Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 45 (2024), No. 4, pp. 1093–1104
DOI : 10.47974/JIOS-1694

Solving neutrosophic minimal cost flow problem using multi-objective linear programming problem

Shubham Kumar Tripathi
Ranjan Kumar *
VIT-AP University
Inavolu
Beside AP Secretariat
Amaravati 522237
Andhra Pradesh
India

Abstract

The Linear Programming (LP) model, renowned for its robust modeling capabilities, is a highly effective tool for solving real-world problems and optimizing tasks with specific objective. A key component of the LP model is the Minimum Cost Flow (MCF), which aims to minimize the transportation cost of a single product through a capacitated network. This article focuses on the MCF problem with neutrosophic arc cost, a scenario that often arises in real-world situations due to the complexity and uncertainty of data. The neutrosophic theory plays a crucial role in handling this vagueness. The primary objective of this paper is to find the optimal solution to the neutrosophic MCF problem using SVTN numbers.

Subject Classification: *62A86, 90C05 and 90C08.*

Keywords: *LPP, Multi-objective problem, Minimal cost flow, SVTN numbers, Triangular neutrosophic MCF problem.*

1. Introduction

Linear Programming (LP) has been recognized as a superior optimization tool, extensively utilized by numerous researchers to address a variety of real-world optimization problems. The classical LP proves particularly beneficial in scenarios where both the objective functions and the constraint functions are deterministic in nature. Özlen and Azizoğlu [1] proposed a comprehensive approach for MOIP problem, capable of

* *E-mail:* **ranjank.nit52@gmail.com**

generating all non-dominated solutions. This was further expanded upon by Perini et al. [2], who introduced the boxed line method, a criterion space approach specifically designed for BOMIP problems. Behera et al. [3] introduced two novel methods for addressing the LP problem within a triangular fuzzy context, utilizing concepts of fuzzy core, center, and radius. The application of these methods extends beyond theoretical programming. For instance, Ahmed et al. [4] utilized a fuzzy MO defuzzification approach to tackle a transportation issue, demonstrating the practical implications of these techniques. Similarly, Ilyas et al. [5] addressed a multi-constraint, MO optimal power flow problem. Additionally to solve the optimal water allocation problem in river basins is presented by Haro et al. [6] and so on [7] [8] [9].

The classical MCF problem is characterized by parameters such as flow cost, supply, demand, and capacity, which are typically fixed. However, real-world scenarios often present these parameters as dynamic due to inherent uncertainties. Zadeh's [10] fuzzy logic, introduced in 1965, has been effectively applied by numerous researchers to manage such uncertainties, along with its extensions like fuzzy theory and extended fuzzy theory. The field of MCF problems has seen significant advancements over the years. Shih and Lee [11] investigated fuzzy multi-level MCF problems with multiple objectives and hierarchies, considering fuzzy costs and capacities in the arcs. This was further developed by Sedeño-Noda et al. [12], who demonstrated that the bi-objective undirected two-commodity MCF problem could be transformed into two standard bi-objective MCF problems using a change of variables approach. Raith and Ehrgott [13] presented a two-phase algorithm for solving MCF problem. Moradi et al. [14] handle the multi-commodity MCF problem. Ghasemishabankareh et al. [15] use the genetic algorithm for solving MOIP with MCF problems. Most recently, Khezri and Khodayifar [16] tackled the MOMCMCF problem using copula theory, where they modeled the coefficients of the capacity constraints as normally distributed random variables and their dependence using an Archimedean copula theory. These studies collectively represent the evolution and progress in the field of MCF problems, each contributing unique methodologies and approaches to address complex and multi-faceted challenges. In light of the literature review, it is observed that in a fuzzy environment, uncertainty is handled usefully by fuzzy logic, but when it comes to dealing with indeterminate and inconsistent information data, it is found to be unable to handle such a situation properly due to which Samarandche [17] introduced the Neutrosophic set theory in the year 1999. There are several works in the field of neutrosophic such as Imran et al. [18] intrduces the neutrosophic crisp i-open sets and Parikh and Sahni's [19] work on logistic differential equations under the neutrosophic environment and so on.

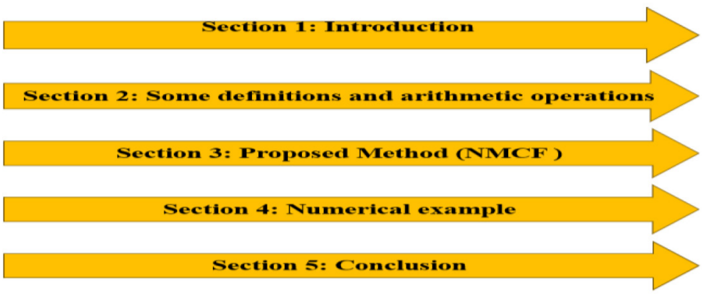
In this article, our primary objective of this article is to address the MCF problem in a neutrosophic environment, specifically where the flow cost parameter is uncertain. Additionally, we aim to solve the numerical problem of the bounded minimal flow problem with neutrosophic flow cost, using triangular neutrosophic numbers. Additionally, the goal is to find the least cost to send commodities through a network. It's a type of network flow problem where the challenge is to meet the needs at each point in the most cost-effective way.

