Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 46 (2025), No. 3, pp. 831–850
DOI : 10.47974/JIOS-1882

Establishing a stochastic model for optimal decision making incorporating a random sum of discounted random variables

Constantinos T. Artikis *
Department of Tourism
Faculty of Economic Sciences
Ionian University
49132 Corfu
Greece

Panagiotis T. Artikis †
Department of Accounting & Finance
School of Management, Economic & Social Sciences
University of West Attica
12244 Egaleo
Athens
Greece

Abstract

Stochastic modelling strongly contributes to the decision making process. The paper establishes a stochastic model by combining a random variable and a random sum. The theoretical importance of the model is derived by evaluating its characteristic function, while the applicability of the model as an indicator for decision making in various disciplines consists its practical contribution. The paper performs a simulation and analyzes the outcomes.

Subject Classification (2020): *60E05, 60E10, 65C20, 90B50.*

Keywords: *Stochastic model, Random sum, Decision making, Simulation.*

* *ORCID-ID:* **0000-0003-2987-4898**
† *ORCID-ID:* **0000-0002-8301-3485**

* *E-mail:* **ctartikis@gmail.com (Corresponding Author)**
† *E-mail:* **ptartikis@gmail.com**

1. Introduction

The use of stochastic modeling is widespread in a wide variety of disciplines due to its ability to capture and analyze uncertainty. Stochastic modeling has valuable applications across several scientific disciplines, such as finance and economics, insurance, supply chain management, operations research, healthcare, environmental science, engineering, telecommunications, marketing and business analytics, biostatistics, urban planning [3, 4, 10, 24, 56, 59, 60]. This illustrates the suitability of stochastic modelling in various disciplines to address complex problems involving uncertainty.

Stochastic models are very powerful tools for decision making in situations that are constantly changing due to their inherent intricate nature. Stochastic models enable decision makers to integrate uncertainty and unpredictability into their analysis. Real life situations often include the effect of random events or the presence of variability, which may affect the conclusive outcomes. Stochastic models significantly contribute in capturing and reflecting uncertainty, hence offering a more accurate description of potential outcomes. Deterministic models may oversimplify problems in complex systems characterized by the interaction of various components. Stochastic models provide a more accurate representation of the intricacy of such systems, resulting in more refined decision support [10, 16, 31]. By evaluating a spectrum of potential outcomes and their corresponding likelihoods, individuals in positions of authority may make more informed decisions that include the inherent uncertainties in a specific circumstance. Stochastic models enable the process of assessing and measuring uncertainties and risks related to various activities [22, 23, 30, 51]. Stochastic models often include simulation and scenario analysis, enabling decision makers to examine a diverse range of potential outcomes. This facilitates a broader understanding of the implications of various choices [31, 33, 46, 50]. In conclusion, stochastic models provide a more comprehensive and realistic framework for decision making by explicitly considering uncertainty. This provides an enhanced, adaptable, and well informed decision making in various practical disciplines.

Random sums are essential in stochastic modelling since several practical operations include the accumulation or aggregation of random variables. Stochastic modelling is applicable to systems that include uncertainty, and random sums contribute to capturing this uncertainty through a mathematical framework [10, 34, 55]. Moreover, random sums allow stochastic models to integrate uncertainty by expressing the cumulative impact of random events over a period of time. In addition, random sums are often used in financial modelling to represent the combined effect of unpredictable market fluctuations on investment portfolios. This is essential for the process of assessing and mitigating risks, as well as making informed choices, within the financial sector [36, 41, 57]. Random sums are flexible and effective methods for modelling the evolution of systems over time. Monte Carlo simulations frequently consist of developing random sums to simulate the characteristics of uncertain processes. Consequently, random sums are versatile and powerful tools in stochastic modeling, allowing researchers and practitioners to represent and analyze the cumulative impact of random events in a wide range of situations.

The main contribution concentrates on the development of a stochastic model, by combining a random sum of present values of continuous r.v. and a positive r.v.. The proposed stochastic model can be interpreted as a probabilistic indicator that significantly contributes to the decision making process.

The following outlines the overall structure. The second section provides an overview of the prior research that is relevant to this study. In the third section, the formulation of the stochastic model is described. The fourth section establishes the theoretical contribution of the stochastic model by calculating the corresponding characteristic function. The fifth section demonstrates applications across various scientific disciplines. The sixth section provides a simulation of the proposed stochastic model. Conclusions and topics for further research are given by the seventh section.

2. Literature Review

The development and investigation of stochastic models encompassing random sums and discounting of random variables, as well as the investigation of ratios generated by random variables has been conducted by many researchers. Applications of such stochastic models has also been established in various scientific disciplines.

Pinsky and Carlin presented the principles and techniques of stochastic modelling to demonstrate the wide range of practical applications in the applied sciences. They also offered examples to apply fundamental stochastic concepts to real world problems. In addition, properties and applications of characteristic functions are also provided [47]. Voudouri and Ntziachristos introduce a random sum of present values, which demonstrates compelling applications in the field of financial economics [58]. An investigation on the tail behavior of random sums in the context of consistent variation was conducted by Aleskeviciene, Leipus, and Siaulys [2]. The investigation conducted by Wang and Wang is also concentrated in a related goal [61]. Artikis, Jerwood, and Voudouri conducted a study on the analysis of discounted binomial random sums and the establishment of the characteristic functions [7]. The present value of a random sum under random timing is considered by Artikis and Malliaris [6]. The research paper presented by Artikis focuses on the formulation of a renewal random sum. Furthermore, a comprehension of the random sum in the field of computational intelligence is also provided [8]. The work presented by Artikis and Artikis is to define the fundamental components necessary for developing two stochastic models [12]. The authors have provided another study that focuses on incorporating discounting into proactive decision making for system operations. The paper presents an additional stochastic model that incorporates three stochastic integrals associated with power contractions, which have practical applications in several areas [11].

