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Applications of connected certified domination in wireless sensor networks

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Abstract

Wireless Sensor Networks (WSNs) are an amazing combination of cutting-edge technology and effective data collecting. WSN technology is more efficient and low power consuming networking technology in the present world. These networks are used for a variety of purposes, including environmental monitoring, automation in factories, healthcare, and smart cities. The fundamental aim of this article is to study the applications of connected certified dominating sets (CCDS) in WSNs. To construct a minimal CCDS, we apply the greedy technique to develop a centralized algorithm GR_CCDS in just one phase, with a time complexity of $O((\Delta^2 + log n)n)$. Subsequently, another method called P_CCDS is designed with a time complexity of $O(n^2)$ to prune unnecessary nodes in the MCCDS generated by GR_CCDS.

Subject Classification: *05CXX, 05C69.*

Keywords: *Connected certified dominating set (CCDS), Wireless sensor networks (WSNs), Construction of CCDS in WSNs.*

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1. Introduction

In contemporary contexts, newly established categorizations of WSNs have led to a diminished demarcation between the underlying network infrastructure and the network's constituent clients. For example, WSNs consist of multiple main stations and numerous cost-effective nodes that incorporate sensors and low-power wireless radios. In parallel fashion, in mobile ad hoc networks (MANETs), regular nodes also handle the task of routing messages. In fact, a MANET usually lacks a traditional network infrastructure, necessitating that all routing and network management tasks must be carried out by regular nodes. Traditional computer networks cannot scale or function efficiently unless they are organized in a hierarchical fashion. Sensor networks and MANETs, on the other hand, are essentially flat since they lack a standard network backbone. In order to surmount these limitations and accomplish both efficiency and scalability, novel techniques have been developed. These techniques make use of a virtual network infrastructure that generate a hierarchy of nodes that allows them to be organized in a structured order [1].

WSN is a burgeoning wireless communication technology that has the potential to contribute significantly to the monitoring of remote areas [2]. WSNs are the networks with sensors that are used for collection and transmission of information to central node for processing [3, 4]. A typical sensor node is made up of a wireless transmitter/receiver, a storage unit, a processor unit, a sensing device, and a power unit. The hardware architecture of a WSN is shown in Figure 1.

One of the most challenging aspects in WSNs is to ensure that every single node in the network is monitored by at least one sensor. This is important because the failure to monitor a critical point can result in catastrophic consequences. Connected certified dominating sets (CCDSs) can be used to address this challenge in WSNs. Wireless ad hoc networks and WSNs both exhibit ad hoc communication patterns. The radio range and network connection of WSNs also fluctuate over time, making them dynamic systems. The idea of connected certified domination was introduced in 2022 [5] and well-studied in [6].

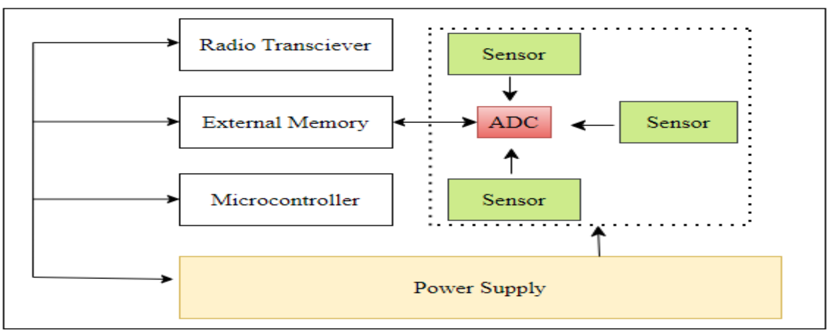


Figure 1
Architecture of a wireless sensor

In a WSN, a CCDS $D_c \subseteq V_\Gamma$ of the network Γ such that:

1. Every sensing unit in the network either belongs to D_c or is adjacent to at least one vertex in the set D_c,
2. For every sensing unit $u \in D_c, |N(u) \cap (V_\Gamma - D_c)|$ is either 0 or at least 2, and
3. The subgraph $\Gamma[D_c]$ is connected.

The CCDS D_c in WSNs can guarantee that every sensor in the network is monitored by at least one sensor in D_c and every sensor in D_c is monitoring 0 or atleast 2 sensors outside D_c. In other words, a CCDS ensures that the network is fully covered by the sensors and that the network remains connected even if some nodes fail. The idea of CCDS is significant for WSNs since it offers a means of ensuring network connection and coverage with a small number of sensors. CCDSs can ensure complete network coverage with smallest number of sensors, minimizing the cost and consumption of energy of the network.

The connected dominating sets (CDSs) has also applications in WSNs and for construction of CDSs in WSNs, several techniques and heuristics have been proposed. To identify a minimal CDS in WSNs, these algorithms often use a variety of methodologies such as graph theory, optimization, and heuristics [7-10]. The use of CCDSs in WSNs can provide a more reliable and efficient way to ensure the full coverage of the network with a minimal number of sensors. The approach of using CCDS instead of CDS is more effective because every sensor in the CCDS is monitoring 0 or atleast 2 sensor nodes outside the CCDS and every weak support and its only leaf neighbor will be in CCDS [5], that is, if a sensor node has only one adjacent sensor in the network then in that case both the sensors will be included in the CCDS. For general graph theoretic terms and notation, we follow [11] and [12].