1.1. *List of abbreviations throughout the article*

BOMIP : "Bi-objective Mixed integer programming"

MOIP: "multi-objective integer programming**"; SVTN: "**Single-valued triangular neutrosophic**"; MO: "**multi-objective**"; NMFP:** "Neutrosophic minimal flow problem"**; MOLPP**: **"**multi-objective linear programming problem**"; SOLPP: "**Single-objective linear programming problem**"; MOMCMCF: "**multi-objective multi-commodity minimum cost flow"

1.2. *Structure of the paper*



2. Preliminaries

Arithmetic Operations on SVTN number [20]: $\tilde{K}_{\eta1} = \langle (u_{11}, u_{21}, u_{31}), (v_{11}, v_{21}, v_{31}), (y_{11}, y_{21}, y_{31})\rangle$ $\tilde{K}_{\eta2} = \langle (g_{12}, g_{22}, g_{32}), (s_{12}, s_{22}, s_{32}), (y_{12}, y_{22}, y_{32})\rangle$ be two SVTN numbers and $\Omega > 0$ then-

I. $$\tilde{K}_{\eta1} \oplus \tilde{K}_{\eta2} = \left\langle \begin{matrix} (u_{11}+g_{12}, u_{21}+g_{22}, u_{31}+g_{32}), (v_{11}+s_{12}, v_{21}+s_{22}, v_{31}+s_{32}), \\ (y_{11}+y_{12}, y_{21}+y_{22}, y_{31}+y_{32}) \end{matrix} \right\rangle$$

II. $\tilde{K}_{\eta 1} \otimes \tilde{K}_{\eta 2} = \left\langle \begin{matrix} (u_{11}.g_{12}, u_{21}.g_{22}, u_{31}.g_{32}), (v_{11}.s_{12}, v_{21}.s_{22}, v_{31}.s_{32}), \\ (y_{11}.y_{12}, y_{21}.y_{22}, y_{31}.y_{32}) \end{matrix} \right\rangle$

III. $\Omega.\tilde{K}_{\eta 1} = \langle (\Omega.u_{11}, \Omega.u_{21}, \Omega.u_{31}), (\Omega.v_{11}, \Omega.v_{21}, \Omega.v_{31}), (\Omega.y_{11}, \Omega.y_{21}, \Omega.y_{31}) \rangle$

Single-valued triangular neutrosophic (SVTN) number: - [21] SVTN number is a special type of SVTN set. A set $\tilde{K}_\eta = \langle (u_{11}, u_{21}, u_{31}), (v_{11}, v_{21}, v_{31}), (y_{11}, y_{21}, y_{31}) \rangle$ is called SVTN number if its define a truth functions $T_{\tilde{K}_\eta}(u)$, indeterminacy function, $I_{\tilde{K}_\eta}(u)$ and the falsity membership function $F_{\tilde{K}_\eta}(u)$ of an element u in $\tilde{K}_\eta$ is defined as,

$$T_{\tilde{K}_\eta}(u) = \begin{cases} \left(\frac{u-u_{11}}{u_{21}-u_{11}} \right) & u_{11} \le u < u_{21} \\ 1 & u_{21} = u \\ \left(\frac{u_{31}-u}{u_{31}-u_{21}} \right) & u_{21} < u \le u_{31} \\ 0 & \text{otherwise} \end{cases}, \quad I_{\tilde{K}_\eta}(u) = \begin{cases} \left(\frac{v_{21}-u}{v_{21}-v_{11}} \right) & v_{11} \le u < v_{21} \\ 0 & v_{21} = u \\ \left(\frac{u-v_{21}}{v_{31}-v_{21}} \right) & v_{21} < u \le v_{31} \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\tilde{K}_\eta}(u) = \begin{cases} \left(\frac{y_{21}-u}{y_{21}-y_{11}} \right) & y_{11} \le u < y_{21} \\ 0 & y_{21} = u \\ \left(\frac{u-y_{21}}{y_{31}-y_{21}} \right) & y_{21} < u \le y_{31} \\ 1 & \text{otherwise} \end{cases}$$

with the condition $0 \le T_{\tilde{K}_\eta}(u) + I_{\tilde{K}_\eta}(u) + F_{\tilde{K}_\eta}(u) \le 3$

3. Proposed model

Let us consider a directed graph $\xi^\eta = (V_\eta, \omega_\eta)$ where $V_\eta = \{1, 2, 3, ..., n\}$ is the set of finite nodes and ω_η is the set of arcs and each arc is denoted by $(i, j) \in \omega_\eta$, a flow x_{ij}, and a neutrosophic cost per unite flow $\tilde{C}_{ij}^\eta$ from arc *I* to *j*. Let us consider the two numbers l_{ij} and u_{ij} as lower capacity and upper capacity respectively. For each node $i \in V_\eta$ assign a number h_i representing the supply, demand or transshipment node. If $h_i = 0$ then the node i is a transshipment node, if $h_i < 0$ then node i is a demand node, and $h_i > 0$ then node i is a supply node.

