

The elements that determine whether distressed insurance firms merge or go bankrupt are examined by BarNiv and Hathorn [3]. They discover that whereas severely strained businesses face insolvency, moderately distressed businesses typically merge. The study may have been impacted by the economic climate of the period, but it provides important information for industry stakeholders and regulators. Through the Malmquist input, output, or productivity index, this phenomenon breaks out by Diewert, Christensen and Caves [6] in the year 1982. Each of the input output vectors (m^t, n^t) at period t are required to be related to the technology T^{t+1} in the following period in order to generate the source-based Malmquist productivity index. In light of this, we calculate the input distance function for an input output vector (m^t, n^t) at period t in relation to the input requirement set T^{t+1} in the subsequent period. We define the input-based Malmquist productivity index as

$$Q_i^{t+1}(n^{t+1}, m^{t+1}, n^t, m^t) = \left[\frac{E_i^t(n^{t+1}, m^{t+1})E_i^{t+1}(n^{t+1}, m^{t+1})}{E_i^t(n^t, m^t)E_i^{t+1}(n^t, m^t)}\right]^{1/2} \tag{1}$$

In the work of Roos, Lindgren, Fare, and Grosskopf [13], the technology has been described as piecewise linear including of inefficiencies. By considering inefficiencies, the productiveness index can be divided into two parts: one that measures technological advancement, alternatively, frontier technology changes, and the other that evaluates efficiency improvements.

$$Q_i^{t+1}(n^{t+1},m^{t+1},n^t,m^t)=\frac{E_i^{t+1}(n^{t+1},m^{t+1})}{E_i^t(n^t,m^t)}\left[\frac{E_i^t(n^{t+1},m^{t+1})E_i^{t+1}(n^{t+1},m^{t+1})}{E_i^t(n^t,m^t)E_i^{t+1}(n^t,m^t)}\right]^{1/2} \quad (2)$$

This article "Productivity changes in Swedish pharamacies 1980-1989 [13]: A non-parametric Malmquist approach" was developed by Roos, Grosskopf, lindgren and Fare in the year of 1992. They constructed a non-parametric, input-based model for evaluating increases in productivity. They composed productivity of total factor Malmquist indexes to the labor productivity measures. In this study, released in 1994 by Zhongyang Zhang, Grosskopf, Mary Norris Shawna and Rolf Fare productivity growth in various nations was examined from 1979 to 1988. [14] Malmquist productivity metrics were calculated the activity analysis. Change in technology along with effectiveness change were the two measures that made up these. They observed that due to technological advancement, productivity growth in the United States is marginally above average. Japan's productivity growth is the highest in the sample, with efficiency change accounting for about half of the increase.

In this research article Lovell and Grifell -Tatje [17] discussed on flaws in the Malmquist index approach. They proved, this method did not measure on productivity changes in the presence of non-constant scale return and also depend on the magnitude of economies of scale. The previous studies based on Malmquist productivity on the period t+1 output oriented is

$$Q^t(x^{t+1},y^{t+1},x^{t+2},y^{t+2})=\frac{E^{t+1}(x^{t+2},y^{t+2})}{E^{t+1}(x^{t+1},y^{t+1})}\frac{E^{t+1}(x^{t+2},y^{t+2})}{E^{t+2}(x^{t+2},y^{t+2})} \quad (3)$$

The period t+2 output related Malmquist productivity index is

$$Q^{t+2}(x^{t+1},y^{t+1},x^{t+2},y^{t+2})=\frac{E^{t+2}(x^{t+2},y^{t+2})}{E^{t+1}(x^{t+1},y^{t+1})}\frac{E^{t+1}(x^{t+2},y^{t+2})}{E^{t+1}(x^{t+1},y^{t+1})} \quad (4)$$

The output-oriented Malmquist productivity indicators of adjacent periods' geometric means are

$$Q^g(x^{t+1},y^{t+1},x^{t+2},y^{t+2})=(E^{t+1}(x^{t+2},n^{t+2})E^{t+2}(m^{t+2},n^{t+2})| \\ E^{t+1}(m^{t+1},n^{t+1})E^{t+2}(m^{t+1},n^{t+1}))^{1/2} \quad (5)$$

Situation with just one input and one output with scaled constant returns an accurate way to gauge productivity change is via the Malmquist productivity index. This methodology's non-constant return to scale, which permits varied returns to scale and provides skewed measurements of productivity changes, is what makes it unique.