2. Network Model and CCDS Applications

Within this section, we embark on a mathematical representation of the networks being discussed and introduce pertinent concepts of graph theory. Furthermore, we investigate the applications of wireless networks that utilize CCDS.

2.1 *Network Model*

In a UDG, an edge is present between two different vertices if and only if the distance between them is equal to or less than 1 unit (which represents the fixed communication range). A WSN can be depicted by a graph $\Gamma = (\mathrm{V}, \mathrm{E})$, where vertices represents sensor nodes and edges indicates connection between them. The following is the graphical representation of WSN with eight vertices representing sensor nodes of the WSN is shown in Figure 2.

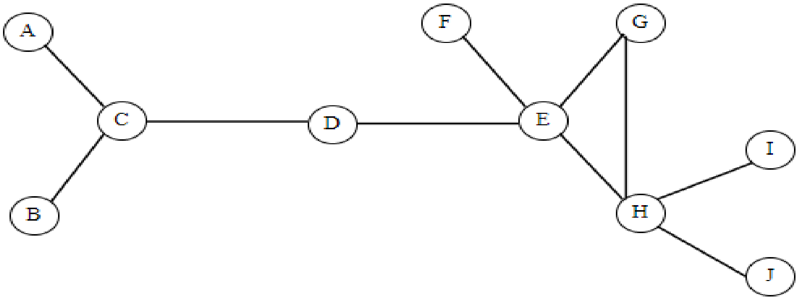


Figure 2
A graphical representation of a WSN with eight sensor nodes

2.2. *Applications of CCDS in WSNs.*

WSNs and (MANETs) possess distinct attributes that necessitate the creation of protocols tailored specifically to their requirements. To enhance efficiency, numerous protocols prioritize the initial formation of certified dominating sets to organize the network. These protocols address various aspects such as network access, data transmission, energy management, and topology control.

A CCDS possess the capability of generating a virtual network backbone for data transmission and control. The data transmission procedure encompasses the transmission of messages from the sink node to an adjacent node within the CCDS. These messages are then routed through the CCDS until they reach the CCDS member closest to the target node, after which they are delivered to the destination node. Limiting the routing within the CCDS leads to a notable decrease in the message overhead linked to routing updates. Additionally, the CCDS can be structured hierarchically, which further reduces the control message overhead. The use of CCDSs can also enhance the effectiveness of multicast/broadcast data transmission. In multicast/broadcast data transmission, a significant issue arises where numerous intermediate sensing units superfluously forward a message, leading to multiple instances of the same message being heard.

CCDS construction can effectively address the task of extracting sensor topology information in extensive, high-density sensor networks. In addition to data transmission, the virtual backbone created by the CFDS can serve various purposes such as propagating "link quality" information to facilitate route selection for multimedia traffic or acting as database servers, among other potential applications.

3. CCDS Construction

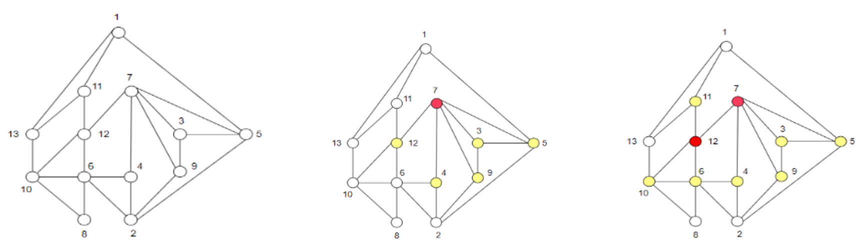
In the following we will discuss greedy construction of CCDS shown in Figure 3. we write it GR_CCDS. It should be noted that the algorithm proposed can be applied to both unit-disk graphs and general graphs, regardless of the specific graph model.

3.1 *Algorithm GR_CCDS*

Let's consider a graph Γ and a subset $\mathfrak{D}_c$ of vertices within Γ. We can categorize all the vertices in G into three classes based on their relationship with $\mathfrak{D}_c$. The first class consists of vertices that belong to $\mathfrak{D}_c$, referred to as red vertices for simplicity. The second class comprises vertices that are not part of $\mathfrak{D}_c$ but are adjacent to at least one vertex in $\mathfrak{D}_c$, known as yellow vertices. The third class consists of vertices that are neither in $\mathfrak{D}_c$ nor adjacent to any vertex in $\mathfrak{D}_c$, referred to as white vertices.

