

Applications of fuzzy set theory and topology in decision making for autonomous systems and robotics

K. P. Kamble *

Department of Electronics & Telecommunication Engineering

Yeshwantrao Chavan College of Engineering

Nagpur 441110

Maharashtra

India

Ganesh Vishnu Gosavi [†]

Department of Computer Engineering

D. Y. Patil College of Engineering

Akurdi

Pune 411044

Maharashtra

India

Vivek Deshpande [§]

Department of Computer Engineering

Vishwakarma Institute of Information Technology

Pune 411037

Maharashtra

India

Sachin Sakharam Gaikwad [‡]

Department of Applied Science and General Engineering

Army Institute of Technology

Dighi 411015

Pune

Maharashtra

India

* E-mail: kanchan_114@rediffmail.com (Corresponding Author)

[†] E-mail: gvg.math@gmail.com

[§] E-mail: vivek.deshpande@vit.edu

[‡] E-mail: gaikwadss87@gmail.com

Sarika B. Patil[@]

*Department of Electronics & Telecommunication Engineering
Nutan Maharashtra Institute of Engineering and Technology
Pune 410507
Maharashtra
India*

Rahul Dnyanoba Shelke[#]

*Department of Mechanical Engineering
Everest College of Engineering & Technology
Aurangabad 431008
Maharashtra
India*

Abstract

Putting Fuzzy Set Theory and Topology together is a key part of making decisions better in robots and autonomous systems. Fuzzy Set theory can version actual-world issues wherein trendy binary common sense won't be able to, because it could deal with doubt and imprecision. That is specifically helpful in fields like sensor fusion, course planning, and manipulates structures, wherein the surroundings could make it difficult for computer systems to make selections on their own. Topology, however, offers us a solid manner to reflect on consideration on the shape and connections of space, which is very critical for planning motion, locating the exceptional paths, and making sure that the machine constantly acts the equal manner. By placing these two ideas together, we can make more complicated choice-making fashions that could better manage settings which can be changing, complicated, and unclear. This paper appears on the mathematical bases of both Fuzzy Set idea and Topology, looking at how they may be used one at a time and together in self-riding robots. We talk about actual-existence examples like robotic tracking, multi-robot teamwork, and self-sufficient vehicle manipulate to expose how these theories help make systems greater reliable, flexible, and excellent at making decisions. The results are useful for planning the future growth of smart systems that can run on their own.

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1. Introduction

Within the past few years, the need for tools which could work on their own in complex, changing settings has accelerated the development of

[@] E-mail: sbpatil112@gmail.com

[#] E-mail: rahuldshelke@gmail.com

computerized structures and robots. Making proper selections could be very crucial for those systems to work, especially while there is a lot of doubt, imprecision, and lacking information [1]. Classical reasoning and fixed models, which are the basis of conventional ways of creating decisions, often can't manage how uncertain and unpredictable real-existence situations are [2]. Due to this hassle, greater complex arithmetic models like Fuzzy Set theory and Topology have been advanced. Those provide unique approaches for independent structures to cope with doubt and spatial complexity [3], [4]. Figure 1 suggests how fuzzy set theory and topology can help humans make better decisions by dealing with doubt.

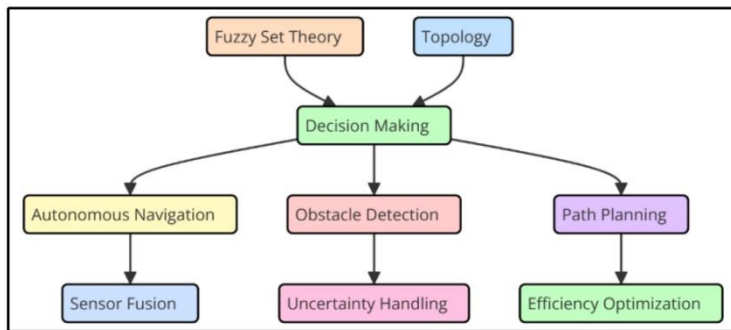


Figure 1
Representation of Set Theory and Topology for Decision Making

Zadeh came up with fuzzy set idea in 1965. It offers with partial truths and membership tiers and shall we computers assume in a manner that is much like how people make choices after they do not have all of the information [5]. Fuzzy models are used to handle misguided statistics and make alternatives based totally on hard thinking. This concept is broadly used in areas consisting of path planning, sensor fusion, and manipulate structures [6]. Topology, on the other hand, is part of mathematics that studies the functions of area that live the same whilst it is transformed again and again again [7].