In addition, several publications have been conducted on the development, theoretical investigation, and practical implementation of ratios that incorporate random variables.

Ahsanullah, Kibria, and Shakil studied the distribution of the ratio X/Y, assuming X and Y are independent and have the same normal distributions [1], Nadarajah and Kotz evaluated the probability distribution of X/Y, where X and Y are independent gamma and Weibull random variables, respectively [45]. Hinkley examines the distribution of the ratio of two random variables that are correlated and follow the normal distribution [32]. Annavajjala, Chockalingam, and Mohammed provided the probability density function and the cumulative distribution function for the ratio of two random sums. Furthermore, the Laplace transform of the pdf is also provided [5]. Finally, Ladoucette, and Teugels conducted a study on the asymptotic behavior of general moments of the ratio between two random sums of independent random variables following a Pareto distribution [40].

3. Model formulation

We assume that N is a discrete r.v. taking values in the set $\mathbf{N} = \{1,2,...\}$, $\{X_n, n = 1,2,...\}$ and $\{T_n, n = 1,2,...\}$ are sequences of positive random variables. We consider the sequence $\{W_n = e^{-rT_n}, n = 1,2,...\}$ taking values in $(0,1)$, where r is a positive real number. We also consider the sequence of random variables $\{C_n = X_n W_n, n = 1,2,...\}$. Finally, we consider the random sum

$$Y = C_1 + C_2 + \cdots + C_N$$

and the positive random variable S. Based upon the aforementioned considerations, we establish the stochastic model

$$L = \frac{Y}{S}$$

or equivalently

$$L = \frac{X_1 e^{-rT_1} + X_2 e^{-rT_2} + \cdots + X_N e^{-rT_N}}{S}.$$

The stochastic model L is interpreted as an indicator that offers crucial insights in decision making. The practical application of the defined model may span across several disciplines, depending on how the random variables of the structural parts of the stochastic model are represented. This observation illustrates the flexibility of the stochastic model and its ability to adjust to specific situational factors.

4. Determining the corresponding characteristic function

The primary objective of this section is to establish the sufficient conditions in order to evaluate the corresponding characteristic function. It simplifies the examination of the sum of random variables and allows the use of Fourier analysis techniques [37, 48].

Theorem: *Let N be a discrete r.v with values in the set* **N** *= {1,2 ,...} and p.g.f $P_N(z), \{X_n, n = 1,2,...\}$ be a sequence of positive and independent r.v. distributed as the r.v. X with c.f. $\varphi_X(u)$, $\{T_n, n = 1,2,...\}$ be a sequence of positive and independent r.v. distributed as the r.v. T with c.d.f $F_T(t)$, and S be a positive r.v with c.d.f $F_S(s)$. We set*

$$W_n = e^{-rT_n}, \quad n = 1,2,...$$

$$C_n = X_n W_n, \quad n = 1,2,...$$

and

$$Y = C_1 + C_2 + \cdots + C_N.$$

If $N, \{X_n, n = 1,2,...\}, \{T_n, n = 1,2,...\}$ and S are independent, then the characteristic function of the stochastic model

$$L = \frac{Y}{S}$$

or equivalently

$$L = \frac{X_1 e^{-rT_1} + X_2 e^{-rT_2} + \cdots + X_N e^{-rT_N}}{S}.$$

is given by

$$\varphi_L(u) = \int_0^\infty \left\{ P_N \left[\int_0^1 \varphi_X\left(\frac{iuw}{s}\right) d\left(1 - F_T\left(-\frac{1}{r} logw\right)\right) \right] \right\} dF_S(s).$$

Proof: The independence of $N, \{X_n, n = 1,2,...\}, \{T_n, n = 1,2,...\}, S$ implies the independence of $\{X_n, n = 1,2,...\}, \{W_n = e^{-rT_n}, \ , n = 1,2,...\}$. The c.d.f. of $W = e^{-rT}$ is given by

$$\begin{aligned} F_W(w) &= P(W \le w) = P(e^{-rT} \le w) = P(loge^{-rT} \le logw) \\ &= P(-rT \le logw) = 1 - P\left(T \le -\frac{1}{r} logw\right) \\ &= 1 - F_T\left(-\frac{1}{r} logw\right), \ 0 < w < 1 \end{aligned} \quad (1)$$

Hence from (1) and the independence of $\{X_n, n = 1,2,...\}$ and $\{W_n = e^{-rT_n}, \ , n = 1,2,...\}$ it can be readily derived that the c.f. of C is given by

$$\begin{aligned} \varphi_C(u) &= E(e^{iuC}) = E\left(E(e^{iuXW} | W = w)\right) \\ &= \int_0^1 E(e^{iuXW} | W = w)\, f_W(w) dw \\ &= \int_0^1 E(e^{iuwX})\, dF_W(w) \\ &= \int_0^1 \varphi_X(uw) dF_W(w) \\ &= \int_0^1 \varphi_X(uw) d\left(1 - F_T\left(-\frac{1}{r} logw\right)\right). \end{aligned} \quad (2)$$

If $\varphi_Y(u)$ is the characteristic function of Y then

$$\begin{aligned} \varphi_Y(u) &= E(e^{iuY}) = E\left(E\left(e^{iu(C_1 + C_2 ... + C_N)} | N = n\right)\right) \\ &= \sum_{n=0}^\infty E\left(e^{iu(C_1 + C_2 ... + C_N)} | N = n\right) P(N = n) \end{aligned}$$

$$= \sum_{n=0}^{\infty} E(e^{iuC_1})(e^{iuC_2}) \dots (e^{iuC_n})P(N = n)$$
$$= \sum_{n=0}^{\infty} \varphi_{C_1}(u)\varphi_{C_2}(u) \dots \varphi_{C_n}(u)P(N = n)$$
$$= \sum_{n=0}^{\infty} \varphi_C^n(u)P(N = n)$$
$$= P_N\big(\varphi_C(u)\big) \tag{3}$$

