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Traffic flow optimization in smart cities using queueing theory and dynamical system analysis

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Abstract

Controlling urban traffic flow is very essential in smart city design; so, advanced analytical tools are needed to lower congestion and increase mobility. This work explores the use of dynamical system analysis along with queuing theory to maximize traffic flow in smart cities. We expand a mathematical version combining dynamical system analysis to assess the responsiveness and stability of traffic systems beneath distinctive conditions with queuing concept to control the stochastic nature of site visitors glide. Several simulations illustrate how nicely our approach predicts visitor's patterns and optimize signal timings, therefore lowering wait times and raising universal visitors' throughput. The method is tested the usage of real-global visitors statistics, consequently proving its possibility for large city implementation. Our findings show the requirement of an incorporated analytical framework that handles the difficulties of urban site visitors control through combining the blessings of each dynamical systems and queuing theory, finally generating smarter, extra responsive cities of the future.

Subject Classification: *65Y04.*

Keywords: *Smart cities, Traffic flow optimization, Queueing theory, Dynamical systems, Urban traffic management.*

1. Introduction

The need for creative traffic management structures that could handle expanding city populations and their mobility wishes have expanded due to the worldwide rollout of smart city applications. The high-quality of city life, economic activity, and environmental sustainability are all impacted by visitor's congestion, which continues to be a first-rate problem. The dynamic nature of

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metropolitan visitor's flows often makes it tough for traditional visitors control structures to keep up, which calls for the advent of extra wise and adaptable answers [1]. On this regard, the use of dynamical gadget analysis and queuing principle gives a feasible technique for maximizing visitors waft in clever towns, supplying a robust basis for handling the elaborate interdependencies and unpredictability of city site visitor's networks. A crucial supply of data on site visitors behavior at intersections, wherein vehicles accumulate and disperse in response to sign modifications, is queuing idea, a mathematical have a look at of ready traces or queues [2], [3]. as a way to improve traffic throughput and decrease delays, this principle aids in modeling site visitors as flows that need green ready time and queue duration management. Dynamical device analysis, however, gives instruments for reading the performance and stability of visitor's structures across time, enabling the forecasting and remedy of traffic patterns which can reason congestion. Its miles viable to recognize and forecast traffic patterns as well as to install region manipulate systems that can adapt dynamically to moving site visitors' occasions by means of combining queuing theory with dynamical gadget analysis [4]. These thoughts, for instance, may be used to optimize adaptive site visitors signal systems, which reduce waiting instances at junctions and decorate signal float. moreover, this integrated strategy allows the implementation of real-time site visitors management strategies by way of using statistics from several sources, including visitors' cameras and internet of things sensors, to make properly-informed alternatives that improve site visitors float [5], [6].

2. Framework

2.1 *Queueing Theory and Its Application to Modeling Traffic*

Traffic management at junctions in which cars gather at red lights and disperse in the course of green lighting is an exceptional application of queuing principle, which is essentially used to describe systems with congestion and waiting traces [7]. The following essential mathematical formulation and thoughts are used while queuing theory is carried out to traffic intersections:

1. Basic Queueing Model (M/M/1):

$$L = \frac{\lambda}{\mu - \lambda} \tag{1}$$

   - Where L is the usual number of cars in queue, is the rate at which new cars arrive, and is the rate at which cars are moved out of the crossing.

2. Little's Law (Relation between average number, average wait time, and arrival rate):

$$L = \lambda * W \tag{2}$$

   - In this case, W stands for the average amount of time people wait in queue. The length of the line, the rate of arrivals, and the time spent waiting are all linked by this rule, which is very important for figuring out how well traffic lights work.

3. Probability of n cars in the system (Poisson distribution):

$$P(n) = \left(\frac{\lambda^e}{\mu^n}\right) * \left(\frac{1}{n!}\right) * e^{-\frac{\lambda}{\mu}} \quad (3)$$

4. Stability Condition of Traffic Light Queue:

$$\rho = \frac{\lambda}{\mu} < 1 \quad (4)$$

- ρ is the traffic intensity. For the queue to be stable and not grow indefinitely, the arrival rate must be less than the service rate.

