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Solving neutrosophic critical path problem using Python

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Abstract

The study observation offers a method for project control known as the Neutrosophic Critical Path Method (NCPM) which combines the traditional Critical Path Method (CPM) with neutrosophic sets. Dealing with actual global situations that frequently pose demanding situations for CPM specifically while initiatives involve interconnected operations. The presence of uncertainties creates hurdles during challenge execution, due to incomplete and uncertain data. By illustrating surroundings for instance this research outlines the transition from CPM to NCPM and highlights its advantages. Numerical examples have been implemented for solving NCPM, that discover solutions in the face of uncertainty. Furthermore, the advanced Python software specifically designed for network evaluation permits willpower of critical route lengths. This synthesis complements selection-making in project control that paves the manner for precise management of complicated situations.

Subject Classification: *62A86, 90C05, 90C08.*

Keywords: *Uncertainty, Neutrosophic, Critical path method (CPM), Neutrosophic critical path problem (NCPM).*

1. Introduction

Several strategies for project management are critical to ensure the overall performance of complex scenarios. An approach that is not planned can result in a loss of preference for expected outcomes or profits. By accessing the computer facility, obligations like design, calculation,

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modeling, control, and checking procedures and tasks can be accomplished effectively. Managing complicated activities includes planning from disciplines that require activity durations at different perspective times. These frequent consequences under severe complications need to be managed efficiently. Having numerous methods, traditional scheduling techniques frequently rely upon Gantt charts [1]. While Gantt charts are still equipped as collectible tools, their utility is restrained to scheduling on a large scale [2]. In uniqueness, these Gantt charts fail to outline the interactions and priority relationships among project activities. Network Analysis offers extensive study in making plans and controlling large projects in major fields. The network offers representations of projects that facilitate tracking progress from the start node to the finish node, within a possible time frame and the study assists in other objectives alike resource allocation for minimization of time, costs, etc.

In Network, the critical path method (CPM) [3] [4] was tested to be a useful tool, within the realm of planning, evaluating, and manipulating complex projects [5]. This method scrutinizes the problem statement, identifies criticality in activity durations, and calculates the time required for project completion [6] [7]. However, the usual activity duration prefers deterministic value and fails to involve incomplete, inconsistent information in estimation. To address this difficulty many tasks are adopting alternative facts resources like analysis or expert-based estimates. Involving such situations, fuzzy sets (FS) [8] are a method for undertaking uncertainty in network analysis. To handle this situation efficaciously, the adoption of fuzzy numbers has grown extensively in the enlargement of the CPM application. Several researchers from the wonderful discipline have defined many approaches to define fuzzy, and their perspectives to apply the principle of uncertainty. However, while sorting numbers in the calculations of early and late finish, new kinds of fuzzy numbers can emerge in the range by comparing fuzzy numbers to deduct the duration of the activity [9]. Moreover, many extended principles have been introduced to replicate the uncertainty followed by intuitionistic fuzzy (IF) [10], and neutrosophic (NS) [11], as the intuitionistic fuzzy doesn't deal with the inadequate situation. In such cases, neutrosophic adapts uncertainty parameters into the fields for more practical study. Furthermore, single-valued neutrosophic (SVN) has been attained as an extension of NS [12]. In recent studies, the extended fuzzy approaches are widely implemented in other respective sub-areas of operations research [13] [14] [15].

The preceding works on CPM undergo main drawbacks involving the classical and fuzzy, as the situations or constraints resulting in a crisp solution disregard involving fuzzy situations. Unfortunately, these advanced works no longer virtually cope with the classical in determining the project completion length and the way unique solutions may additionally emerge. As per the observation from the study of CPM, the technique has not been extensively implemented in the neutrosophic logic in research and application. In such case, our objective has proposed to work on a trapezoidal neutrosophic framework using the CPM approach, to resolve complexity networks that handle uncertainty from disagreements, while having multiple possible outcomes. Integrating CPM into the neutrosophic framework can potentially yield novel insights and enhance versatility by enabling optimal solutions by differentiating critical and non-critical activities of the network.

The structure of the paper is as follows: Section 1 presents the introduction; Section 2 presents preliminary concepts on fuzzy sets and their extended fuzzy principles; Section 3 discusses the notations; Section 4 introduces the proposed neutrosophic network problem; Section 5 presents numerical examples: solving trapezoidal neutrosophic; Section 6 discusses the conclusion and then references.