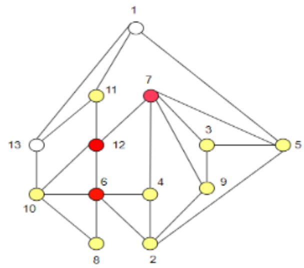
Initially, all nodes are marked white; this is referred to as the initial state of the network. A global parameter, denoted by μ, is introduced with a starting value of n, or the total number of nodes in the network Γ, keeps track of how many white nodes there are in the network. To initiate the construction of CCDS, we initially identify a node v that has the highest degree. This selected node x is then transformed into the initial node by changing its color to red. In addition, at this stage, all the neighbouring nodes connected to x are colored yellow. In spite of the color of node x, μ is reduced by 1. The node x is removed from set $N(y)$ for any node $y \in N(x)$, that is, $N(y) = N(y)\backslash x$. Then every node in $N(y)$ is white. On a continuous basis, the yellow node x with the maximal number of white adjacent nodes is selected as long as $\mu > 0$. This node x that has been selected is then added as a new node to the CCDS and colored as red. Subsequently, all of its white neighbors turn into yellow. In the event that two or more yellow nodes have the same number of adjacent white nodes that are more than the others, we select the yellow node with the largest degree as the new member of the CCDS. The process will be carried out continuously until all the nodes have been colored red or yellow in such a way that every element in CCDS or every red node must have 0 or at least 2 adjacent yellow nodes.

Figure 3 shows the outcomes of the CCDS construction procedure by implementing the GR_CCDS algorithm. Node 7 is colored red at first because it possesses the greatest degree among all its neighbouring nodes. Nodes 3, 4, 5, 9, and 12 are then colored yellow since they're adjacent to node 7. Subsequently, node 12 is colored red mainly because it has the highest count of neighbouring white nodes when compared to all other nodes already colored yellow. As a result, its white neighbors, specifically nodes 6, 10, and 11, are then colored yellow. Afterwards, the node 6 and 11 are colored red because both nodes are having exactly the same number (which is 2) of white neighbors.

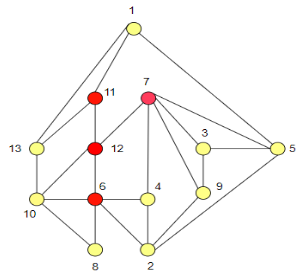


A. Initial stage. B. Node 7 becomes red. C. Node 12 becomes red.

Contd…



D. Node 6 becomes red



E. Final CCDS.

Figure 3
Example showing construction of CCDS.

Lemma 1: *CCDS is the exact connected certified dominating set.*

Proof: We will use the deduction to prove that the set CCDS is connected certified dominating set. Assume that CCDS $= \{u_1, u_2, u_3 \cdots u_\sigma\}$, where σ is the number of elements in CCDS, and that the element $u_i \in$ CCDS does not correspond with any of the other elements in CCDS. The element u_i is then colored red, whereas all of its adjacent elements are colored yellow. According to this approach, whenever node u_i is white, there should be yellow node with at least two white neighbors, i.e., the value of the count should be greater than 0. Consequently, at least one of its neighbouring elements must be red, indicating that each of the red element has a pathway to connect with other red elements in CCDS.

Theorem 4: *The complexity time of the GR_CCDS algorithm is* $O\big((\Delta^2 + log\ n)n\big)$.

Proof: In the first stage, choose the element with the largest degree as the starting point of the process, and the time required to identify all the elements is $O(logn)$.

First choose the element with the largest degree as the starting point of the process and the time required to identify all the elements is $O(logn)$. In the second stage, the time required to remove the colored adjacent node for each neighbor is $O(\Delta^2)$. After that, because of the starting value of count n, the duration for the completion of the while loop is n. Furthermore, within the iteration, we have a yellow node u with the maximal count of adjacent white nodes as a new element in the CCDS; therefore, the duration needed to arrange all yellow nodes is $O(log\ n)$**.** Finally, when node V is turned black, then the process of eliminating its colored neighbors consumes O(log n) time for every neighbouring node. Therefore, the time complexity of the GR_CCDS algorithm for constructing CCDS is $\mathrm{O}\big((\Delta^2 + log\ n)n\big)$.

4.1 *Algorithm P_CCDS*

In order to reduce the size of the obtained CCDS constructed using GR_CCDS algorithm, we now develop algorithm P_CCDS (Pruning for CCDS), which has a time complexity of $O(n^2)$. First of all, this algorithm is a preparation of making a decision on every element $u \in$ CCDS. This element u is colored yellow if all of its yellow neighbor nodes are adjacent with any other red element say v in CCDS such that $|N(v) \cap V_\Gamma \setminus \text{CCDS}| \geq 2$, and there is still a connected path between all its red elements when the element u is deleted because every red element in CCDS are connected with each other by a path. In such a scenario, certain redundant red nodes can be deleted from the CCDS, and the remaining portion of the CCDS is called as concise CCDS. It can be proved in a similar way as of theorem 2 that completion time of P_CCDS algorithm is $O(n^2)$.

Theorem 3: *The completion time of P_CCDS algorithm is* $O(n^2)$.

5. Conclusion

In this article we have discussed the applications of CCDS in WSNs. We have proposed greedy algorithm GR_CCDS for the construction of CCDS in WSNs to get a reasonably optimal size of the CCDS with complexity time $O\big((\Delta^2 + log\ n)n\big)$. Further to reduce the size of CCDS, another algorithm P_CCDS is proposed with complexity time $O(n^2)$.

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