Efficiency and consolidation in the US life insurance market sector have been studied in this research article by Cummins, Tennyson, and Weiss [10]. Researchers addressed the connection between mergers and acquisitions, efficiency, and economies of scale in the US life insurance market. Using analysis of data envelopment, they measured cost and revenue efficiency for the years nineteen eighty-eight to nineteen ninety. Employing the Malmquist approach, efficiency evolution is tracked throughout time. They observed that companies that have been acquired see better efficiency benefits than companies that have not undergone mergers or acquisitions. Businesses with increasing scale revenues & businesses that pose a risk to the economy are more likely to be sought after for purchase. The non-parametric Malmquist Index method is used in this study to try and gauge how successful the life insurance sector is. According to the results, productivity growth in the insurance sector was still relatively low when compared to Malaysia's real economic growth. The capacities of this industry to compete effectively will determine its future growth, just like the manufacturing sector. An essential source of comparative advantage in the age of internationalization would be the ability to provide products and services in the most effective way possible. The research results implied that technical advancement and technical efficacy both contribute to the industry's overall rise in productivity [26].

In Japan the Productivity & Efficiency change of general insurance companies [16] this paper measured the efficiency from linear programming problem which did not allow output slack in output-based analysis of data envelopment. This analysis used data on Japanese non-life insurance firms for the period of nineteen eighty-three to nineteen ninety-four. They used the result is total factor productivity change is equal to the efficiency change multiplied with the technological change. The development clarified how changes in technology and technical efficiency, which are employed by insurance companies' envelopment analysis techniques, affect overall production. They found that some companies faced an increase in productivity and others was experienced with lower productivity due to managerial policy. They were analyzed research period is nineteen ninety-five to two thousand one (Assessing insurance businesses' productivity and efficiency using the Malmquist index: Portugal 2005 case study authored by Barros, Barroso and Borges [4]).

The Greek Insurance Industry in the year of 2010 using A Malmquist Index, this article written by the following authors are Nektarios and Barros [24]. The original article employed the Malmquist methodology to test three distinct types of insurance companies: non-life, life, and mixed. They discovered that the non-life sector experienced an average annual productivity growth of six percentage points due to pure technical, scale efficiency, and technology changes; the life sector, on the other hand, saw an average annual productivity growth of sixteen percentage points, and mixed insurance companies had the lowest productivity at three percentage points. Another study focused on Malaysia life insurance industry's efficiency change and technical change by using the method of analysis of data envelopment. They used eleven sample of life insurance firm's data from the year of two thousand to two thousand five & used as two inputs, two outputs for testing. They found on average of the productivity of total factor in life insurance industry focused on both efficiency and technical change, mainly scale efficiency rather than pure efficiency. [27] [25] Efficiency and Productivity of Indian Life Insurance Industry authored by Chakraborty, Dutta and Sengupta [8] in the year of 2012, this article investigated that Indian Life insurance industries' technical and productivity growth used by the model of DEA and Malmquist productivity index approach. The industry was proposed to be more efficient in non-life and life business than in life insurance. It was also proposed that company size, distribution ratio, and claim ratio had a favourable long-term impact on technical efficiency. The above mentioned authored again testing that Indian non-life insurance industry by the method of stochastic frontier inefficiency-effect model for testing productive efficiency & Malmquist index for dynamic productivity index. Also, they interpreted the difference between parametric and non-parametric approaches. Their conclusion and suggestion about the analysis is on positive relation with net premium earned to net claims, operating expenses & total investment and also eight firms were earned gain in total factor productivity growth [8].