From (2) and (3) we get

$$\varphi_Y(u) = P_N\left[\int_0^1 \varphi_X(uw)d\left(1 - F_T\left(-\frac{1}{r}logw\right)\right)\right]. \tag{4}$$

The independence of $N, \{X_n, n = 1,2, \dots\}, \{T_n, n = 1,2, \dots\}, S$ implies the independence of Y and S. If the characteristic function of L is $\varphi_L(u)$ is then

$$\varphi_L(u) = E(e^{iuL}) = E\left(E\left(e^{iu\left(\frac{Y}{S}\right)}|S = s\right)\right)$$
$$= \int_0^{\infty} E\left(e^{iu\left(\frac{Y}{S}\right)}|S = s\right)f_S(s)ds$$
$$= \int_0^{\infty} E\left(e^{i\frac{u}{s}Y}\right)dF_S(s)$$
$$= \int_0^{\infty} \varphi_Y\left(\frac{u}{s}\right)dF_S(s). \tag{5}$$

From (4) and (5) we get

$$\varphi_L(u) = \int_0^{\infty}\left\{P_N\left[\int_0^1 \varphi_X\left(\frac{uw}{s}\right)d\left(1 - F_T\left(-\frac{1}{r}logw\right)\right)\right]\right\}dF_S(s).$$

5. Practical applicability of the stochastic model in decision making process

The utilization of the stochastic model as a probabilistic indicator can provide critical information in decision making across various fields.

In the realm of business, an indicator is a measurable or observable factor that offers valuable information about the performance, well being, or evaluation of a corporation or its particular components. Indicators are employed to evaluate and track diverse activities, fiscal state, and overall achievement of a company. They contribute in facilitating well informed decision making, identifying patterns, and assessing the efficiency of strategies. Utilizing indicators effectively enables firms to monitor progress, identify areas for improvements and make decisions based on data. Organizations must extensively determine indicators which are in line with their objectives. Furthermore, the analysis of indicators should consider the wider context and any interconnections among various activities of the organization [20, 27, 44, 49].

Below, we examine three interpretations in the decision making process in the disciplines of investment analysis, risk management, and supply chain management.

Stochastic models are crucial in investment analysis as they incorporate uncertainty in decision making. Their objective is to enable investors to make well informed decisions, efficiently handle uncertainty, and successfully navigate the constantly evolving investing environment [13, 35, 38, 42, 50, 62, 63]. We consider that N denotes the number of future cash flows generated by an investment. Each positive and independent random variable of $\{X_n, n = 1,2, \ldots\}$ represents the size of the cash flow generated by the investment. Furthermore, each positive and independent random variable of $\{T_n, n = 1,2, \ldots\}$ denotes the arrival time of the cash flow generated by the investment. Hence, each random variable of $\{C_n = X_n W_n, n = 1,2, \ldots\}$ denotes the present value of the cash flow generated by the investment, where r denotes the rate of interest. From these assumptions, it is easily understood that $Y = C_1 + C_2 + \cdots + C_N$ denotes the aggregate of the present values of N cash flows generated by the investment. We also consider that S represents the initial investment cost. Consequently, the stochastic model L serves as an indicator that provides crucial information to support investment decisions. This includes important details about the required number, size, and timing of cash flows generated by the investment, the initial capital needed for each investment, and the cost of capital that determines the profitability of the investment.

It is generally accepted that stochastic models provide a framework for incorporating uncertainty into risk management operations, enabling businesses and financial institutions to make more informed decisions and manage their exposure to different risks effectively [9, 18, 26, 52, 54]. Stochastic models support optimized risk management decision and aid in developing strategies that are efficient against unpredictable events and variations. Therefore, we suppose that N represents the number of risk occurrences that the organism is exposed. Each positive and independent random variable of $\{X_n, n = 1,2, \ldots\}$ indicates the cost associated with the risk occurrence. Moreover, each positive and independent random variable of $\{T_n, n = 1,2, \ldots\}$ denotes the time of the risk occurrence. Hence, each random variable of $\{C_n = X_n W_n, n = 1,2, \ldots\}$ represents the present value of the cost resulting from a risk occurrence, where r denotes the risk free rate of interest. Based on these assumptions, it is clear that $Y = C_1 + C_2 + \cdots + C_N$ denotes the sum of the present values of the costs resulted by the N risk occurrences. We also consider that S represents the amount of capital reserves held by the organization to absorb costs caused from the occurrences of the risk. It is readily understood, that the stochastic model L can be used as an indicator that discloses essential information regarding the number of risk occurrences, the cost derived from these risk occurrences and the required level of capital reserves that the organization must retain in order to absorb the occurring losses without interrupting its activities and deviating from its strategic goals and threatening its viability.