Step 1: The general formulation of the mathematical model for the NMFP with neutrosophic cost is as:

$$\text{Minimise } \tilde{\mathbb{Z}}^{\eta} \approx \sum_{(i,j)\in\omega_{\eta}} \tilde{C}_{ij}^{\eta} x_{ij} \tag{1}$$

$$\text{Subject to} \sum_{j:(i,j)\in\omega_{\eta}} x_{ij} - \sum_{j:(j,i)\in\omega_{\eta}} x_{ji} = h_i, \tag{2}$$

$$0 \le l_{ij} \le x_{ij} \le u_{ij} \quad \forall\ (i,j)\in\omega_{\eta} \tag{3}$$

Here we consider the

$$\tilde{C}_{ij}^{\eta} = \langle(\tilde{\Phi}_{1ij}^{\eta}, \tilde{\Phi}_{2ij}^{\eta}, \tilde{\Phi}_{3ij}^{\eta}), (\tilde{\Phi}_{4ij}^{\eta}, \tilde{\Phi}_{5ij}^{\eta}, \tilde{\Phi}_{6ij}^{\eta}), (\tilde{\Phi}_{7ij}^{\eta}, \tilde{\Phi}_{8ij}^{\eta}, \tilde{\Phi}_{9ij}^{\eta})\rangle$$

Then the NMFP with neutrosophic cost $\tilde{C}_{ij}^{\eta}$, the objective function (1) will be

$$\text{Minimise } \tilde{\mathbb{Z}}^{\eta} \approx \sum_{(i,j)\in\omega_{\eta}} \langle(\tilde{\Phi}_{1ij}^{\eta}, \tilde{\Phi}_{2ij}^{\eta}, \tilde{\Phi}_{3ij}^{\eta}), (\tilde{\Phi}_{4ij}^{\eta}, \tilde{\Phi}_{5ij}^{\eta}, \tilde{\Phi}_{6ij}^{\eta}), (\tilde{\Phi}_{7ij}^{\eta}, \tilde{\Phi}_{8ij}^{\eta}, \tilde{\Phi}_{9ij}^{\eta})\rangle x_{ij} \tag{4}$$

Step 2: The above objective function can now, be expressed as nine distinct classical objective functions: (refer to equations 5 to 13 respectively)

$$\left.\begin{array}{l}
Min(\tilde{\mathbb{Z}}_1^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{1ij}^{\eta} x_{ij};\, Min(\tilde{\mathbb{Z}}_2^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{2ij}^{\eta} x_{ij};\, Min(\tilde{\mathbb{Z}}_3^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{3ij}^{\eta} x_{ij}, \\
Min(\tilde{\mathbb{Z}}_4^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{4ij}^{\eta} x_{ij};\, Min(\tilde{\mathbb{Z}}_5^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{5ij}^{\eta} x_{ij};\, Min(\tilde{\mathbb{Z}}_6^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{6ij}^{\eta} x_{ij}, \\
Min(\tilde{\mathbb{Z}}_7^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{7ij}^{\eta} x_{ij};\, Min(\tilde{\mathbb{Z}}_8^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{8ij}^{\eta} x_{ij};\, Min\ (\tilde{\mathbb{Z}}_9^{\eta}) = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{9ij}^{\eta} x_{ij}
\end{array}\right] \tag{5-13}$$

With the same subject to constraints (2) and (3).

The steps of NMFP with neutrosophic costs $\tilde{C}_{ij}^{\eta}$ are as follows,

Step 2a: The classical MOLPP is solved as a SOLPP. Each time, it focuses on just one objective while ignoring the others.

Step 2b: Based on the obtained results, in every solution from step 1, the values of the objectives are derived column-wise, which are presented in the following Table 1.