"Impact of privatisation on productivity: A non-parametric analysis of the Indian insurance sector," Dutta compared both life and non-life insurance sectors excluding public sector in India using Malmquist Indices approach. [12] The article's ultimate conclusion was that, for various reasons, non-life insurance expanded while the productivity of the life insurance business deteriorated. He also suggested that technological improvement very much needed for Indian life insurance firms to improve the efficiency and productivity & also to improve the penetration level of non-life insurance sector to concentrate on scale efficiency. The pure technical efficiency will need to improve by both sectors using the physical efficiency of inputs. Finally, he concluded that to improve productivity and gain of both firms should focus on technological improvement, market expansion with higher level of penetration, skilled employees and marketing strategies. Tanuj Mathur and Ujjwal Kanti Paul's [23] book Performance Appraisal of Indian Non-Life Insurance Companies: A DEA Approach. Their work was based on a year-long observation of twenty Indian non-life insurers. The technical efficiency of each was tested using the DEA method, and the total technical efficiency was tested using the ordinary least square regression approach. The author gave suggestion to inefficiency insurers will focus on productive scale size and they will require to improve operating size. To check the overall technical efficiency, technical reserve & net earnings were positive impact and management expense was negative impact. They finally concluded that one insurer had adequacy margin, remaining insurers had lower ratios. To overcome this issue all insurer must looking the management of working capital.

This research (2015) used the DEA approach to analyse the non-life insurance market in South Africa from 2007 to 2012 [1]. The focus was on efficiency, productivity, and returns to scale economies. It represented the efficiency and productivity growth of non-life insurer with Malmquist index. The output of this paper, fifty percent of insurers was inefficiencies & twenty percent of insurers was in optimal scale. These insurance firms will consider the technological changes will lead to improve the productivity. The regression test revealed that product diversification, reinsurance & leverage had significant role in efficiency and constant returns to scale. The non-life insurer growth level based on management & regulation of the industry. Efficiency and productivity analyses of general insurance firms in India [7]: A Malmquist approach this article published in the year of 2018. This author collected sample of twelve insurers of general in India for the time period 2006 to 2015 based on two input and output framework. This article applied the techniques like DEA & Malmquist TFP index. The result of this article, technical efficiency and scale efficiency scores was more in public sector than private sectors. But using Malmquist index test suggested that private sectors are more efficient than public sectors.

Ilyas and Rajasekaran [18] authored an empirical inquiry on productivity and efficiency in the Indian non-life insurance industry. They used three methods like the DEA bootstrap, bootstrap regression and bootstrapped Malmquist index to test efficiency and productivity of non-life insurance market. The first method's output was the non-life insurance sector, which was more cost-effective for public sectors than for private ones and was also somewhat technological, scalable, economical, and allocatively efficient. The second approach comes to the conclusion that insurer productivity was a result of using the best technology. The last method suggested that size & reinsurance is had negative relationship with efficiency and also positive relationship with age efficiency. The first method's output was the non-life insurance sector, which was more cost-effective for public sectors than for private ones and was also somewhat technological, scalable, economical, and allocatively efficient. The second approach comes to the conclusion that insurer productivity was a result of using the best technology. Also, they tested the growth in productivity by the method of Mann-Whitney U between public & private sectors. The insurance sector's extremely low level of productivity during the observation period was the result of the first method. According to the second technique, state-owned insurers' productivity growth rate was lower than that of the private sector. Ilyas and Rajasekaran [19] use the Färe–Primont index approach to assess the productivity of India's non-life insurance industry. Their research highlights the impact of management techniques and technical advancements in productivity growth by exposing differences in efficiency throughout organisations. Sarab, Amirteimoori, Malek and Kordrostami [28], they talk about the cost efficiency analysis using DEA framework in the sugar industries. This study identifies a certain convex cone and a subset to control annual inflation to keep unit cost efficiency under fluctuating input costs. A new DEA model is presented in this study to assess DMU cost efficiency in non-competitive marketplaces, where DMUs could affect input costs rather than assume they are constant. Because the model establishes an "acceptable set" of input costs, effective DMUs can continue to operate efficiently within this range. This set must be determined, cost changes must be evaluated both mathematically and visually, and the model's efficacy must be illustrated using real-world examples. Decision-makers can strategically regulate input prices without compromising output with this method, which helps businesses retain efficiency in the face of cost volatility. This might be expanded to include situations with erratic output prices in future studies.