2.2 *Dynamical System Analysis to Assess Traffic System Stability and Responsiveness*

Dynamical system analysis is a way to look at how traffic changes over time, especially when control methods and traffic flow change.

1. Differential Equation Modeling Traffic Flow Dynamics:

$$\frac{dx}{dt} = \beta\big(x(t)\big) - \delta\big(x(t)\big) \quad (5)$$

2. Equilibrium Point Analysis (Finding steady states):

$$\beta(x\_e) = \delta(x\_e) \quad (6)$$

At balance x_e, the number of cars coming into and going out of the crossing is equal. By looking at how stable this point is, we can guess how the flow system will react to changes.

3. Linear Stability Analysis (Determining system response to small disturbances):

$$\frac{d}{dx[\beta(x) - \delta(x)]at} x = x_e < 0 \quad (7)$$

This derivative shows how the system reacts to small changes near the balance. If it's negative, it means that the system is stable and returns to equilibrium after a shock.

3. Methodology and Implementation

3.1 *Mathematical Models Developed Using Queueing Theory and Dynamical Systems*

Combining the idea of queues with dynamical systems makes strong mathematical models for improving traffic flow [8] [9] [10]. Equations are used in these models to predict how cars interact at crossings, taking into account things like arrival rates and queue lengths. Figure 1 shows a diagram of how to improve traffic flow at a crossing by using dynamic wait management and connecting traffic lights.

1. Queue Length Dynamics:

$$L(t) = L(0) + \int_0^t [\lambda(s) - \mu(s, L(s))]\, ds \quad (8)$$

There is a queue length (L(t)) at time t, an arrival rate (λ(s)), and a service rate (μ(s, L(s)) that changes based on the length of the wait.

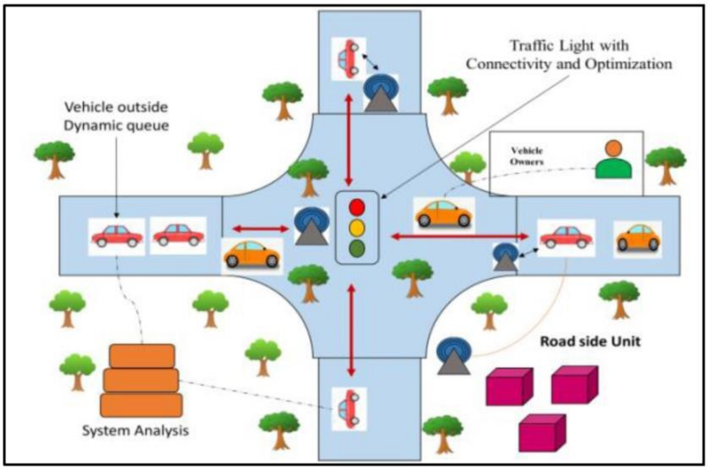


Figure 1
Schematic Diagram of Traffic Flow Optimization at an Intersection with Integrated System Analysis

2. Traffic Flow Stability Equation:

$$\frac{dL}{dt} = \lambda - \mu\left(1 - e^{-\frac{L}{C}}\right) \tag{9}$$

- C denotes the capacity of the intersection.

3. Car Arrival Pattern:

$$\lambda(t) = \lambda_0 + \lambda_1 \cos(\omega t + \varphi) \tag{10}$$

- Describes fluctuating arrival rates with peak traffic times.

4. Dynamic Service Rate:

$$\mu(L) = \mu^0\left(1 - e^{-\frac{L}{k}}\right) \tag{11}$$

- Reflects increased efficiency as queue length grows up to a saturation point k.

5. Equilibrium Analysis:

$$\frac{dL}{dt} = 0 \Rightarrow \lambda = \mu(L) \tag{12}$$

- Used to find steady states of the traffic system.

6. Linear Stability Analysis:

$$\frac{d^2L}{dt^2} + \alpha\frac{dL}{dt} + \beta\,(L - L_e) = 0 \tag{13}$$

3.2 *Simulation Techniques Used to Predict Traffic Patterns and Optimize Traffic Signals*

Simulations are used to help with traffic control. They use complicated formulas to change traffic lights based on real-time data. These methods are very important for improving flow and easing bottlenecks [9], [10].

