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Optimal route planning for multi-level multi-choice transportation problem with uncertain parameters using Lagrange's interpolating polynomial

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Abstract

This study introduces a multi-level multi-choice multi-objective linear transportation problem (ML-MCMOLTP) that involves uncertain variables. Multi-level programming addresses scenarios in which multiple decision makers are involved in modeling

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decentralized planning problems inside a hierarchical system. In the model described in this paper, the objective is to maximize profit by selecting the most suitable choice from a set of options for the cost coefficients of the objective function. This is achieved by introducing Lagrange's interpolating polynomial. It is important to note that all parameters involved, such as supply and demand, are treated as uncertain variables. Additionally, I will provide a numerical illustration of a three-level uncertain linear transit problem.

Subject Classification: *90B06, 90C29.*

Keywords: *Transportation problem, Multi-level programming, Multi-choice, Multi-objective programming, Lagrange's interpolating polynomial, Fuzzy goal programming.*

1. Introduction

The transportation problem is an optimization problem that seeks to reduce the total cost of carrying commodities from multiple origins to multiple destinations, while ensuring that supply and demand restrictions are satisfied. There are three parameters involved: cost coefficient, supply parameter, and demand parameter [1]. Hierarchical optimization, or multi-level programming, deals with optimization problems that involve several decision-makers organized in a hierarchical structure [2]. Its objective is to synchronize the choices made by many decision-makers in order to attain an optimal result, hence preventing decision impasses that may emerge due to opposing preferences. Lagrange's interpolation polynomial [3] is a mathematical method employed to determine a specific option from a set of several options in transportation problems, particularly advantageous for managing uncertain factors.

Modified fuzzy goal programming is the effective alternative solution method for ML-MOLFPPs that is suggested in this study [4]. To address multi-choice, multi-objective linear programming problems, an analogous mathematical model utilizing nonlinear programming methods is suggested [5]. Joshi et al. [6] Addressing complex multi-objective linear and linear fractional transport issues and presenting a new, roughly parameterized method. Using a linear-linear fractional objective function, this research investigates a transportation problem with the goal of minimizing damage and costs [7]. Solution preference problems and deadlocks in decision-making are eliminated by the paper's simplified modified FGP technique to ML-MOLFPP solving [2]. The study presents a fuzzy-based solution strategy and a multi-level model for the solid transportation issue with uncertain variables [8].

2. Model formulation

An expansion of the traditional multi-level transportation issue, the ML-MCMOLTP has a set of objective functions in the form of linear functions at each level. From a mathematical perspective, the problem can be expressed in the following manner:

ML-MCMOLTP	Fuzzy goal programming formulation
Model 1 $Max_{\bar{X}_1}\ \{Z_{11}, Z_{12}, \dots, Z_{1M}\}$ $Max_{\bar{X}_2}\ \{Z_{21}, Z_{22}, \dots, Z_{2M}\}$ . . $Max_{\bar{X}_K}\ \{Z_{K1}, Z_{K2}, \dots, Z_{KM}\}$ Subject to $\sum_{j=1}^{J} X_{ij} \leq \widetilde{a_i}, \sum_{i=1}^{I} X_{ij} \geq \widetilde{b_j}$, $X_{ij} \geq 0, i = 1,2 \dots, I\ \ j = 1,2, \dots, J$ Where $\widetilde{a_i}$ and $\widetilde{b_j}$ are crisp value [9] of uncertain supply and demand. • $\boldsymbol{Z_{KM}}$: Objective function • $\boldsymbol{\widetilde{a_i}}$: Uncertain product availability at i • $\boldsymbol{\widetilde{b_j}}$: Uncertain product requirement at j • $\boldsymbol{X_{ij}}$: Quantity of product shipped from origin i to destination j • $\boldsymbol{\bar{X}_t}$: Decision vector controlled by t^{th} level DM	Model 2 $$min\ \lambda = \sum_{t=1}^{K}\sum_{r=1}^{R} d_{tr} + \sum_{t=1}^{K} d_t^-$$ Subject to $-\bar{Z}_{tr} + Z_{tr} + d_{tr}(\bar{Z}_{tr} - \underline{Z}_{tr}) \geq 0,$ $t = 1,2, \dots, K; r = 1,2, \dots, M$ $-\bar{X}_{ij} + X_{ij} + d_t(\bar{X}_{ij} - \underline{X}_{ij}) \geq 0,$ $i = 1,2, \dots, I - 1, j = 1,2, \dots, J$ $\sum_{j=1}^{J} X_{ij} \leq \widetilde{a_i}$, $\sum_{i=1}^{I} X_{ij} \geq \widetilde{b_j}$, $X_{ij} \geq 0, i = 1,2 \dots, I\ \ j = 1,2, \dots, J$ • $\boldsymbol{\bar{Z}_{tr}}$: highest value of objective function • $\boldsymbol{\underline{Z}_{tr}}$: lowest value of objective function • $\boldsymbol{\bar{X}_{ij}}$: highest value of variables • $\boldsymbol{\underline{X}_{ij}}$: Lowest value of variables