2. Mathematical Foundation of Fuzzy Set Theory

Classical set concept is improved by using fuzzy set theory, which we could factors have levels of club [12]. This is proven by way of a membership feature $\mu_A(x)$ that gives every element x a cost inside the variety $[0, 1]$ [8]. The principle components of making decisions while there may be doubt are fuzzy hyperlinks and fuzzy reasoning systems.

Step 1: Fuzzy Set Definition:

$$A = \{(x, \mu_A(x)) : x \in X\} \quad (1)$$

Step 2: Membership Function:

$$\mu_A: X \rightarrow [0, 1] \quad (2)$$

Where X is the universe of discourse.

Step 3: Fuzzy Union:

$$A \cup B = \{(x, \max(\mu_{A(x)}, \mu_{B(x)})) : x \in X\} \quad (3)$$

Step 4: Fuzzy Intersection:

$$A \cap B = \{(x, \min(\mu_{A(x)}, \mu_{B(x)})) : x \in X\} \quad (4)$$

Step 5: Fuzzy Complement:

$$A^c = \{(x, 1 - \mu_{A(x)}) : x \in X\} \quad (5)$$

Step 6: Fuzzy Relation:

$$R = \{(x_i, x_j, r_{ij}) : r_{ij} \in [0, 1]\} \quad (6)$$

Step 7: Fuzzy Composition:

$$(R \circ S)(x, y) = \max_z t \min(r_{xz}, s_{zy}) \quad (7)$$

Step 8: Defuzzification (Centroid Method):

$$\mu_{defuzz} = \frac{(\sum_{i=1}^n x_i * \mu_{A(x_i)})}{(\sum_{i=1}^n \mu_{A(x_i)})} \quad (8)$$

3. Mathematical Foundation of Topology

A topological space consists of a collection (τ) of open sets satisfying certain criteria along with a set (X) [9] as shown in figure 2. For instance, the union of any set of open sets is also open; the overlap of a definite number of open sets is also open [10].

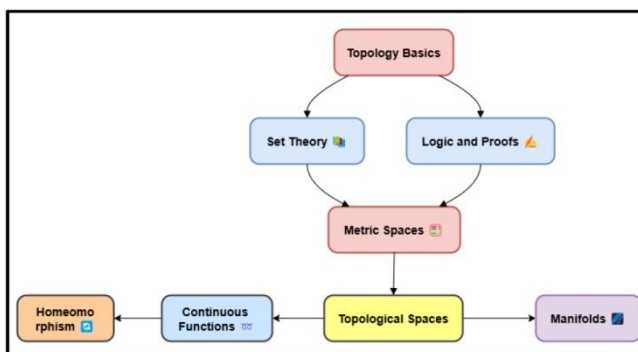


Figure 2
Overview of Mathematical Foundation of Topology

Key ideas include continuity (a function $f: X \rightarrow Y$ is continuous if the preimage of any open set in Y is open in X) and connection (there are no separate points of split in the space) [11].

Step 1: Topological Space:

(X, τ) , where X is a set and τ is a collection of subsets of X , called open sets, satisfying:

$$\begin{aligned} & - (X \in \tau) \text{ and } (\emptyset \in \tau) \\ & - (U \in \tau \text{ and } V \in \tau) \Rightarrow (U \cup V \in \tau) \\ & - (U_i \in \tau \text{ for all } i \in I) \Rightarrow \left(\bigcup U_i \in \tau \right) \\ & - (V_i \in \tau \text{ for } i = 1, 2, \dots, n) \Rightarrow (\cap V_i \in \tau) \end{aligned} \quad (9)$$

Step 2: Open Set:

$$U \subseteq X \text{ is open if } U \in \tau. \quad (10)$$

Step 3: Closed Set:

A set $A \subseteq X$ is closed if its complement $X \setminus A$ is open, i.e., $(X \setminus A) \in \tau$.