Author: Shoaib Alam Siddiqui [29]; Total factor productivity increase of Indian life insurance companies: A Malmquist method (2021). This author tested the productivity growth of 24 Indian life insurance company over the period of 2012-2016 IRDAI annual report data. The output of the result indicated that private life insurers had high productivity growth than public sector. He also suggested that average total factor productivity of life insurer was increased over the study period that is improvement in efficiency and technology. Quantifying the overall factor productivity of the non-life insurance market in China Provincial: A Malmquist index approach using the DEA in 2021, written by Yu et al. [30] This paper investigated 31 non-life insurance companies from China in the period of 2004-2017. This article tested by using the method of DEA- Malmquist the total amount of factor productivity in non-life insurers increased by the observation time period. Also interpretated that technological progress in western region had rapid growth compared with eastern region and based on these results non-life insurance market to frame the business and development strategies.

Changes in total amount of factor productivity of ensures non-life in India: A bootstrapped Malmquist approach authored by Ilyas & Rajasekaran [20]. They were used three different types of methods to test the non-life Indian insurers. Using the greatest available technology has led to an improvement in productivity, as indicated by the results of the bootstrapped Malmquist index model. The following model, a truncated regression, indicated that reinsurance and size had a favourable effect on age productivity growth but a negative influence on productivity. Finally, Mann-Whitney U, Wilcoxon W and Kolmogorov -Smirnov Z test method interpreted that there was no significant different between both public and private insurer's productivity growth. In 2023, Reza Mahmoudi and Ali Emrouznejad wrote a multi-period performance analysis of airlines using a Malmquist index approach and a game called SBM-NDEA [22]. They were used bargaining game theory, network data envelopment analysis and slack based measure methodology. They were tested the overall efficiency scores, operational efficiency of Iranian airlines industry over observation period. Slack and surplus values were supplied by this model for every inefficient DMU. These data show how the owners of the inefficient DMU were able to pinpoint their areas of weakness and, by using their reference sets, come up with superior fixes. This model will be combined with the Malmquist index in order to compute changes in efficiency over the course of the observation period. Using Data Envelopment Analysis (DEA) and the Malmquist Index, Bağcı and Koçyiğit [2] investigate how decentralisation affects the technical efficiency of public hospitals in Turkey. According to their findings, decentralisation increases efficiency, mostly due to technical developments rather than structural adjustments. The study offers insightful policy recommendations, highlighting the necessity of ongoing workforce development and technological investments. Despite having a solid methodology, it is devoid of qualitative analysis and could not be as applicable to different healthcare systems. All things considered, the study is well-done and has significant ramifications for hospital administration and policy-making. A metafrontier-biennial cost Malmquist productivity index is used by Du et al. [11] to evaluate the productivity of China's urban water supply sector. Their results draw attention to regional differences in efficiency and stress the importance of cost control and technology in raising production. Li et al. [21] use the Malmquist Index and super-efficiency DEA to evaluate the effectiveness and determinants of sustainable forestry development in China. Their results draw attention to regional variations in efficiency and stress the importance of policy assistance and technology developments in enhancing sustainability.

In 2023, Capeletti, Amado, and Santos [5] used data envelopment analysis and the quality-adjusted Malmquist index to examine the performance of primary health care services. The Malmquist productivity index was applied in the area of productivity and quality of the primary care services, and the author used the DEA methodology to investigate the volume of activities performed and taking into consideration the impact of these activities by using a quality indicator directly related to primary health care outcomes. The outcome of the study was that primary health care services needed to be appropriately planned for and structured, taking into account the range of services provided.

Summary of previous research studies related to Indian non-life insurer using the method of DEA & Malmquist index.

Previous study & year	Objective of the study	Methodology	Insurance industry	No. of DMU's	Study period	Input samples	Output samples	Result
Sinha & 2007b	Productivity, Technical, scale efficiency, & pure technical change	DEA & Malmquist index	Non -life	12	2003-2004	Operational costs, agent count, and equity capital	Income from net premiums and gross income	**1.** Private insurance firms have slightly higher pure technical efficiency than public insurance companies, but public insurance companies outperform them in terms of technical efficiency. **2.** Compared to public insurers, private insurers have had a high rate of productivity increase.
Dutta [12]	Productivity changes	Malmquist index	Both Life & non-life	13	2005-2009	Labour costs, material and business service costs, and overall investment expenses	Income from investments and premiums received	**1.** Life insurers' production has decreased as a result of both pure technical inefficiency and technology inefficiency. **2.** The advancement of technology has led to an increase in non-life insurers' productivity.