3. Numerical example

Consider the following (Table 1, Table 2 and Table 3) multi-level multi-choice multi-objective linear transportation programming problem. We use Roy [3] approach to convert multichoice into single choice.

Table 1
Profit matrix for the first-level multi-choice multi-objective transportation problem

	$b_1 = (10,1.5)$	$b_2 = (33,1)$	$b_3 = (38,1.5)$	
$a_1 = (20,2)$	{16,19,20}, {10,11,12}	{17,19}, {21,23,25}	{12,15,17,19}, {14,16,17,19}	$\widetilde{a_1} = 17.58$
$a_2 = (35,1)$	{14,16}, {15,16, }	{20,22,23}, {20,21,23,26}	{13,15,16}, {9,10,11}	$\widetilde{a_2} = 32.79$
$a_3 = (29,1.5)$	{12,15,17,19,20}, {18,19}	{14,17,19,22}, {14,17,19,20,21}	{15,16}, {14,16}	$\widetilde{a_3} = 27.18$
	$\widetilde{b_1} = 8.18$	$\widetilde{b_2} = 31.79$	$\widetilde{b_3} = 36.18$	***CL**=0.9

***CL=** Confidence level

Table 2

Profit matrix for the second-level multi-choice multi-objective transportation problem

	$b_1 = (10{,}1.5)$	$b_2 = (33{,}1)$	$b_3 = (38{,}1.5)$	
$a_1 = (20{,}2)$	{20,21,22,25}, {15,18,20}, {20,25}	{14,16,17}, {15,16,18,19}, {15,16,17}	{10,12,13,15}, {10,15}, {12,15,17,21,23}	$\widetilde{a_1} = 17.58$
$a_2 = (35{,}1)$	{15,18,20}, {25,27,28,30}, {15,18}	{14,16}, {10,11,13}, {18,22,25}	{20,22,25}, {14,17,19,22}, {10,12,13,15}	$\widetilde{a_2} = 32.79$
$a_3 = (29{,}1.5)$	{16,19,22}, {11,18,20}, {10,12}	{12,15,17,21}, {20,21,22,25}, {7,8,10,11}	{12,15,17,21}, {20,21,22,25}, {7,8,10,11}	$\widetilde{a_3} = 27.18$
	$\widetilde{b_1} = 8.18$	$\widetilde{b_2} = 31.79$	$\widetilde{b_3} = 36.18$	***CL**=0.9

Table 3

Profit matrix for the third-level multi-choice multi-objective transportation problem