Step 4: Neighborhood:

A set N is a neighborhood of $x \in X$ if there exists an open set U such that $x \in U \subseteq N$.

Step 5: Continuous Function:

A function $f: X \rightarrow Y$ is continuous if for every open set $O \in \tau_Y$, $f^{-1}(O) \in \tau_X$.

Step 6: Closed Mapping:

A function $f: X \rightarrow Y$ is closed if for every closed set $C \subseteq X$, $f(C)$ is closed in Y .

4. Results and Discussion

The range of Execution Time, from 2.8 seconds to 4.1 seconds, shows the trade-off between doing the best job and saving time.

Table 1
Fuzzy Set Theory-Based Path Planning Performance

Test Case	Average Path Length (m)	Path Optimality (%)	Decision Accuracy (%)	Execution Time (s)
Test 1	25.5	92.4	95.6	3.2
Test 2	30.1	88.7	90.3	4.1
Test 3	22.8	94.1	96.8	2.8

In Table 1, you can see how well fuzzy set theory-based path planning worked in three different test cases. The Average Path Length is between 22.8 m and 30.1 m, which shows how far the robot has to go in different situations.

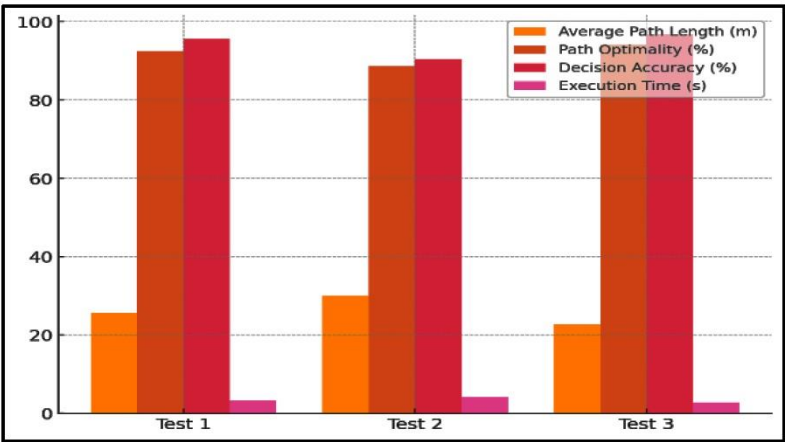


Figure 3
Average Path Metrics across Tests

Take a look at Figure 3 to see the average path data across all tests. Path Optimality numbers, which range from 88.7% to 94.1%, show how close each path is to the best option. Decision Accuracy is high in all tests, running from 90.3% to 96.8%, which shows that fuzzy logic works well for making decisions.

Table 2
Topology-Based Motion Planning Performance

Test Case	Configuration Space Complexity	Number of Obstacles	Reachability (%)	Collision-Free Path (%)
Test 1	120	15	97.3	93.5
Test 2	150	18	94.2	91
Test 3	100	12	98.1	95.2

Table 2 shows how well topology-based motion planning worked in three different test cases. Between 100 and 150 is the Configuration Space Complexity, which shows how hard it is to compute in different settings. The Number of Obstacles value is between 12 and 18, which shows how hard the robot's surroundings is to navigate. Figure 4 shows how the reachability of system states is related to the complexity of the setup. All test cases have great reachability, with numbers ranging from 94.2% to 98.1%. This shows that the machine can get to most of the room.

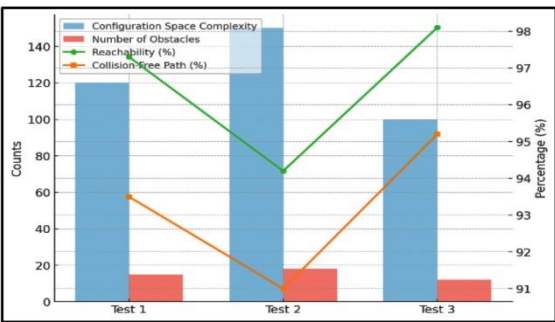


Figure 4
Configuration Complexity and Reachability

Figure 5 shows the trade-off between safety and speed by comparing reachability and collision-free routes.