Table 1

$\tilde{\mathbb{Z}}_1^{*\eta}$	$\tilde{\mathbb{Z}}_1^{2\eta}$	$\tilde{\mathbb{Z}}_1^{3\eta}$	$\tilde{\mathbb{Z}}_1^{4\eta}$	$\tilde{\mathbb{Z}}_1^{5\eta}$	$\tilde{\mathbb{Z}}_1^{6\eta}$	$\tilde{\mathbb{Z}}_1^{7\eta}$	$\tilde{\mathbb{Z}}_1^{8\eta}$	$\tilde{\mathbb{Z}}_1^{9\eta}$
$\tilde{\mathbb{Z}}_2^{1\eta}$	$\tilde{\mathbb{Z}}_2^{*\eta}$	$\tilde{\mathbb{Z}}_2^{3\eta}$	$\tilde{\mathbb{Z}}_2^{4\eta}$	$\tilde{\mathbb{Z}}_2^{5\eta}$	$\tilde{\mathbb{Z}}_2^{6\eta}$	$\tilde{\mathbb{Z}}_2^{7\eta}$	$\tilde{\mathbb{Z}}_2^{8\eta}$	$\tilde{\mathbb{Z}}_2^{9\eta}$
$\tilde{\mathbb{Z}}_3^{1\eta}$	$\tilde{\mathbb{Z}}_3^{2\eta}$	$\tilde{\mathbb{Z}}_3^{*\eta}$	$\tilde{\mathbb{Z}}_3^{4\eta}$	$\tilde{\mathbb{Z}}_3^{5\eta}$	$\tilde{\mathbb{Z}}_3^{6\eta}$	$\tilde{\mathbb{Z}}_3^{7\eta}$	$\tilde{\mathbb{Z}}_3^{8\eta}$	$\tilde{\mathbb{Z}}_3^{9\eta}$
$\tilde{\mathbb{Z}}_4^{1\eta}$	$\tilde{\mathbb{Z}}_4^{2\eta}$	$\tilde{\mathbb{Z}}_4^{3\eta}$	$\tilde{\mathbb{Z}}_4^{*\eta}$	$\tilde{\mathbb{Z}}_4^{5\eta}$	$\tilde{\mathbb{Z}}_4^{6\eta}$	$\tilde{\mathbb{Z}}_4^{7\eta}$	$\tilde{\mathbb{Z}}_4^{8\eta}$	$\tilde{\mathbb{Z}}_4^{9\eta}$
$\tilde{\mathbb{Z}}_5^{1\eta}$	$\tilde{\mathbb{Z}}_5^{2\eta}$	$\tilde{\mathbb{Z}}_5^{3\eta}$	$\tilde{\mathbb{Z}}_5^{4\eta}$	$\tilde{\mathbb{Z}}_5^{*\eta}$	$\tilde{\mathbb{Z}}_5^{6\eta}$	$\tilde{\mathbb{Z}}_5^{7\eta}$	$\tilde{\mathbb{Z}}_5^{8\eta}$	$\tilde{\mathbb{Z}}_5^{9\eta}$
$\tilde{\mathbb{Z}}_6^{1\eta}$	$\tilde{\mathbb{Z}}_6^{2\eta}$	$\tilde{\mathbb{Z}}_6^{3\eta}$	$\tilde{\mathbb{Z}}_6^{4\eta}$	$\tilde{\mathbb{Z}}_6^{5\eta}$	$\tilde{\mathbb{Z}}_6^{*\eta}$	$\tilde{\mathbb{Z}}_6^{7\eta}$	$\tilde{\mathbb{Z}}_6^{8\eta}$	$\tilde{\mathbb{Z}}_6^{9\eta}$
$\tilde{\mathbb{Z}}_7^{1\eta}$	$\tilde{\mathbb{Z}}_7^{2\eta}$	$\tilde{\mathbb{Z}}_7^{3\eta}$	$\tilde{\mathbb{Z}}_7^{4\eta}$	$\tilde{\mathbb{Z}}_7^{5\eta}$	$\tilde{\mathbb{Z}}_7^{6\eta}$	$\tilde{\mathbb{Z}}_7^{*\eta}$	$\tilde{\mathbb{Z}}_7^{8\eta}$	$\tilde{\mathbb{Z}}_7^{9\eta}$
$\tilde{\mathbb{Z}}_8^{1\eta}$	$\tilde{\mathbb{Z}}_8^{2\eta}$	$\tilde{\mathbb{Z}}_8^{3\eta}$	$\tilde{\mathbb{Z}}_8^{4\eta}$	$\tilde{\mathbb{Z}}_8^{5\eta}$	$\tilde{\mathbb{Z}}_8^{6\eta}$	$\tilde{\mathbb{Z}}_8^{7\eta}$	$\tilde{\mathbb{Z}}_8^{*\eta}$	$\tilde{\mathbb{Z}}_8^{9\eta}$
$\tilde{\mathbb{Z}}_9^{1\eta}$	$\tilde{\mathbb{Z}}_9^{2\eta}$	$\tilde{\mathbb{Z}}_9^{3\eta}$	$\tilde{\mathbb{Z}}_9^{4\eta}$	$\tilde{\mathbb{Z}}_9^{5\eta}$	$\tilde{\mathbb{Z}}_9^{6\eta}$	$\tilde{\mathbb{Z}}_9^{7\eta}$	$\tilde{\mathbb{Z}}_9^{8\eta}$	$\tilde{\mathbb{Z}}_9^{*\eta}$

Step 2c: Now our objective is to find the lower $(\tilde{\mathbb{Z}}_\varepsilon^\eta)^L$ and upper $(\tilde{\mathbb{Z}}_\varepsilon^\eta)^U$ values. For that purpose, we use the below two equations (14) and (15)

$$(\tilde{\mathbb{Z}}_\varepsilon^\eta)^L = \min\{\tilde{\mathbb{Z}}_\varepsilon^{*\eta}, \tilde{\mathbb{Z}}_\varepsilon^{2\eta}, \tilde{\mathbb{Z}}_\varepsilon^{3\eta}, \tilde{\mathbb{Z}}_\varepsilon^{4\eta}, \tilde{\mathbb{Z}}_\varepsilon^{5\eta}, \tilde{\mathbb{Z}}_\varepsilon^{6\eta}, \tilde{\mathbb{Z}}_\varepsilon^{7\eta}, \tilde{\mathbb{Z}}_\varepsilon^{8\eta}, \tilde{\mathbb{Z}}_\varepsilon^{9\eta}\} \tag{14}$$

$$(\tilde{\mathbb{Z}}_\varepsilon^\eta)^U = \max\{\tilde{\mathbb{Z}}_\varepsilon^{*\eta}, \tilde{\mathbb{Z}}_\varepsilon^{2\eta}, \tilde{\mathbb{Z}}_\varepsilon^{3\eta}, \tilde{\mathbb{Z}}_\varepsilon^{4\eta}, \tilde{\mathbb{Z}}_\varepsilon^{5\eta}, \tilde{\mathbb{Z}}_\varepsilon^{6\eta}, \tilde{\mathbb{Z}}_\varepsilon^{7\eta}, \tilde{\mathbb{Z}}_\varepsilon^{8\eta}, \tilde{\mathbb{Z}}_\varepsilon^{9\eta}\} \tag{15}$$