	$b_1 = (10{,}1.5)$	$b_2 = (33{,}1)$	$b_3 = (38{,}1.5)$	
$a_1 = (20{,}2)$	{12,15,17}, {16,17,19}	{9,11,13,15}, {14,17,19,21}	{19,21}, {13,15}	$\widetilde{a_1} = 17.58$
$a_2 = (35{,}1)$	{22,24,25}, {12,15,17,19}	{14,18}, {24,28}	{12,15,17,19,20}, {11,13,15}	$\widetilde{a_2} = 32.79$
$a_3 = (29{,}1.5)$	{9,11,13}, {17,19}	{12,15,19}, {10,12,15,17}	{23,25}, {11,15,17,19}	$\widetilde{a_3} = 27.18$
	$\widetilde{b_1} = 8.18$	$\widetilde{b_2} = 31.79$	$\widetilde{b_3} = 36.18$	***CL**=0.9

$$
\begin{aligned}
Z_{11} = {} & \{16,19,20\}X_{11} + \{17,19\}X_{12} + \{12,15,17,19\}X_{13} + \{14,16\}X_{21} \\
& + \{20,22,23\}X_{22} + \{13,15,16\}X_{23} + \{12,15,17,19,20\}X_{31} \\
& + \{14,17,19,22\}X_{32} + \{15,16\}X_{33}
\end{aligned}
$$

$$
\begin{aligned}
Z_{12} = {} & \{10,11,12\}X_{11} + \{21,23,25\}X_{12} + \{14,16,17,19\}X_{13} + \{15,16\}X_{21} \\
& + \{20,21,23,26\}X_{22} + \{9,10,11\}X_{23} + \{18,19\}X_{31} \\
& + \{14,17,19,20,21\}X_{32} + \{14,16\}X_{33}
\end{aligned}
$$

$$
\begin{aligned}
Z_{21} = {} & \{20,21,22,25\}X_{11} + \{14,16,17\}X_{12} + \{10,12,13,15\}X_{13} \\
& + \{15,18,20\}X_{21} + \{14,16\}X_{22} + \{20,22,25\}X_{23} \\
& + \{16,19,22\}X_{31} + \{12,15,17,21\}X_{32} + \{14,18,22\}X_{33}
\end{aligned}
$$

$$
\begin{aligned}
Z_{22} = {} & \{15,18,20\}X_{11} + \{15,16,18,19\}X_{12} + \{10,15\}X_{13} + \{25,27,28,30\}X_{21} \\
& + \{10,11,13\}X_{22} + \{14,17,19,22\}X_{23} + \{11,18,20\}X_{31} \\
& + \{20,21,22,25\}X_{32} + \{18,22\}X_{33}
\end{aligned}
$$

$$
\begin{aligned}
Z_{23} = {} & \{20,25\}X_{11} + \{15,16,17\}X_{12} + \{12,15,17,21,23\}X_{13} + \{15,18\}X_{21} \\
& + \{18,22,25\}X_{22} + \{10,12,13,15\}X_{23} + \{10,12\}X_{31} \\
& + \{7,8,10,11\}X_{32} + \{12,17,19\}X_{33}
\end{aligned}
$$

$$Z_{31} = \{12,15,17\}X_{11} + \{9,11,13,15\}X_{12} + \{19,21\}X_{13} + \{22,24,25\}X_{21} + \{14,18\}X_{22} + \{12,15,17,19,20\}X_{23} + \{9,11,13\}X_{31} + \{12,15,19\}X_{32} + \{23,25\}X_{33}$$

$$Z_{32} = \{16,17,19\}X_{11} + \{14,17,19,21\}X_{12} + \{13,15\}X_{13} + \{12,15,17,19\}X_{21} + \{24,28\}X_{22} + \{11,13,15\}X_{23} + \{17,19\}X_{31} + \{10,12,15,17\}X_{32} + \{11,15,17,19\}X_{33}$$

I level DM will regulate the decision variables $\bar{X}_1 = \{X_{11}, X_{12}, X_{13}\}$, Decision variables to be managed by II level DM are denoted by $\bar{X}_2 = \{X_{21}, X_{22}, X_{23}\}$. Decision variables that are under the jurisdiction of III level DM are denoted by $\bar{X}_3 = \{X_{31}, X_{32}, X_{33}\}$.

This graph shown in Figure 1 compromise solutions close to the ideal value. That proposed model is the best.