2. Preliminary

The study on FS, NS, and SVN and their corresponding arithmetic operations are defined for our subsequent study development.

Definition 2.1 [16]: Fuzzy set $(\widetilde{fs})$ is defined as $\widetilde{fs} = \{\xi,\ \lambda_{\widetilde{fs}}(\xi);\ \xi \in v\}$, where v the universal discourse and $\lambda_{\widetilde{fs}}(\xi) \in [0,1]$ as it defines the membership function (MF) of ξ in $\widetilde{fs}$ by satisfying the condition $0 \le \lambda_{\widetilde{fs}}(\xi) \le 1$.

Definition 2.2 [17]: A fuzzy trapezoidal number by its MF $\mu_{\widetilde{TP}}(\xi)$ is represented in the form of $\widetilde{TP} = (tp_1,\ tp_2,\ tp_3,\ tp_4)$ as follows:

$$\mu_{\widetilde{TP}}(\xi) = \begin{cases} \frac{\xi - tp_1}{tp_2 - tp_1}, & tp_1 \le \xi \le tp_2; \\ 1, & tp_2 \le \xi \le tp_3; \\ \frac{tp_4 - \xi}{tp_3 - tp_4}, & tp_3 \le \xi \le tp_4; \\ 0, & \text{else.} \end{cases}$$

where $tp_1, tp_2, tp_3, tp_4 \in \mathbb{R}$

Definition 2.3 [18]: Let v be the universal discourse represented as $(n_s)^N$, which defines the elements of truth MF, indeterminacy MF, and falsity MF by satisfying the condition $(n_s)^N = \{\langle \xi; [\psi_{(n_s)^N}(\xi) + \pi_{(n_s)^N}(\xi) + \rho_{(n_s)^N}(\xi)]\rangle \vdots \xi \in \nu\}$, where the limit lies $\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi) : \nu \to [0,1]$. The MF of Truth $\psi_{(n_s)^N}(\xi)$, Indeterminacy $\pi_{(n_s)^N}(\xi)$, and falsity $\rho_{(n_s)^N}(\xi)$ are real and subsets of [0, 1] as follows by the condition:

$$0 \le \psi_{(n_s)^N}(\xi) + \pi_{(n_s)^N}(\xi) + \rho_{(n_s)^N}(\xi) \le 3 \ \forall \xi \in \nu$$

Definition 2.4 [19]: The single-valued trapezoidal neutrosophic number (SVTrN) on the real line $\mathbb{R}$ forms a unique representation that is illustrated $\left\langle \begin{matrix} (tp_1, tp_2, tp_3, tp_4); \\ \psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi) \end{matrix} \right\rangle$ with truth MF, Indeterminacy MF, and falsity MF as follows:

$$\psi_{(n_s)^N}(\xi) = \begin{cases} \kappa_{(NS)^N}\left(\frac{\xi - tp_1}{tp_2 - tp_1}\right), & tp_1 \le \xi \le tp_2; \\ \kappa_{(NS)^N}, & tp_2 \le \xi \le tp_3; \\ \kappa_{(NS)^N}\left(\frac{tp_4 - \xi}{tp_4 - tp_3}\right), & tp_3 \le \xi \le tp_4; \\ 0, & else. \end{cases} \quad \pi_{(n_s)^N}(\xi)\_ = \begin{cases} \frac{(tp_2 - \xi + o_{(NS)^N}(\xi - tp_1))}{(tp_2 - tp_1)}, & tp_1 \le \xi < tp_2; \\ o_{(NS)^N}, & tp_2 \le \xi \le tp_3; \\ \frac{(\xi - tp_3 + o_{(NS)^N}(tp_4 - \xi))}{(tp_4 - tp_3)}, & tp_3 < \xi \le tp_4; \\ 1, & else. \end{cases}$$

$$\rho_{(n_s)^N}(\xi) = \begin{cases} \frac{tp_2 - \xi + \varphi_{(NS)^N}(\xi - tp_1)}{(tp_2 - tp_1)}, & tp_1 \le \xi < tp_2; \\ \varphi_{(NS)^N}, & tp_2 \le \xi \le tp_3; \\ \frac{\xi - tp_1 + \varphi_{(NS)^N}(tp_4 - \xi)}{(tp_4 - tp_3)}, & tp_3 < \xi \le tp_4; \\ 1, & \text{else.} \end{cases}$$