For each objective function, primary model can be written in terms of the aspiration levels $\boldsymbol{\Omega}$. To obtained the objective value $\tilde{\mathbb{Z}}_\varepsilon^\eta$ for each ε^{th} objective function and optimal solution $x_{ij}, \forall\ (i, j) \in \omega_\eta$ along with the constraints (5-13) under the condition $\tilde{\mathbb{Z}}_\varepsilon^\eta \le (\tilde{\mathbb{Z}}_\varepsilon^\eta)^L$ for ε^{th} objective functions. Since all the objective functions here are of the minimization type so membership functions $\mu_{\tilde{\mathbb{Z}}_\varepsilon^\eta}(x_{ij})$ are defined as

$$\mu_{\tilde{\mathbb{Z}}_\varepsilon^\eta}(x_{ij}) = \left\{\begin{array}{ll} 1 & \tilde{\mathbb{Z}}_\varepsilon^\eta \le (\tilde{\mathbb{Z}}_\varepsilon^\eta)^L \\ 1 - \frac{\tilde{\mathbb{Z}}_\varepsilon^\eta - (\tilde{\mathbb{Z}}_\varepsilon^\eta)^L}{(\tilde{\mathbb{Z}}_\varepsilon^\eta)^U - (\tilde{\mathbb{Z}}_\varepsilon^\eta)^L} & (\tilde{\mathbb{Z}}_\varepsilon^\eta)^L < \tilde{\mathbb{Z}}_\varepsilon^\eta < (\tilde{\mathbb{Z}}_\varepsilon^\eta)^U \\ 0 & \tilde{\mathbb{Z}}_\varepsilon^\eta \ge (\tilde{\mathbb{Z}}_\varepsilon^\eta)^U \end{array}\right\}$$

The equivalent LPP can be written as

$Max \quad \Omega$

$$\text{Subject to } \Omega \le \frac{(\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{U} - \tilde{\mathbb{Z}}_{\varepsilon}^{\eta}}{(\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{U} - (\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{L}} \qquad \forall\ \varepsilon = 1,2,....,9$$

where $\Omega = \min\{\mu_{\tilde{\mathbb{Z}}_{\varepsilon}^{\eta}}(x_{ij})\}$

Step 3: *Max* Ω represented as below-

Max Ω

$$\text{Subject to } \tilde{\mathbb{Z}}_{\varepsilon}^{\eta} + \Omega((\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{U} - (\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{L}) \le (\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{U} \quad \forall \varepsilon = 1,2,....,9 \text{ and } \Omega \ge 0 \tag{16}$$

$$\left.\begin{aligned} &\tilde{\mathbb{Z}}_{1}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{1ij}^{\eta} x_{ij}; \tilde{\mathbb{Z}}_{2}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{2ij}^{\eta} x_{ij}; \tilde{\mathbb{Z}}_{3}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{3ij}^{\eta} x_{ij}, \\ &\tilde{\mathbb{Z}}_{4}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{4ij}^{\eta} x_{ij}; \tilde{\mathbb{Z}}_{5}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{5ij}^{\eta} x_{ij}; \tilde{\mathbb{Z}}_{6}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{6ij}^{\eta} x_{ij}, \\ &\tilde{\mathbb{Z}}_{7}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{7ij}^{\eta} x_{ij}; \tilde{\mathbb{Z}}_{8}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{8ij}^{\eta} x_{ij}; \tilde{\mathbb{Z}}_{9}^{\eta} = \sum_{(i,j)\in\omega_{\eta}} \tilde{\Phi}_{9ij}^{\eta} x_{ij} \end{aligned}\right] \tag{17}$$

Along with the same subject to constraints (2) and (3).

Using LINGO 18.0 software can easily solve the above model equations (16) and (17) we get the optimal solution.