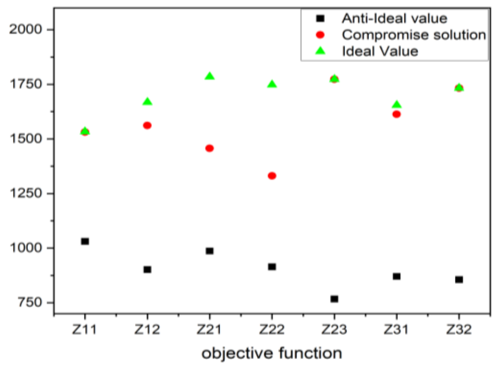

Figure 1
Graphical representation of comparison of ideal value, anti-ideal value and stability of compromise solution

4. Results and discussion

We calculate the value of each objective function at each level individually. Then we obtain $\overline{Z_{11}} = 1533.66$, $\underline{Z_{11}} = 1030.78$, $\overline{Z_{12}} = 1668.37$, $\underline{Z_{12}} = 901.7$, $\overline{Z_{21}} = 1784.66$, $\underline{Z_{21}} = 986.8$, $\overline{Z_{22}} = 1747.84$, $\underline{Z_{22}} = 914.48$, $\overline{Z_{23}} = 1773.10$, $\underline{Z_{23}} = 766.92$, $\overline{Z_{31}} = 1654.48$, $\underline{Z_{31}} = 870.42$, and $\overline{Z_{32}} = 1733.34$, $\underline{Z_{32}} = 855.62$. The compromise optimal solution of ML-MCMOLTP is as follows: $\lambda = 1.108113$, $Z_{11} = 1531.66$, $Z_{12} = 1561.37$, $Z_{21} = 1457.09$, $Z_{22} = 1330.83$, $Z_{23} = 1773.10$, $Z_{31} = 1613.16$, $Z_{32} = 1732.54$. The compromise solutions close to the ideal value (figure 1). Results clearly shows the superiority of our algorithm over the previous algorithms.

5. Conclusion

This paper introduces a network planning decision model for ML-MCMOLTP that incorporates independent uncertain variables. The uncertain

MCMOLTP is formulated as a multi-level programming problem, a novel approach presented in this paper. The problem becomes convoluted due to the presence of uncertain variables, and to address this complexity, the concept of expected constraint programming is employed to define the problem. Additionally, a numerical example is provided to illustrate the model clearly, utilizing the LINGO 18.0 optimization software. For future research, algorithm can be combined with other complex transportation scenarios for better compromise optimal solution.

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References

[1] F. Hitchcock, "The distribution of a product from several sources to numerous localities," *J. Math. Phys.*, vol. 20, pp. 224–230 (1941).

[2] K. Lachhwani, "Modified FGP approach for multi-level multi-objective linear fractional programming problems," *Appl. Math. Comput.*, vol. 266, pp. 1038–1049 (2015).

[3] S. K. Roy, "Lagrange's interpolating polynomial approach to solve multi-choice transportation problem," *Int. J. Appl. Comput. Math.*, vol. 1, pp. 639–649 (2015).

[4] V. D. Joshi, K. Lachhwani, J. Singh, and K. S. Nisar, "Alternate solution approach for multi-level multi-objective linear fractional programming problems," *J. Interdiscip. Math.*, vol. 25, no. 1, pp. 183–194 (2022).

[5] M. P. Biswal and S. Acharya, "Multi-choice multi-objective linear programming problem," *J. Interdiscip. Math.*, vol. 12, no. 5, pp. 606–637 (2009).

[6] V. D. Joshi, S. S. Naharwar, J. Singh, and K. S. Nisar, "A new solution procedure for multi-objective linear fractional transportation problem with rough parameters," *J. Interdiscip. Math.*, vol. 26, no. 1, pp. 121–132 (2023).

[7] V. D. Joshi, S. S. Naharwar, J. Singh, and K. S. Nisar, "An improved solution approach for solving multi-objective linear and linear fractional transportation problem," *J. Interdiscip. Math.*, vol. 26, no. 1, pp. 133–143 (2023).

[8] S. A. Jalil, S. Javaid, and S. M. Muneeb, "A decentralized multi-level decision-making model for solid transportation problem with uncertainty," *Int. J. Syst. Assur. Eng. Manag.*, vol. 9, pp. 1022–1033 (2018).

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