Another utilization is demonstrated in the field of supply chain management. By integrating stochastic modelling into supply chain decision making, organizations can enhance their decision making process by considering the inherent uncertainties in the supply chain environment, resulting in better informed and resilient decisions. Stochastic modelling is highly advantageous in addressing variables such as demand fluctuations, lead time variability, inventory variability, and other uncertainties that might influence decision making in supply chain operations [14, 19, 29, 43, 64, 65]. We consider that N denotes the number of orders for finished goods, work in progress or raw materials held as inventory to satisfy customer demand. Each positive and independent random variable of $\{X_n, n = 1,2, \dots\}$ represents the size of an order in monetary value. Furthermore, each positive and independent random variable of $\{T_n, n = 1,2, \dots\}$ denotes the arrival time of the nth order. Hence, each random variable of $\{C_n = X_n W_n, n = 1,2, \dots\}$ denotes the present value of the monetary value of the nth order, where r denotes the discount rate. Based on the preceding assumptions, it is evident that $Y = C_1 + C_2 + \cdots + C_N$ represents the sum of the discounted monetary values of the N orders. We also consider that S represents the monetary value of the safety stock used as a cushion for unexpected demand or uncertainties within the supply chain. Hence, the stochastic model L can serve as an indicator for determining the appropriate size of the safety stock that the organisation needs to maintain in order to handle variations in demand, provide important information considering the size and time of reordering to replenish the stocks. Additionally, the stochastic model can provide valuable insights into the circumstances in which the safety stock can be utilized to satisfy demand.

The aforementioned interpretations support the general acceptance that probabilistic indicators serve as vital tools for businesses to navigate and handle uncertainties that are inherent in different parts of their operations, investment decisions, and strategic planning. They offer a method based on probability to comprehend and reduce risks, ultimately assisting in making more accurate and resilient decisions. These indicators have a substantial impact on multiple facets of business, serving as benchmarks, metrics, and points of reference that assist organizations in making well informed choices, evaluating performance, and managing their operations [25]. They constitute crucial instruments in domains where uncertainty and probability have a substantial impact.

6. Performing simulation and displaying the outcomes

The next section illustrates the simulation outcomes of the formulated stochastic model. Simulation offers decision makers an opportunity to evaluate different approaches or strategies across multiple situations. It enables the assessment of the resilience of decisions in the context of uncertainty. The simulation was implemented using the MATLAB programming and computing environment, version 8.5.0.197613 [15, 21, 53]. A total of 500 iterations were performed. In addition, the simulation of the stochastic model was implemented based on the following assumptions. We assume that N follows the Poisson distribution, with parameter $\lambda = 10$. The sequence $\{X_n, n = 1,2, \dots\}$ distributed as X follows the normal distribution with parameters $\mu = 10.000$ and $\sigma = 2.000$, the sequence $\{T_n, n = 1,2, \dots\}$ distributed as T follows the exponential distribution with parameter $\lambda = 10$, and S follows the normal distribution with parameters $\mu = 100.000$ and $\sigma = 1.000$. By selecting a discount rate of $r = 0.05$ and performing 500 iterations, the frequency table and descriptive statistics of N, $\{X_n, n = 1,2,\ldots\}$, $\{T_n, n = 1,2,\ldots\}$, $Y = C_1 + C_2 + \cdots + C_N$, S, and L are displayed in Table I-VI, respectively.

Table I
Frequency Table and Descriptive Statistics of N

BinStart	BinEnd	Frequency
2	3	3
3	4	4
4	5	8
5	6	17
6	7	41
7	8	51
8	9	54
9	10	78
10	11	53
11	12	38
12	13	34
13	14	43
14	15	37
15	16	17
16	17	9
17	18	8
18	19	5
Mean	9,86	
Median	9	
Standard Deviaton	3,2014	
Variance	10,2489	
Minimum	2	
Maximum	18	
Range	16	
Interquantile Range (IQR)	4	

Table II
Frequency Table and Descriptive Statistics of Xn, n=1,2,...

BinStart	BinEnd	Frequency
2000	6000	126
6000	10000	2343
10000	14000	2348
14000	18000	113
Mean	10000,0209	

Median	9995,3425
Standard Deviaton	2012,6619
Variance	4050807,797
Minimum	2168,621
Maximum	17312,549
Range	15143,928
Interquantile Range (IQR)	2702,678

Table III
Frequency Table and Descriptive Statistics of Tn, n=1,2,…

BinStart	BinEnd	Frequency
0	0,1	3075
0,1	0,2	1154
0,2	0,3	450
0,3	0,4	156
0,4	0,5	61
0,5	0,6	22
0,6	0,7	6
0,7	0,8	3
0,8	0,9	1
0,9	1	2
Mean	0,10177	
Median	0,069	
Standard Deviaton	0,10188	
Variance	0,01038	
Minimum	0	
Maximum	0,956	
Range	0,956	
Interquantile Range (IQR)	0,112	

Table IV
Frequency Table and Descriptive Statistics of Y = C1+C2+…+CN

BinStart	BinEnd	Frequency
15000	50000	26
50000	85000	155
85000	120000	189
120000	155000	107
155000	190000	23
Mean	98100,9883	
Median	94468,295	
Standard Deviaton	31757,5834	
Variance	1008544106,347	
Minimum	17323,294	
Maximum	187684,654	
Range	170361,36	
Interquantile Range (IQR)	46344,438	

Table V
Frequency Table and Descriptive Statistics of S

BinStart	BinEnd	Frequency
97000	98000	7
98000	99000	69
99000	100000	192
100000	101000	157
101000	102000	67
102000	103000	8
Mean	99962,2354	
Median	99895,3845	
Standard Deviaton	958,3834	
Variance	918498,781	
Minimum	97213,453	
Maximum	102431,92	
Range	5218,467	
Interquantile Range (IQR)	1275,242	

Table VI
Frequency Table and Descriptive Statistics of L

BinStart	BinEnd	Frequency
0	0,2	2
0,2	0,4	11
0,4	0,6	42
0,6	0,8	91
0,8	1	135
1	1,2	87
1,2	1,4	84
1,4	1,6	32
1,6	1,8	13
1,8	2	3
Mean	0,98123	
Median	0,9415	
Standard Deviaton	0,31681	
Variance	0,10037	
Minimum	0,174	
Maximum	1,867	
Range	1,693	
Interquantile Range (IQR)	0,4675	

The corresponding frequency histogram and c.d.f. of the stochastic model *L* are displayed in Figure 1 and Figure 2, respectively.