4. Numerical example

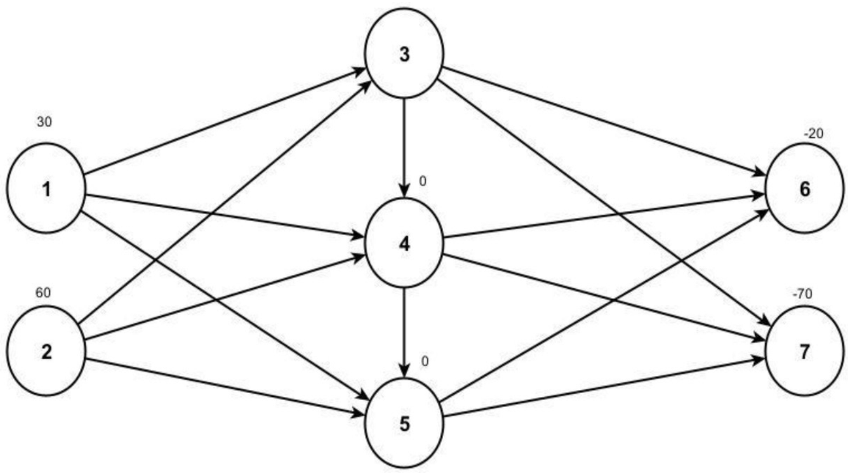


Figure 1

Represent the network for solving NMCFP

Table 2

Consider the data set for solving NMCFP

T	H	Neutrosophic cost $(\tilde{C}_{ij}^{\eta})$	C	T	H	Neutrosophic cost $(\tilde{C}_{ij}^{\eta})$	C
1	3	<(1, 4, 7), (1, 3, 5), (3.5, 6, 7.5) >	40	3	6	<(1, 5, 8), (1.5, 4.5, 7.5) (4, 6.5, 9)>	100
1	4	<(0.5, 2.5, 4.5), (1, 2, 3), (1.5, 3.5, 5.5)>	10	3	7	<(1, 5, 8), (1.5, 3, 6.5), (4, 7, 9)>	90
1	5	<(1, 3, 5), (0.5, 1.5, 3.5), (3, 4, 6)>	15	4	5	<(10, 15, 20), (14, 16, 22), (12, 15, 19)>	15
2	3	<(1, 2, 3), (0.5, 1.5, 2.5), (1.5, 2.5, 3.5)>	50	4	6	<(20, 25, 30), (24, 26, 32), (22, 25, 29)>	30
2	4	<(1, 1.5, 4), (0.5, 1, 2.5), (2.25, 3, 4.25)>	80	4	7	<(15, 20, 25), (19, 21, 27), (17, 20, 24)>	20
2	5	<(1.5, 2.5, 3.5), (1, 1.5, 3), (2, 3, 4)>	40	5	6	<(13, 18, 23), (17, 19, 25), (15, 18, 22)>	15
3	4	<(2, 4, 6), (1.5, 2.5, 4.5), (3, 5, 7)>	20	5	7	<(15, 20, 25), (19, 21, 27), (17, 20, 24)>	100

Where C denotes "capacity" T denotes "Tail node" and H denotes "Head node"

Solution: Step-1 The general formulation of the mathematical model for the NMFP with neutrosophic cost by using data from the Figure 1 and Table 2 is –

$$
\begin{aligned}
&Min\ (\widetilde{Z^{\eta}}) = \\
&<(1,4,7),(1,3,5),(3.5,6,7.5)> x_{13} + <(0.5,2.5,4.5),(1,2,3),(1.5,3.5,5.5)> x_{14} \\
&+<(1,3,5),(0.5,1.5,3.5),(3,4,6)> x_{15} + <(1,2,3),(0.5,1.5,2.5),(1.5,2.5,3.5)> x_{23} \\
&+<(1,1.5,4),(0.5,1,2.5),(2.25,3,4.25)> x_{24} + <(1.5,2.5,3.5),(1,1.5,3),(2,3,4)> x_{25} \\
&+<(2,4,6),(1.5,2.5,4.5),(3,5,7)> x_{34} + <(1,5,8),(1.5,4.5,7.5),(4,6.5,9)> x_{36} \\
&+<(1,5,8),(1.5,3,6.5),(4,7,9)> x_{37} + <(10,15,20),(14,16,22),(12,15,19)> x_{45} \\
&+<(20,25,30),(24,26,32),(22,25,29)> x_{46} + <(15,20,25),(19,21,27),(17,20,24)> x_{47} \\
&+<(13,18,23),(17,19,25),(15,18,22)> x_{56} + <(15,20,25),(19,21,27),(17,20,24)> x_{57}.
\end{aligned}
$$

Subject to constraints equation ………… (**)

$$
\begin{aligned}
&x_{15} + x_{14} + x_{13} = 30;\ 0 \le x_{14} \le 10;\ x_{25} + x_{24} + x_{23} = 60;\ 0 \le x_{15} \le 15; \\
&0 \le x_{13} \le 40;\ 0 \le x_{34} \le 20;\ x_{37} + x_{36} + x_{34} - x_{23} - x_{13} = 0;\ 0 \le x_{36} \le 100; \\
&0 \le x_{23} \le 50;\ 0 \le x_{25} \le 40;\ -x_{36} - x_{46} - x_{56} = -20;\ 0 \le x_{24} \le 80; \\
&0 \le x_{37} \le 90;\ x_{45} + x_{46} + x_{47} - x_{14} - x_{24} - x_{34} = 0;\ 0 \le x_{56} \le 15;\ 0 \le x_{45} \le 15; \\
&-x_{37} - x_{47} - x_{57} = -70;\ -x_{37} - x_{47} - x_{57} = -70;\ 0 \le x_{46} \le 30; \\
&x_{57} + x_{56} - x_{45} - x_{25} - x_{15} = 0;\ 0 \le x_{47} \le 20;\ 0 \le x_{57} \le 100
\end{aligned}
$$