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Sinha and Khan & 2014	Productivity change	Malmquist index	Non-life	12	2004-2012	Commission and Management Expenses	Gross Investment Income; Net Premium	**1.** 56% insurers show a growth in the productivity but the growth rate is low. **2.** public sectors are not performed well. **3.** need for the investment in technology to improve productivity.
Mandal and Dastidar &2014	Technical efficiency, purely technical	DEA	Non-life	12	2006-2009	operating expense; commissions paid	Incurred Claims; premium earned	**1.** private non-life insurers are severely affected by the economic crisis of 2008. **2.** public insurers performed better than their private companies.
Bawa and Kaur &2014	Scale efficiency, technical efficiency, and only technical efficiency	DEA	Non-life	4	1991-2011	shareholders fund, operating expenses	Total income	In the time leading up to the change, the four public insurers performed better.

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Chakraborty &2016 b	Technical efficiency, purely technical efficiency, scale efficiency and Productivity change	DEA and Malmquist Index	Non-life	4	2008-2014	running costs and investment	Income from investments and net premium	**1.** The United Indian Insurance Company outperforms the other three public insurers in terms of scale and technology. **2.** The productivity of the national insurance firm surpasses that of the other three public insurers.
Sinha &2017a	Technical efficiency	DEA	Non-life	16	2013	Total premium income as well as operational expenses	Operating earnings, assets under management, and benefits obtained	**1.** In terms of technological efficiency, state insurers outperform private insurance. **2.** There exists a robust correlation between the insurer's level of regulatory solvency ratio and technical efficiency.
Sinha &2017b	Technical efficiency	DEA	Non-life	15	2009-2013	Net premium income and operating expenses	Benefits received; managed asset	**1.** The efficiency of state insurers surpasses that of private insurers. **2.** technical efficiency and the level of the regulator solvency ratio has a strong relationship.

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Chakraborty and Harper & 2017	Productivity change	Malmquist Index	Non- Life and life	18non-life &14life	2008 to 2015	Total investments, equity capital, and operating expenses	Net premium, investment income	**1.** The productivity regression in the life and non-life insurance sectors is 11.8 percent and 0.4 percent, respectively. **2.** The impact of the claim ratio, distribution ratio, and input cost ratio on the productivity of life and non-life insurers is comparable.
Chakraborty [7]	Productivity change	Malmquist index	Non-life	12	2006-2015	Operating Costs and Capital Shares.	Earned net premium and investment income	**1.** The efficiency of public insurers surpasses that of private insurers. **2.** Private insurers' mean productivity growth has outpaced that of public insurers.
Shoaib Alam Siddiqui & 2021	Total factor Productivity Growth	Malmquist	Life	24	2013-2017	Commission paid, equity capital, debt capital, operating expenses	Claims paid, income from investment	**1.** Life insurance companies average annual growth is 27.6% **2.** Insurance industry growth depends on Private life insurance companies
Ashiq Ilyas & 2022	Productivity changes and efficiency	Bootstrap Malmquist	Non-life	15	2005-2016	Labour and business services, equity capital, debt capital	Net claims, Investments	**1.** Indian non-life insurers have significant progress in total factor productivity. **2.** Public insurers less productivity growth than private insurers.

As per the previous studies concern, authors (Chakraborty [7], Sinha &2017b) are tested the productivity and efficiency of Indian non-life insurers both public and private insurer using the model of DEA & Malmquist index. In this particular study only to test the private players in insurance industry of Indian using the method of DEA & Malmquist index. The main objective of this research work to test the efficiency level and technological changes of private insurers. Because they will easily adopt any new technologies to reach the highest market level. Indian private insurers are entered into the insurance market in the year of 2001 onwards. They are all have the equal experience than public sectors [20]. So, in this article analyses and test the efficiency level Indian private insurers.