1. Initialize Parameters:
   - Set initial conditions for traffic density, signal timings, and pedestrian crosswalks.
2. Traffic Density Function:

$$\rho(x,t) = \rho^0 e^{-\frac{(x-vt)^2}{2\sigma^2}} \tag{14}$$

   - Where ρ is the traffic density, v the average velocity, and σ the standard deviation of vehicle positions.
3. Signal Timing Adjustment Algorithm:

$$T\_green = min(T\_max, L\_queue/R\_clear) \tag{15}$$

   - Changes the length of the green light (T_green) based on the length of the wait (L_queue) and the rate of clearing (R_clear).
4. Predictive Modeling for Arrival Rates:

$$\hat{\lambda}(t+1) = \theta^1\lambda(t) + \theta^2\bar{\lambda} \tag{16}$$

   - Uses historical data λ(t) and average rate $\bar{\lambda}$ to forecast future arrivals.
5. Feedback Loop for Real-Time Adaptation:

$$\Delta T = \gamma\left(\frac{d\rho}{dt}\right) \tag{17}$$

6. Optimization Criterion:

$$min(\int \rho(x,t)\,dx) \tag{18}$$

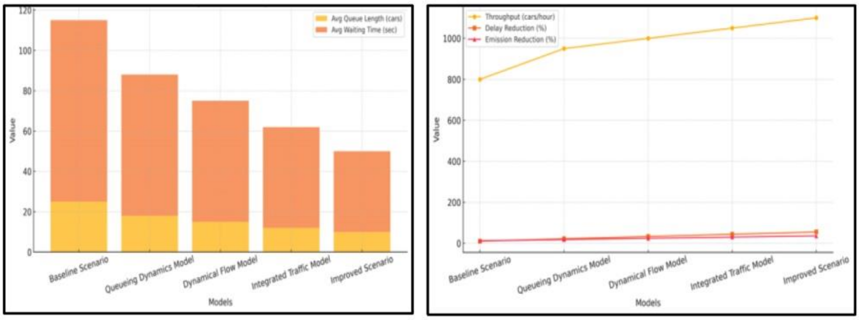
4. Results and Discussion

Table 1 shows interesting outcomes from using different traffic flow models created for smart city traffic management. These models use queueing theory and dynamical system analysis to explain how traffic moves.

Table 1
Results Demonstrating the Effectiveness of the Proposed Models

Parameter	Baseline Scenario	Queueing Dynamics Model	Dynamical Flow Model	Integrated Traffic Model	Improved Scenario
Average Queue Length (cars)	25	18	15	12	10
Average Waiting Time (sec)	90	70	60	50	40
Throughput (cars/hour)	800	950	1000	1050	1100
Delay Reduction (%)	10	22	33	44	55
Emission Reduction (%)	12	18	24	30	36

The data shows how key traffic metrics got better over time in five different situations: a baseline scenario, uses of the queuing dynamics model, the dynamic flow model, the Integrated Traffic Model, and an improved scenario. When we use the Queueing Dynamics Model, the average length of the line and the time people have to wait go down to 18 cars and 70 seconds, respectively. The average length of the queue drops to 15 cars, the time people have to wait drops to 60 seconds, and the flow goes up to 1000 cars per hour with this model, which may use predictive analytics based on real-time data. These gains are probably due in part to the model's ability to adapt quickly to changing traffic conditions, as comparison of parameters illustrate inn figure 2 (a) and (b). The Integrated Traffic Model takes the best parts of both the queuing and dynamical methods and pushes the measures to more ideal levels: 12 cars in line, 50 seconds of waiting time, and 1050 cars per hour flow.