Definition 2.5 [20]:

Let $\tilde{\alpha} = \left\langle \begin{matrix} (tp_1, tp_2, tp_3, tp_4); \\ \widetilde{p_1}\kappa_{(n_s)^N}, \widetilde{p_2}o_{(n_s)^N}, \widetilde{p_3}\varphi_{(n_s)^N} \end{matrix} \right\rangle$ and $\tilde{\beta} = \left\langle \begin{matrix} (tp_{11}, tp_{22}, tp_{33}, tp_{44}); \\ \widetilde{q_1}\kappa_{(n_s)^N}, \widetilde{q_2}o_{(n_s)^N}, \widetilde{q_3}\varphi_{(n_s)^N} \end{matrix} \right\rangle$ be the two single-valued trapezoidal neutrosophic numbers then,

$$\tilde{\alpha} + \tilde{\beta} = \left\langle \begin{matrix} (tp_1 + tp_{11}, tp_2 + tp_{22}, tp_3 + tp_{33}, tp_4 + tp_{44}); \\ \widetilde{p_1}\kappa_{(n_s)^N} \wedge \widetilde{q_1}\kappa_{(n_s)^N}, \widetilde{p_2}o_{(n_s)^N} \vee \widetilde{q_2}o_{(n_s)^N}, \widetilde{p_3}\varphi_{(n_s)^N} \vee \widetilde{q_3}\varphi_{(n_s)^N} \end{matrix} \right\rangle$$

$$\tilde{\alpha} - \tilde{\beta} = \left\langle \begin{matrix} (tp_1 - tp_{11}, tp_2 - tp_{22}, tp_3 - tp_{33}, tp_4 - tp_{44}); \\ \widetilde{p_1}\kappa_{(n_s)^N} \wedge \widetilde{q_1}\kappa_{(n_s)^N}, \widetilde{p_2}o_{(n_s)^N} \vee \widetilde{q_2}o_{(n_s)^N}, \widetilde{p_3}\varphi_{(n_s)^N} \vee \widetilde{q_3}\varphi_{(n_s)^N} \end{matrix} \right\rangle$$

The score and accuracy function provides a convenient method for comparing the two different (SVTrN).

Definition 2.6 [21]

Let $\widehat{n_1} = \left\langle \begin{matrix} (tp_1, tp_2, tp_3, tp_4); \\ \widetilde{p_1}\kappa_{(n_s)^N}, \widetilde{p_2}o_{(n_s)^N}, \widetilde{p_3}\varphi_{(n_s)^N} \end{matrix} \right\rangle$ and $\widehat{n_2} = \left\langle \begin{matrix} (tp_{11}, tp_{22}, tp_{33}, tp_{44}); \\ \widetilde{q_1}\kappa_{(n_s)^N}, \widetilde{q_2}o_{(n_s)^N}, \widetilde{q_3}\varphi_{(n_s)^N} \end{matrix} \right\rangle$ be the two SVTrN Neutrosophic Numbers, such that

1 If then $E(\widehat{n_1}) < E(\widehat{n_2})$, $\widehat{n_1}$ is bigger than $\widehat{n_2}$, denoted by $\widehat{n_1} < \widehat{n_2}$;

2 If $E(\widehat{n_1}) = E(\widehat{n_2})$,

a. $A(\widehat{n_1}) < A(\widehat{n_2})$, then $\widehat{n_1}$ is bigger than $\widehat{n_2}$, denoted by $\widehat{n_1} < \widehat{n_2}$;

b. $A(\widehat{n_1}) = A(\widehat{n_2})$,

i. $C(\widehat{n_1}) < C(\widehat{n_2})$, then then $\widehat{n_1}$ is bigger than $\widehat{n_2}$, denoted by $\widehat{n_1} < \widehat{n_2}$;

Where, $g_j = \langle (tp_j, tp_j, tp_j, tp_j); T_{\widehat{g_j}}, I_{\widehat{g_j}}, F_{\widehat{g_j}} \rangle$