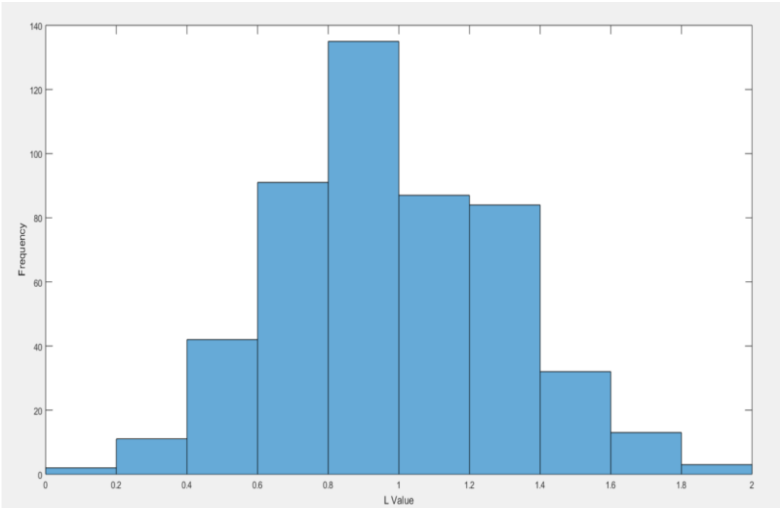

Figure 1
Frequency Histogram

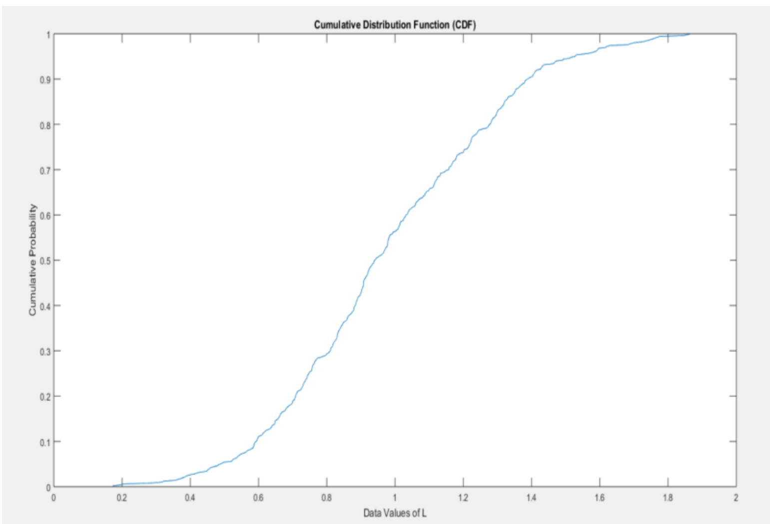

Figure 2
Cumulative Distribution Function

Based on the aforementioned distribution function, it is evident that there is a probability of approximately fifty eight percent that Y is less than or equal to S., i.e. $P(L \leq 1) = 0{,}58$. Interpreting the result of the above simulation, in the context of the practical interpretations provided in the previous section, it is justifiable to assert that there is a probability of approximately fifty eight percent that, the sum of the present values of N cash flows generated by the investment will not cover the initial cost of the investment, the sum of the discounted costs resulted by the N risk occurrences will not surpass the amount of capital reserves held by the organization, and the demand resulting from the N orders will not exceed the safety stock.

7. Conclusion and topics for further research

The development and implementation of stochastic models in decision making is widely acknowledged and adopted. Stochastic models are essential instruments that offer crucial insights for both proactive and reactive strategies in several domains such as investment analysis, risk management, project management, health, engineering, and other scientific disciplines.

The stochastic model is built on the computation of the relevant characteristic function, which forms the theoretical basis. The model finds practical applications in investment analysis, risk management, and inventory management.

The novelty and adaptability of the model are demonstrated by the computation of the characteristic function, which completely defines the probability distribution. The applicability of the model is demonstrated by its utilization of random variables, which can effectively represent quantities relevant to the subject which is under study. Furthermore, the representation of the model as a ratio supports its use as a stochastic indicator for organizations to make informed decisions under the presence of effectiveness, efficiency, and uncertainty.

Further investigation could focus on the choice of other probability distributions for the random variables of the stochastic model, in order to analyze the probabilistic characteristics of the outcomes. Employing sensitivity analysis is also important, as it offers structural insights for decision making. Sensitivity analysis results can be utilized by decision makers to effectively prioritize efforts, allocate resources, and concentrate on managing or monitoring parameters that exert the greatest influence on outcomes [17, 28, 39].

References

[1] M. Ahsanullah, BG. Kibria, and M. Shakil, ‘Sum, Product and Ratio for the Normal Random Variables’, *Normal and Student´s t Distributions and Their Applications,* pp. 63-79, (2014). doi: https://doi.org/10.2991/978-94-6239-061-4_4.

[2] A. Aleškevičienė, R. Leipus, and J. Šiaulys, ‘Tail behavior of random sums under consistent variation with applications to the compound renewal risk model’, *Extremes,* Vo. 11, pp. 261-279, (2008) doi: https://doi.org/10.1007/s10687-008-0057-3.

[3] I. Al Kattan, A. Al Nunu , and K. Saleh, 'A stochastic model for improving information security in supply chain systems', *International Journal of Information Systems and Supply Chain Management* (IJISSCM), Vol. 2, No.3, pp. 35-49, (2009) doi:10.4018/jisscm.2009070103.