Step 2: The above objective function can now, be expressed as nine distinct classical objective functions

$$Min\ (\tilde{\mathbb{Z}}_1^\eta) = 1x_{13} + 0.5x_{14} + 1x_{15} + 1x_{23} + 1x_{24} + 1.5x_{25} + 2x_{34} + 1x_{36} + 1x_{37} + 10x_{45} + 20x_{46} + 14x_{47} + 13x_{56} + 15x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_2^\eta) = 4x_{13} + 2.5x_{14} + 3x_{15} + 2x_{23} + 1.5x_{24} + 2.5x_{25} + 4x_{34} + 5x_{36} + 5x_{37} + 15x_{45} + 25x_{46} + 20x_{47} + 18x_{56} + 20x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_3^\eta) = 7x_{13} + 4.5x_{14} + 5x_{15} + 3x_{23} + 4x_{24} + 3.5x_{25} + 6x_{34} + 8x_{36} + 8x_{37} + 20x_{45} + 30x_{46} + 25x_{47} + 23x_{56} + 25x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_4^\eta) = 1x_{13} + 1x_{14} + 0.5x_{15} + 0.5x_{23} + 0.5x_{24} + 1x_{25} + 1.5x_{34} + 1.5x_{36} + 1.5x_{37} + 14x_{45} + 24x_{46} + 19x_{47} + 17x_{56} + 19x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_5^\eta) = 3x_{13} + 2x_{14} + 1.5x_{15} + 1.5x_{23} + 1x_{24} + 1.5x_{25} + 2.5x_{34} + 4.5x_{36} + 3x_{37} + 16x_{45} + 26x_{46} + 21x_{47} + 19x_{56} + 21x_{57}.$$

$$Min(\tilde{\mathbb{Z}}_6^\eta) = 5x_{13} + 3x_{14} + 3.5x_{15} + 2.5x_{23} + 2.5x_{24} + 3x_{25} + 4.5x_{34} + 7.5x_{36} + 6.5x_{37} + 22x_{45} + 32x_{46} + 27x_{47} + 25x_{56} + 27x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_7^\eta) = 3.5x_{13} + 1.5x_{14} + 3x_{15} + 1.5x_{23} + 2.25x_{24} + 2x_{25} + 3x_{34} + 4x_{36} + 4x_{37} + 12x_{45} + 22x_{46} + 17x_{47} + 15x_{56} + 17x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_8^\eta) = 6x_{13} + 3.5x_{14} + 4x_{15} + 2.5x_{23} + 3x_{24} + 3x_{25} + 5x_{34} + 6.5x_{36} + 7x_{37} + 15x_{45} + 25x_{46} + 20x_{47} + 18x_{56} + 20x_{57}.$$

$$Min\ (\tilde{\mathbb{Z}}_9^\eta) = 7.5x_{13} + 5.5x_{14} + 6x_{15} + 3.5x_{23} + 4.25x_{24} + 4x_{25} + 7x_{34} + 9x_{36} + 9x_{37} + 19x_{45} + 29x_{46} + 24x_{47} + 22x_{56} + 24x_{57}.$$

Along with the same constraints (**)

After solving step 2a, we get the solutions- $\tilde{\mathbb{Z}}_1^\eta = 305$; $\tilde{\mathbb{Z}}_2^\eta = 825$; $\tilde{\mathbb{Z}}_3^\eta = 1265$; $\tilde{\mathbb{Z}}_4^\eta = 355$; $\tilde{\mathbb{Z}}_5^\eta = 625$; $\tilde{\mathbb{Z}}_6^\eta = 1085$; $\tilde{\mathbb{Z}}_7^\eta = 670$; $\tilde{\mathbb{Z}}_8^\eta = 1070$; $\tilde{\mathbb{Z}}_9^\eta = 1380$; and $x_{13} = 30,\ x_{14} = 0,\ x_{15} = 0,\ x_{23} = 50,\ x_{24} = 0,\ x_{25} = 10,\ x_{34} = 0,\ x_{36} = 10,\ x_{37} = 70,\ x_{45} = 0,\ x_{46} = 0,\ x_{47} = 0,\ x_{56} = 10,\ x_{57} = 0$

After solving step-2b and step-2c using the two equations (14) and (15), we get the lower $(\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{L}=305$ and upper $(\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{U}=1380.$ Now, the lower $(\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{L}$ and upper $(\tilde{\mathbb{Z}}_{\varepsilon}^{\eta})^{U}$ value step 2 can be written in the form of step 3.

Step-3: Now, a objective function are form as-
$Max\ \ \Omega$

Subject to-

$$\tilde{\mathbb{Z}}_{1}^{\eta}+1075\Omega\le 1380;\ \tilde{\mathbb{Z}}_{2}^{\eta}+1075\Omega\le 1380;\ \tilde{\mathbb{Z}}_{3}^{\eta}+1075\Omega\le 1380;$$

$$\tilde{\mathbb{Z}}_{4}^{\eta}+1075\Omega\le 1380;\ \tilde{\mathbb{Z}}_{5}^{\eta}+1075\Omega\le 1380;\ \tilde{\mathbb{Z}}_{6}^{\eta}+1075\Omega\le 1380;$$

$$\tilde{\mathbb{Z}}_{7}^{\eta}+1075\Omega\le 1380;\ \tilde{\mathbb{Z}}_{8}^{\eta}+1075\Omega\le 1380;\ \tilde{\mathbb{Z}}_{9}^{\eta}+1075\Omega\le 1380;\ \ \Omega\ge 0$$

and along with the same constraints (**).

Now using LINGO 18.0, to solve the above LP model we get the optimal solution.