Methodology

This study measures changes in productivity and breaks down those changes into technical and technical efficiency changes [14] using the DEA linear programme approach [9] and [13] a Malmquist total factor productivity index (Fare et al., 1992). A Malmquist productivity change index based on output as

$$P_o(N_{t+2}, M_{t+2}, N_{t+1}, M_{t+1}) = \left[\frac{Q_o^{t+1}(M_{t+2}, N_{t+2})}{Q_o^{t+1}(M_{t+1}, N_{t+1})} X \frac{Q_o^{t+2}(M_{t+2}, N_{t+2})}{Q_o^{t+2}(M_{t+1}, N_{t+1})} \right]^{\wedge 1/2} \tag{6}$$

In relation to the production point (M_{t+1}, N_{t+1}), this shows the productivity of the production point (M_{t+2}, N_{t+2}). Positive growth in total factor productivity from t+1 to t+2 is indicated by a value greater than one. Actually, according to Caves, Christensen and Diewert [6], this measure is the geometric mean of two output-based Malmquist total factor productivity indices. Period t+1 technology is used by one index, and period t+2 technology is used by the other. We need to compute the component distance functions using equation (6), which requires solving linear programming issues.

By assuming return of Constant scale technology, related to output linear model to calculate $Q_o^{t+1}(M_{t+1}, N_{t+1})$ is identical to

maximum of $_{\phi,\lambda}\, \phi$

$$\begin{aligned} -\phi y_i + Y\lambda &\geq 0, \\ X_i - X\lambda &\geq 0, \\ \lambda &\geq 0. \end{aligned} \tag{7}$$

Except that the convexity variable return to scale, limitation lifted, and time subscripts are now present

$$[Q_o^{t+1}(M_{t+1}, N_{t+1})]^{-1} = \text{maximum of}_{\phi,\lambda}\ \phi$$
$$-\phi y_{it+1} + Y_{t+1}\lambda \geq 0,$$
$$X_{it+1} - X_{t+1}\lambda \geq 0,$$
$$\lambda \geq 0. \quad (8)$$

Rest 3 linear programming models are simple variants of this:

$$[Q_o^{t+2}(M_{t+2}, N_{t+2})]^{-1} = \text{maximum of}_{\phi,\lambda}\ \phi,$$
$$-\phi y_{it+2} + Y_{t+2}\lambda \geq 0,$$
$$X_{it+2} - X_{t+2}\lambda \geq 0,$$
$$\lambda \geq 0. \quad (9)$$

$$[Q_o^{t}(M_{t+2}, N_{t+2})]^{-1} = \text{maximum of}_{\phi,\lambda}\ \phi,$$
$$-\phi y_{i,t+2} + Y_{t+1}\lambda \geq 0,$$
$$X_{i,t+2} - X_{t+1}\lambda \geq 0,$$
$$\lambda \geq 0. \quad (10)$$

$$[Q_o^{t+2}(M_{t+1}, N_{t+1})]^{-1} = \text{maximum of}_{\phi,\lambda}\ \phi,$$
$$-\phi y_{it+1} + Y_{t+2}\lambda \geq 0,$$
$$X_{it+1} - X_{t+2}\lambda \geq 0,$$
$$\lambda \geq 0. \quad (11)$$

When comparing production points to technology from different eras in linear programming equations 10 and 11, the ϕ parameter does not have to be > 1, as it is required when calculating efficiency. The point might be located above the range of possible productions. This will probably happen in equation 10 when comparing the technology of period t+1 to the production point from period t+2. In the event of technological advancement, a value of ϕ<1 could exist. It should be noted that although it is less common, it might also happen in linear programming equation 11 if technical regress has happened. It's important to remember that in the linear programming above, the values of the ϕ & λ are probably going to vary. Moreover, keep in mind that each company in the sample needs to have the aforesaid linear programming calculated. Also take note of the fact that each firm will require a separate linear programming calculation as more periods are added. With N=20 businesses and T=10 time periods, we would get 560 linear programming if we were to compute (3T-2) linear programming if we had T time periods.

Scale Efficiency

Dividing the constant rise in technical efficiency into two parts: pure variable technical efficiency and scale efficiency is one way to build upon this tactic. To do this, two more linear programming calculations must be made. These would entail performing linear programming again while adding convexity constraints to each iteration. In other words, one would figure out the distance functions in relation to VRS technology. The scale efficiency effect can then be calculated residually using the methods described above, utilizing the CRS & VRS numbers. In this scenario, there would be more linear programming for N firms and T time periods.