[4] M.C. Andrade, P.T. Urgilés, and M.A. Estrella, 'Information and communication technologies in the development of stochastic models applied to the health sector', *Medicina*, Vol. 80, pp. 31-38, (2020), PMID: 32044739 .

[5] R. Annavajjala, A. Chockalingam, and SK. Mohammed, 'On a ratio of functions of exponential random variables and some applications', *IEEE Transactions on Communications*, Vol. 58, pp. 3091-3097, (2010), doi: 10.1109/TCOMM.2010.091710.100038.

[6] T.P. Artikis and A.G. Malliaris, 'Discounting certain random sums', *Scandinavian Actuarial Journal*, Vol.1, pp. 34-38, (1990), doi: https://doi.org/10.1080/03461238.1990.10413871.

[7] T. Artikis, D. Jerwood, and A. Voudouri, 'Analytical and simulation techniques for discounting binomial random sums', *Mathematical and computer modelling*, Vol. 16, pp. 53-58, (1992), doi: https://doi.org/10.1016/0895-7177(92)90045-M.

[8] P.T. Artikis, 'Incorporating a discrete renewal random sum in computational intelligence and cindynics', *Multimedia Services in Intelligent Environments: Software Development Challenges and Solutions*, pp.173-186, (2010), doi: https://doi.org/10.1007/978-3-642-13355-8_11.

[9] P.T. Artikis and C.T. Artikis, 'Stochastic models for risk control programs of organizations', *Kybernetes*, Vol. 39 No. 4, pp. 570-577, (2010), doi: https://doi.org/10.1108/03684921011036790.

[10] C.T. Artikis, and P.T Artikis, 'Formulation and incorporation of a stochastic model for supporting optimal decision making in global risk governance', *Journal of Information and Optimization Sciences*, Vol. 42., pp. 1-9, (2021) doi: https://doi.org/10.1080/02522667.2020.1857904.

[11] C.T. Artikis and P.T. Artikis, 'Discounting for proactive decision making in systemics', *International Journal of Applied Systemic Studies*, Vol. 10, No.4, pp.343-352, (2023) doi: https://doi.org/10.1504/IJASS.2023.135729.

[12] P.T. Artikis and C.T. Artikis, 'Proactive decision making by incorporation of discrete random sums', *Intelligent Decision Technologies*, Vol.17, No. 4, pp.119-1206, (2023), doi:10.3233/IDT-230268.

[13] S.H. Begg, R.B. Bratvold and J.M. Campbell, 'Improving investment decisions using a stochastic integrated asset model' In SPE Annual Technical Conference and Exhibition, SPE-71414, (2001), doi: https://doi.org/10.2118/71414-MS.

[14] O. Berman and E. Kim, 'Stochastic models for inventory management at service facilities', *Stochastic Models,* Vol. 15, No.4, pp.695-718, (1999), doi: https://doi.org/10.1080/15326349908807558.

[15] P. Bhattacharya, S. Paul and K.S. Choudhury, 'Different Types of Epidemic Models and their Characteristic Behaviour by using Matlab', *Journal of Interdisciplinary Mathematics*, 18(5), pp. 569–592, (2015), https://doi.org/10.1080/09720502.2014.914280

[16] J. P. N. Bishwal, 'Sequential Monte Carlo methods for stochastic volatility models: a review', *Journal of Interdisciplinary Mathematics,* Vol. 13, pp. 619–635, (2010). https://doi.org/10.1080/09720502.2010.10700723

[17] E. Borgonovo and L. Peccati, 'Sensitivity analysis in decision making: a consistent approach', Advances in decision making under risk and uncertainty. Springer Berlin Heidelberg, pp. 65-89, (2008), available at: https://link.springer.com/content/pdf/10.1007/978-3-540-68437-4_5.pdf

[18] CL. Chang, JÁ. Jiménez-Martín, E. Maasoumi and T. Pérez-Amaral T., 'A stochastic dominance approach to financial risk management strategies', *Journal of Econometrics,* Vol.187 No.2, pp.472-485, (2015) doi: https://doi.org/10.1016/j.jeconom.2015.02.032.

[19] J. P. C. Chuang, W. S. Chou, and P. Julianne, 'Inventory models with stochastic ramp type demand', *Journal of Interdisciplinary Mathematics,* Vol. 19(1), pp. 55–64, (2016), https://doi.org/10.1080/09720502.2015.1084775

[20] G. D'Amico, F. Petroni and F. Prattico, 'Economic performance indicators of wind energy based on wind speed stochastic modeling', *Applied Energy*, Vol. 154, pp. 290-297, (2015), doi: https://doi.org/10.1016/j.apenergy.2015.04.124.

[21] E.S.S De Cursi and R. Sampaio, Uncertainty quantification and stochastic modeling with matlab, Elsevier, (2015), doi: https://doi.org/10.1016/C2014-0-04713-2.

[22] R. A. de Freitas, E. P. Vogel, A. L. Korzenowski, and L. A. O. Rocha, 'Stochastic model to aid decision making on investments

in renewable energy generation: Portfolio diffusion and investor risk aversion', *Renewable energy,* Vol. 162, pp. 1161-1176, (2020), doi: https://doi.org/10.1016/j.renene.2020.08.012.

[23] A. Diederich, 'Dynamic stochastic models for decision making under time constraints', *Journal of Mathematical Psychology,* Vol. 41, No. 3, pp. 260-274, (1997), doi: https://doi.org/10.1006/jmps.1997.1167.

[24] E. Di Lorenzo, M. Sibillo and G. Tessitore, 'A stochastic model for financial evaluation: applications to actuarial contracts', *Applied Stochastic Models in Business and Industry,* Vol. 15, No. 4, pp. 269-275, (1999), doi: https://doi.org/10.1002/(SICI)1526-4025(199910/12)15:4%3C269::AID-ASMB392%3E3.0.CO;2-F.