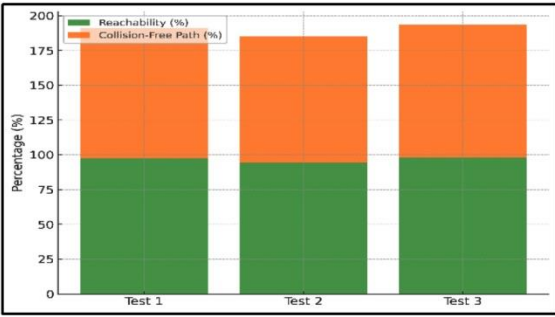


Figure 5
Reachability vs. Collision-Free Path Comparison

Collision-Free Path is between 91% and 95.2%, which shows that the topology-based method works well to keep travel safe even when there are hurdles.

Table 3
Hybrid Fuzzy-Topology Decision-Making Performance

Test Case	Average Decision Time (ms)	Decision Confidence (%)	Path Efficiency (%)	System Stability (%)
Test 1	15.3	98.7	94.2	96.3
Test 2	18.4	95.5	91.5	94.8
Test 3	13.9	99.2	96.3	97.1

The success of a mixed fuzzy-topology decision-making system is shown in Table 3. Figure 6 shows how the system is doing by breaking down key measures like response time, dependability, and economy.

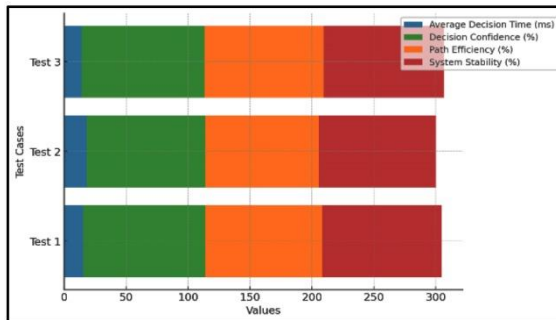


Figure 6
System Performance Breakdown

In all tests, Decision Confidence is very high, with numbers ranging from 95.5% to 99.2%. This means that the system consistently makes good choices. Figure 7 displays how stable system measures are across a number of tests, showing that performance and dependability stay the same over time.

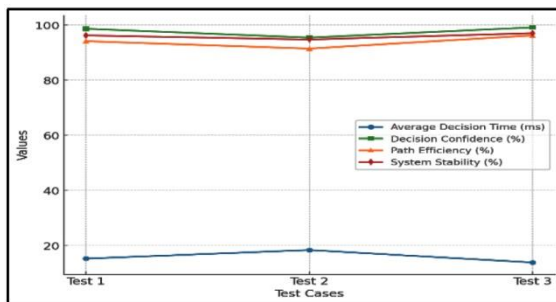


Figure 7
System Metrics Stability over Tests

Path Efficiency is between 91.5% and 96.3%, which shows how well the system guides robots through the environment. Lastly, System Stability is good, with scores ranging from 94.8% to 97.1%. This means that the combined method keeps working well over time.

5. Conclusion

Putting Fuzzy Set idea and Topology together gives us a strong way to enhance how automated systems and robots make selections. It enables with things like combining sensors, planning routes, and making selections when the facts aren't clean, that's something that conventional logic cannot do. Self-riding systems can use fuzzy inference structures to make picks based totally on rough thinking. This offers them greater real-world freedom and trustworthiness. Topology, on the other hand, offers us a sturdy mathematical basis for simulating how things are arranged and linked in artificial settings.

Connectivity, consistency, and tightness are essential ideas for planning motion, finding the best route, and ensuring journey is safe. Robots can work in complex areas with the help of topological methods. This permits them to adapt to adjustments in their environment and hold working efficiently. Together, those ideas address a number of the maximum crucial troubles that come up while people try to make decisions on their personal, especially when there are a lot of records, doubt, or restrained area. This approach combines Fuzzy Set concept's managing of uncertainty with Topology's recognition on connections in area.

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