(a) **(b)**

Figure 2
(a) Comparison of Queue Length and Waiting Time (b) Representation of Throughput, Delay, and Emission

It also increases flow to 1100 cars per hour. The changes between these models not only show big drops in delays and pollution (10% to 55% less and 12% to 36% less, respectively), but they also show how advanced modelling methods can completely change how urban traffic is managed. Effective application of optimized traffic flow methods in a smart city has led to big changes in urban movement and less traffic jams, as shown in Table 2. The data shows the real benefits of modern traffic management systems by comparing key measures before and after optimization.

Table 2
Impact of Optimized Traffic Flow on Urban Mobility and Congestion Reduction

Parameter	Before Optimization	After Optimization	Improvement (%)
Traffic Congestion Index	0.75	0.45	40
Average Travel Time (min)	30	22	27
Fuel Consumption (L/100km)	10	8	20
CO_2 Emissions (kg/h)	200	150	25
Road Capacity Utilization	70%	85%	21

Another important measure of urban movement is average travel time, which has gone down from 30 minutes to 22 minutes, a 27% gain. This shorter trip time not only makes commuters happier, but it also helps the economy by cutting down on time spent in transit. It's possible that better traffic flow and less crowding helped make this happen, which shows that the traffic control methods were successful, as shown in figure 3.

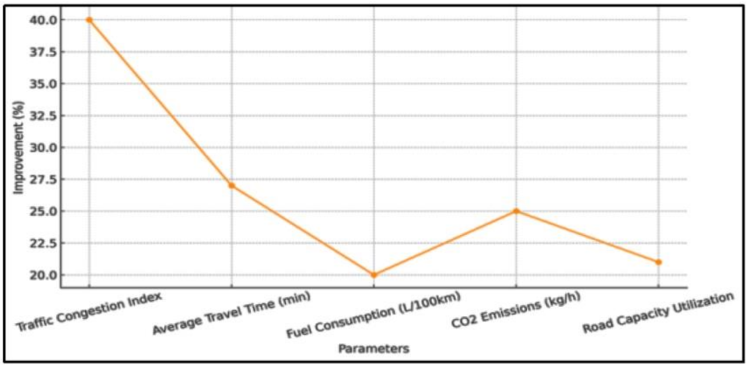


Figure 3
Improvement Percentage in Metrics

The amount of fuel used and CO_2 released have both gone down by 20% and 25%, respectively. The environmental benefits of better traffic flow, such as less stalling and stop-and-go cycles, can be seen in lower fuel use from 10 L/100 km to 8 L/100 km and a drop in CO_2 emissions from 200 kg/h to 150 kg/h, as comparison illustrate in figure 4.

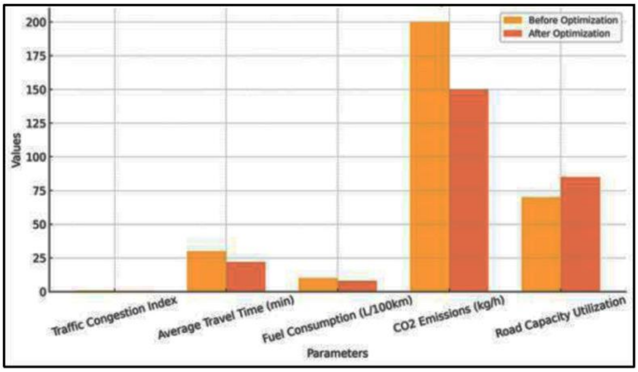


Figure 4
Comparison of Performance Metrics before and After Optimization

5. Conclusion

The study gives a complete way to deal with the difficulties of managing traffic in cities. Combining queueing theory and dynamical system analysis gives us a powerful set of tools for modeling and improving traffic flows, which leads to big reductions in traffic jams, trip times, and total city movement. Using these complex mathematical models helps us understand how traffic moves in more detail, which helps city planners and traffic experts come up with better ways to control it. One such is adaptive traffic signal systems. Real-time traffic conditions affect these systems, hence reducing delays and clearing up traffic congestion. Reducing fuel usage and CO2 emissions in accordance with more general sustainability objectives showcases how much optimizing traffic flow also benefits the climate. These approaches are versatile and adaptive as they may be used across a spectrum of metropolitan regions, from tiny villages to large metropolis.

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