[25] E. Domínguez, B. Pérez, ÁL .Rubio and M.A. Zapata, 'A taxonomy for key performance indicators management', *Computer Standards & Interface,* Vol. 64, pp. 24-40, (2019), doi: https://doi.org/10.1016/j.csi.2018.12.001.

[26] C.H. Fouche, J. Mukuddem-Petersen and M.A. Petersen, 'Continuous-time stochastic modelling of capital adequacy ratios for banks', *Applied Stochastic Models in Business and Industry,* Vol. 22, No.1, pp. 41-71, (2006), doi: https://doi.org/10.1002/asmb.609.

[27] E. Fregonara, R. Giordano, D.G. Ferrando and S. Pattono, 'Economic-environmental indicators to support investment decisions: A focus on the buildings' end-of-life stage', *Buildings,* Vol. 7, No. 3, p. 65, (2017), doi: https://doi.org/10.3390/buildings7030065.

[28] S. French, 'Modelling, making inferences and making decisions: the roles of sensitivity analysis', Top, Vol. 11, pp. 229-251, (2003), doi: https://doi.org/10.1007/BF02579043.

[29] M. Goh, J.Y. Lim and F. Meng, 'A stochastic model for risk management in global supply chain networks', *European Journal of Operational Research,* Vol. 182, No. 1, pp. 164-173, (2007), doi: https://doi.org/10.1016/j.ejor.2006.08.028.

[30] R.W. Grubbström and Z. Wang, 'A stochastic model of multi-level/multi-stage capacity-constrained production–inventory systems', *International journal of production economics,* Vol.81, pp. 483-494, (2003), doi: https://doi.org/10.1016/S0925-5273(02)00358-4.

[31] C.H. Hesse, 'A general procedure for stochastic modelling of systems consisting of a large number of interacting components', *Applied stochastic models and data analysis,* Vol. 13(3-4),

pp. 249-258, (1997), doi: https://doi.org/10.1002/(SICI)1099-0747(199709/12)13:3/4%3C249::AID-ASM319%3E3.0.CO;2-N

[32] D.V. Hinkley, 'On the ratio of two correlated normal random variables', *Biometrika,* Vol. 56, No. 3, pp. 635-639, (1969), doi: https://doi.org/10.1093/biomet/56.3.635.

[33] K. Høyland, E. Ranberg . and S.W. Wallace, 'Developing and Implementing a Stochastic Decision-Support Model Within an Organizational Context: Part II—The Organization', *Journal of Risk Finance,* Vol. 5 No. 2, pp. 58-63, (2004), doi: https://doi.org/10.1108/eb022987.

[34] V. Kaarnioja, F.Y. Kuo and I.H. Sloan, 'Uncertainty quantification using periodic random variables', *SIAM Journal on Numerical Analysis,* Vol. 58, No. 2, pp. 1068-1091, (2020), doi: https://doi.org/10.1137/19M1262796.

[35] M. Khodabakhshi . and M. Ahmadi, 'An approach to cost-benefit analysis by competitive advantages with stochastic data', *Journal of Modelling in Management,* Vol. 17 No. 1, pp. 297-308, (2022), doi: https://doi.org/10.1108/JM2-07-2020-0185.

[36] C. Klüppelberg and T. Mikosch, "Large deviations of heavy-tailed random sums with applications in insurance and finance", *Journal of Applied Probability,* Vol. 34, No. 2, pp. 293-308, (1997), doi: https://doi.org/10.2307/3215371.

[37] H. Ko and S. Lee, 'Fourier transform and regularity of characteristic functions', *Proceedings of the American Mathematical Society*, Vol. 145, No. 3, pp. 1097-1107, (2017), doi: https://doi.org/10.1090/proc/13435.

[38] R. Korn, Optimal portfolios: stochastic models for optimal investment and risk management in continuous time, World scientific, (1997), Handle: RePEc:wsi:wsbook:3548.

[39] P. Kumar, R. Shankar and S.S. Yadav, 'Flexibility in global supply chain: modeling the enablers', *Journal of Modelling in Management,* Vol. 3 No. 3, pp. 277-297, (2008), doi: https://doi.org/10.1108/17465660810920609.

[40] S.A. Ladoucette and J.J. Teugels, 'Limit distributions for the ratio of the random sum of squares to the square of the random sum with applications to risk measures', Publications de l'Institut Mathematique, Vol. 80, No. 94, pp. 219-240, (2006), doi: https://doi.org/10.2298/PIM0694219L.

[41] L. Le Cam, ‘On the distribution of sums of independent random variables”, In Bernoulli 1713, Bayes 1763, Laplace 1813: Anniversary Volume. Proceedings of an International Research Seminar Statistical Laboratory University of California, Berkeley 1963, pp. 179-202, Springer Berlin Heidelberg (1965), doi: https://doi.org/10.1007/978-3-642-49749-0_10.

[42] X. Z. Liao, L. H. Peng, L. X. Wu, M. W. Tian, and S. R. Yan, ‘Research n the optimization of modern portfolio model based on optimal investment weight’, *Journal of Interdisciplinary Mathematics,* Vol. 21(5), pp. 1157–1162, (2018), https://doi.org/10.1080/09720502.2018.1494313

[43] R. Lin, J.S. Lin, K. Chen and P.C. Julian, “Note on inventory model with net present value”, *Journal of Interdisciplinary Mathematics,* Vol. 10(4), pp. 587–592, (2007), https://doi.org/10.1080/09720502.2007.10700518

[44] G. Marsden, C. Kelly and C. Snell, ‘Selecting indicators for strategic performance management’, *Transportation research record*, Vol. 1956, No. 1, pp. 21-29, (2006), doi: https://doi.org/10.1177/0361198106195600103.