Hence, the solution $\langle(305,825,1265),(355,625,1085),(670,1070,1380)\rangle$ and $x_{13}=30,\ x_{14}=0,\ x_{15}=0,\ x_{23}=50,\ x_{24}=0,\ x_{25}=10,\ x_{34}=0,\ x_{36}=10,\ x_{37}=70,\ x_{45}=0,\ x_{46}=0,\ x_{47}=0,\ x_{56}=10,\ x_{57}=0$

5. Conclusion

The Classical MCF problem is characterized by parameters such as flow cost, supply, demand, and capacity, which are typically fixed. In the field of MCF problems, each contributes unique methodologies and approaches to address complex and multi-faceted challenges This article explores the MCF problem to obtaining the optimal solution in terms of uncertainty. Our aim to solve the numerical problem of the bounded minimal flow problem with neutrosophic flow cost, using triangular neutrosophic numbers and LINGO 18.0 software.

References

[1] M. Özlen and M. Azizoğlu, "Multi-objective integer programming: A general approach for generating all non-dominated solutions," *European Journal of Operational Research*, vol. 199, p. 25–35, 2009.

[2] T. Perini, N. Boland, D. Pecin and M. Savelsbergh, "A criterion space method for biobjective mixed integer programming: The boxed line method," *INFORMS Journal on Computing*, vol. 32, p. 16–39 (2020).

[3] D. Behera, K. Peters and S. A. Edalatpanah, "Alternative methods for linear programming problem under triangular fuzzy uncertainty," *Journal of Statistics and Management Systems*, vol. 25, p. 521–539 (2022).

[4] J. S. Ahmed, H. J. Mohammed and I. Z. Chaloob, "Application of a fuzzy multi-objective defuzzification method to solve a transportation problem," Mater. Today Proc (2021).

[5] M. A. Ilyas, G. Abbas, T. Alquthami, M. Awais and M. B. Rasheed, "Multi-objective optimal power flow with integration of renewable energy sources using fuzzy membership function," IEEE Access, vol. 8, p. 143185–143200 (2020).

[6] D. Haro, J. Paredes, A. Solera and J. Andreu, "A model for solving the optimal water allocation problem in river basins with network flow programming when introducing non-linearities," Water resources management, vol. 26, p. 4059–4071 (2012).

[7] S. M. Stefanov, "Linear programming problems and matrix games," *Journal of Interdisciplinary Mathematics*, vol. 27, p. 57–65 (2024).

[8] S. F. Tantawy and O. M. Saad, "A new method for solving the sum of linear and linear fractional programming problems," *Journal of Information and Optimization Sciences*, vol. 29, p. 849–857 (2008).

[9] C. O. Pieume, L. P. Fotso and P. Siarry, "A method for solving bilevel linear programming problems," *Journal of Information and Optimization Sciences*, vol. 29, p. 335–338 (2008).

[10] L. A. Zadeh, Fuzzy sets, Information and Control, Vol. 8, pp. 338–353, 1965, Reproduced with permission of the copyright owner. Further reproduction … (1965).

[11] H.-S. Shih and E. S. Lee, "Fuzzy multi-level minimum cost flow problems," *Fuzzy Sets and Systems*, vol. 107, p. 159–176 (1999).

[12] A. Sedeno-Noda, C. González-Martın and J. Gutiérrez, "The biobjective undirected two-commodity minimum cost flow problem," *European Journal Of Operational Research*, vol. 164, p. 89–103 (2005).

[13] A. Raith and M. Ehrgott, "A two-phase algorithm for the biobjective integer minimum cost flow problem," *Computers & Operations Research*, vol. 36, p. 1945–1954 (2009).

[14] S. Moradi, A. Raith and M. Ehrgott, "A bi-objective column generation algorithm for the multi-commodity minimum cost flow problem," *European Journal of Operational Research*, vol. 244, p. 369–378 (2015).

[15] B. Ghasemishabankareh, M. Ozlen and X. Li, "NSGA-II for solving multiobjective integer minimum cost flow problem with probabilistic tree-based representation," in Evolutionary Multi-Criterion Optimization: 10th International Conference, EMO 2019, East Lansing, MI, USA (2019).

[16] S. Khezri and S. Khodayifar, "Joint chance-constrained multi-objective multi-commodity minimum cost network flow problem with copula theory," *Computers & Operations Research*, vol. 156, p. 106260 (2023).

[17] F. Smarandache, "A unifying field in Logics: Neutrosophic Logic.," in Philosophy, American Research Press, p. 1–141 (1999).

[18] Q. H. Imran, K. S. Tanak and A. H. M. Al-Obaidi, "On new concepts of neutrosophic crisp open sets," *Journal of Interdisciplinary Mathematics*, vol. 25, pp. 563-572 (2022).

[19] M. Parikh and M. Sahni, "Solution of logistic differential equation in an uncertain environment using neutrosophic numbers," *Journal of Interdisciplinary Mathematics*, vol. 27, p. 145–169 (2024).

[20] M. Abdel-Basset, M. Gunasekaran, M. Mohamed and F. Smarandache, "A novel method for solving the fully neutrosophic linear programming problems," *Neural Computing and Applications*, vol. 31, p. 1595–1605 (2019).

[21] A. Chakraborty, S. P. Mondal, A. Ahmadian, N. Senu, S. Alam and S. Salahshour, "Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications," *Symmetry*, vol. 10, p. 327 (2018).