Data, inputs & outputs

A study by Ashiq Mohd Ilyas and Rajasekaran [20] examined and evaluated samples obtained from non-life insurers in India that are privately held and state-owned. Since state-owned insurers have greater experience than private insurers in their particular industry, these samples are not homogeneous. The private insurer performance analysis and testing period from 2016 to 2021 was the primary focus of this study. The information gathered from the IRDAI annual report for 14 Indian private companies, with the sample period spanning from 2016 to 2021. With the assistance of experts, all Indian non-life insurers are utilizing technological breakthroughs across several fields throughout this period. Therefore, this is a sample period for examining general insurer performance. Three phases of the performance analysis of the technologies employed by various Indian insurers will be tested in this article. [12] The author, Dutta & 2013, discussed inputs such as labour costs, business service costs, and total investments. However, the study's goal is to use inputs and outputs that are related to finances, marketability, and customer satisfaction in order of priority. [20] Two outputs are used in this research article (Ashiq Ilyas & 2022), but the outputs in this specific research work are investment, interest/dividend and rent received, current liabilities, profit/loss on sale/redemption of investment, net incurred claim, profit/loss transfer to shareholders account, and status of grievances resolved. Operating costs, equity capital, and total investments were used in this article (Chakraborty & Harper 2018). The study work expands the inputs are equity & debt capital, operating costs, net premium earned, number of offices distributed, number of policies approved by IRDAI, and status of grievances reported [7].

Empirical analysis and interpretation

Table 1
Summary of statistics for the study period of 2016-2021

Outputs	**Interest/ dividend &rent received**	**Current liabilities**	**Profit/loss on sale/redemption on investment**	**Investments**	**Net incurred claim**	**Profit/ loss transfer to S.H account**	**Status of grievances resolved**
Average	460.6	4519.953	83.48988	7413.431	2484.873	396.4023	1052.429
Sample Std. Dev.	345.2024	3362.941	95.88617	6141.188	1940.156	446.416	961.7792
Maximum	1759.4	14226.18	560.5	30892.18	9781.9	1955.44	4263
Minimum	7.26	23.94654	0.27	255.6683	25.84	7.79	1
Inputs	**Equity &debt capital**	**Solvency ratio**	**Operating expenses**	**Net premium earned**	**No. of distribution of offices**	**No. of policies approved by IRDAI**	**Status of grievances reported**
Average	1969.247	2.207509	952.0885	3260.481	145.131	54.45238	1077.536
Sample Std. Dev.	1694.224	0.698185	729.5214	2592.512	71.36058	70.5147	952.36
Maximum	8600.984	4.4375	3920.12	13032.08	340	466	4406
Minimum	207.2874	1.5025	17.35	37.46	7	1	1

In this specific study, fourteen inputs and outputs are employed, as shown in table 1. This statistical overview demonstrates that we incorporated more inputs and outputs, such as the status of grievances settled, the number of offices distributed, the number of policies approved by IRDAI, and the status of grievances reported, in accordance with the previous reviews. This is an additional benefit of identifying the businesses that operate more effectively.

Table 2

Malmquist index summary of annual mean for the study period of 2016-2021

Year	Technical Efficiency Changes	Technological Changes	Pure Technical Efficiency Changes	Scale Efficiency Changes	Total Factor Productivity Changes
2017	1.005	.845	1.002	1.002	0.849
2018	1.010	.845	1.000	1.010	.853
2019	1.000	1.028	1.000	1.000	1.028
2020	.997	.955	.999	.998	.952
2021	.990	1.033	1.001	.989	1.023
Mean	**1.000**	**.937**	**1.000**	**1.000**	**.938**

Ilyas and Rajasekaran [20], demonstrated in the Malmquist test that there has been a notable improvement in technical efficiency. From the table 2 also demonstrated in this article: all firms' technological efficiency increased in 2017, 2018, and 2019 but decreased in the subsequent years. an overall estimate of the projected value of the progression's technical changes. Improvements in technology have occurred in 2019 and 2021. In particular, there was an improvement of 3.3% in 2021, and 2.8% in 2019. With an emphasis on the variable return to scale technology, pure technical efficiency shows the greatest growth in 2017, at 0.2%, and the lowest change in 2021, at 0.1%. There has been a 1% improvement in scale efficiency changes in 2018 compared to a decline in all preceding years. In conclusion, there have been improvements in total factor productivity in 2019 (2.8%), and in 2021 (2.3%). In general, technical efficiency, pure technical efficiency, and scale efficiency have increased throughout the years, while the remaining metrics have declined during the study period.