[45] S. Nadarajah and S. Kotz, ‘On the product and ratio of Gamma and Weibull random variables’, *Econometric Theory,* Vol. 22, No. 2, pp. 338-344, (2006), doi: https://doi.org/10.1017/S0266466606060154.

[46] B.L. Nelson, Stochastic modeling: analysis & simulation, Courier Corporation, (2010), ISBN (Print): 978-0070462137.

[47] M. Pinsky and S. Karlin, An introduction to stochastic modeling. Academic press, BL. (2010), ISBN (Print): 9780123814166.

[48] A. .N. Podkorytov and M. Van Minh, ‘On the inversion formula of the Fourier transform of the characteristic function of several variables’, *Journal of Mathematical Sciences*, Vol. 120, No. 2, pp. 1191-1194, (2004), doi: https://doi.org/10.1023/B:JOTH.0000014846.04894.

[49] C. Powell , Y. Lawryshyn and T. Bender, ‘Using stochastic models to determine financial indicators and technical objectives for organic solar cells’, *Solar energy materials and solar cells*. Vol.107, pp. 236-247, (2012) doi: https://doi.org/10.1016/j.solmat.2012.06.038.

[50] D. Prajapati, S. Mondal and D. Kundu, ‘Optimal decision-theoretic sampling plan for two exponential distributions under joint censor-

ing scheme', *Applied Stochastic Models in Business and Industry,* Vol. 37, No. 3, pp. 560-576, (2021), doi: https://doi.org/10.1002/asmb.2595.

[51] G.D. Rubin and B.N. Patel, 'Financial forecasting and stochastic modeling: predicting the impact of business decisions', *Radiology,* Vol. 283, No. 2, pp. 342-358, (2017), doi: https://doi.org/10.1148/radiol.2017161800

[52] N. Savelli and G.P. Clemente, 'A Risk-Theory Model to Assess the Capital Requirement for Mortality and Longevity Risk', *Journal of Interdisciplinary Mathematics,* Vol. 16, pp. 397–429, (2017), https://doi.org/10.1080/09720502.2013.849943

[53] S. Sharma and A.J. Obaid, A. J. 'Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox', *Journal of Interdisciplinary Mathematics,* Vol. 23, pp. 843–849, (2020) https://doi.org/10.1080/09720502.2020.1727611

[54] S.S. Shin, K.S. Lee and H.G. Cho, 'An objective method of risk assessment based on stochastic modelling', *Journal of Korean Society for Quality Management,* Vol. 41, No. 3, pp. 465, (2013), doi: https://doi.org/10.7469/JKSQM.2013.41.3.465.

[55] J.S. Song, 'The effect of leadtime uncertainty in a simple stochastic inventory model', *Management Science,* Vol. 40, No. 5, pp. 603-613, (1994), doi: https://doi.org/10.1287/mnsc.40.5.603.

[56] C.S. Tapiero, Applied stochastic models and control for finance and insurance., Springer Science & Business Media, (2012), doi: https://doi.org/10.1007/978-1-4615-5823-1.

[57] S. Vanduffel, T. Hoedemakers and J. Dhaene, 'Comparing approximations for risk measures of sums of nonindependent lognormal random variables', *North American Actuarial Journal*, Vol. 9, No. 4, pp. 71-82, (2005), doi: https://doi.org/10.1080/10920277.2005.10596226.

[58] A. Voudouri and E. Ntziachristos, 'A binomial random sum of present value models in investment analysis', *SPOUDAI-Journal of Economics and Business,* Vol. 47, pp. 15-20, (1997), doi: https://spoudai.unipi.gr/index.php/spoudai/article/viewFile/1069/1148.

[59] T. Vrouwenvelder, 'Stochastic modelling of extreme action events in structural engineering', *Probabilistic Engineering Mechanics,* Vol.

15, No. 1, pp.109-117, (2000), doi: https://doi.org/10.1016/S0266-8920(99)00014-4.

[60] U. Wagner and A. Taudes, 'Stochastic models of consumer behaviour', *European Journal of Operational Research,* Vol. 29, No. 1, pp. 1-23, (1987), doi: https://doi.org/10.1016/0377-2217(87)90189-5.

[61] S. Wang and W. Wang, 'Precise large deviations for sums of random variables with consistently varying tails in multi-risk models', *Journal of Applied Probability,* Vol. 44, No. 4, pp. 889-900, (2007), doi: https://doi.org/10.1239/jap/1197908812.

[62] Y. Yakoubov, M. Teeger and D.B. Duval, 'A stochastic investment model for asset and liability management', In Proceedings of the 9th International AFIR Colloquium, pp. 237-66, (1999), available at: https://www.actuaries.org/AFIR/Colloquia/Tokyo/Yaboubov_Teeger_Duval.pdf.

[63] Y. Yang and Y. Zhao, "Modified PROMETHEEII for venture capital investment selection decision-making towards SMEs", *Journal of Interdisciplinary Mathematics,* Vol. 21, pp. 1017–1029, (2018). https://doi.org/10.1080/09720502.2018.1456824.

[64] Q. Zhao and Q. Li, 'Supply chain optimization decision based on mixed component commonality under stochastic constraints', *Journal of Interdisciplinary Mathematics,* Vol. 21(4), pp. 941–948, (2018). https://doi.org/10.1080/09720502.2018.1481587.

[65] L. Zhu, "Supply chain inventory model for perishable items under delayed payment" *Journal of Interdisciplinary Mathematics,* Vol. 21, pp. 849–857, (2018), https://doi.org/10.1080/09720502.2018.1475063.

Received May, 2024