Table 3

Malmquist index summary of firms mean for the study period of 2016-2021

Firm	Technical Efficiency Changes	Technological Changes	Pure Technical Efficiency Changes	Scale Efficiency Changes	Total Factor Productivity Changes
1 Bajaj	1.000	1.014	1.000	1.000	1.014
2 Cholamandalam	1.000	1.018	1.000	1.000	1.018
3 Future	.984	.848	1.000	.984	.835
4 HDFC	1.000	.948	1.000	1.000	.948
5 ICICI	1.000	.836	1.000	1.000	.836
6 IFFCO	1.000	1.003	1.000	1.000	1.003
7 Liberty	.981	.966	1.000	.981	.947
8 Magma HDI	1.032	.999	1.000	1.032	1.032
9 Raheja	1.000	.719	1.000	1.000	.719
10 Reliance	1.000	1.038	1.000	1.000	1.038
11 Royal Sundaram	1.000	.976	1.000	1.000	.976
12 SBI	1.008	.898	1.007	1.002	.905
13 Shriram	1.000	1.008	1.000	1.000	1.008
14 Tata AIG	1.000	.911	1.000	1.000	.911
Mean	1.000	.937	1.000	1.000	.938

From the table 3 shows that the technology, pure, scale, technical efficiency, and total factor productivity changes are arranged in Malmquist index testing. The mean technical efficiency, pure technical efficiency, and scale efficiency all fluctuate throughout the course of the study when matched with every non-life insurer. Only two of the fourteen insurers Magma and SBI show an improvement in technical efficiency changes of 3.2% and 0.8%. When it comes to technical advancements, the surviving insurers are at a standstill. Technical efficiency changes for two of the companies (Future & Liberty) declined by 1.6% and 1.9%, respectively. Five firms demonstrated improvements in technological advancements, with Reliance producing the biggest improvement at a rate of 3.8%. A single company has improved its pure technical efficiency changes (0.7%, SBI) during the course of my analysis. In terms of overall productivity, six firms are displayed, with Reliance receiving the highest score of 3.8%. Lastly, the average impact of technical advancements is a poor efficiency of

6.3%. Therefore, every company will be focusing on areas where technology is improving. The 6.2% average decrease in total factor productivity across all enterprises is also a concern. Companies will focus on strengthening the underwriting process based on the outcome.

Recap and Recommendations

The amount of efficiency exhibited by private insurance providers in India was revealed by this study. This research only looks at India's major private players. Every private non-life insurance provider is in perfect competition with one another. With the resources at their disposal, they demonstrate their level of efficiency. With various inputs and outputs collected between 2016 and 2021, the Malmquist index approach methodology is used to verify these players' proficiencies. The primary inputs consist of commission paid, reinsurance premium, operating expenses, and agents hired. The results include assets under management, investment profit income, net premium earned, and net claims. This model links firm- and year-level outcomes to show how private players will advance technical advancements in the near future. After a few years, technology adoption may result in higher costs, but this will lead to improved cost efficiency. This outcome will shape opinions towards technology adoption. This may simplify the reserving procedure, provide an easy estimation of responsibility, and expedite the settlement of claims. These applications have the potential to draw in clients and tap into India's largest market. The private insurance sector is able to quickly adopt new technologies since it has its own domestic expertise with technology management. In the upcoming year, general insurers' next priority will be raising their overall factor productivity. Insurance firms use several inputs to achieve maximum output, which is represented by their total factor productivity. Throughout the research period, there has been a decline in the overall factor productivity's efficiency level. Five players stand out among the 14 competitors, demonstrating their efficacy across all categories, including technical, technological, pure, scale, and overall productivity. The examination of 14 DMUs revealed a movement towards the effective modifications is that general insurance companies reached in the year of 2019. To minimize costs or make the best use of scarce resources in light of technological improvements, the companies will continue to explore the study and use growth theory or the cost efficiency model.

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