1. The score function $E(\widehat{n_j})$ is given as

$$E(\widehat{n_j}) = \frac{(tp_{1j} + 2tp_{2j} + 2tp_{3j} + tp_{4j})}{6} * \frac{(2 + T_{\widehat{n_j}} - I_{\widehat{n_j}} - F_{\widehat{n_j}})}{3}$$

2. Accuracy function $A(\widehat{n_j})$ is given as

$$E(\widehat{n_j}) = \frac{(tp_{1j} + 2tp_{2j} + 2tp_{3j} + tp_{4j})}{6} * (T_{\widehat{n_j}} - I_{\widehat{n_j}})$$

3. Certainty function is given as

$$C(\widehat{n_j}) = \frac{(tp_{1j} + 2tp_{2j} + 2tp_{3j} + tp_{4j})}{6} * T_{\widehat{n_j}}$$

3. Notations

i : Tail event; j : Head event

E^N, L^N : Early time and the Latest time for the tail event i and head event j

k : Head event of the succeeding activity

t_{ij}^{N} : Activity duration time

es_{ij}^{N} : Start time of the earliest activity

ef_{ij}^{N} : Finish time of the earliest activity

ls_{ij}^{N} : Start time of the latest activity

lf_{ij}^{N} : Finish time of the latest activity

4. Proposed Method of CPM under Neutrosophic Environment

In this section, the pseudo-code gives the study of the CPM technique in a neutrosophic environment:

Pseudo code: Neutrosophic Environment in CPM

Step 1: Start

Step 2: Identify the project activities and dependencies.

Step 3: Determining the project duration (t_{ij}^{N})

Step 4: Using steps 2 and 3; construct the network diagram.

Step 5: Performing the start time of the earliest activity (es_{ij}^{N}) and the finish time of the earliest activity (ef_{ij}^{N}) yields the forward pass method.

Initializing $es_{ij}^{N} = (0,0,0,0;1,0,0)$

Determining the earliest finishing

$$ef_{j}^{N} = \max_{j \in pred(i)} (e_{j}^{N} + t_{ij}^{N})$$

Step 6: Performing the start time of the latest activity (ls_{ij}^{N}) and the finish time of the latest activity (lf_{i}^{N}) yields to backward pass method.

For the ending event assume; $e^{N} = l^{N}$

Determining the latest finish $lf_{i}^{N} = \min_{k \in succ(j)} (lf_{k}^{N} - t_{ij}^{N})$

In the below Figure 1 below, the flow chart of CPM provides the visual representation of all tasks involved in the project by showing the sequence and dependencies of each respective task.

Step 7: Calculating the early and the late activity.

Step 8: Calculate the total float(slack) for each activity to define the critical and non-critical activity

$$\text{Total float (slack)}_i^N = (lf_i^N - ef_i^N)$$

If $\text{Total float (slack)}_i^N = 0,$ then the activity is on the critical path (critical activity)

If $\text{Total float (slack)}_i^N > 0$ so, then the activity has some slack and is not on the critical path (non-critical activity).

Step 9: Identifying the following conditions for critical path time

$$ef_i^N = lf_i^N$$

$$lf_j^N - ef_j^N = 0$$

Step 10: Obtaining the crisp project completion time using definition 2.6.

Step 11: End

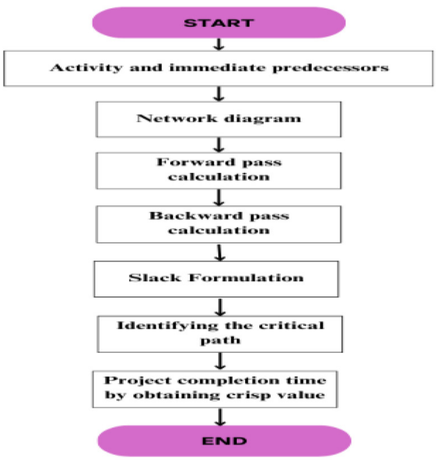

Figure 1

Flow chart of Network Analysis

5. Network Analysis Using Numerical Examples

In the numerical section, the implementation of the network [22] was projected in Figure 2. In the network diagram, where A, B, C, D,.., O represents the activity node (Act) having the trapezoidal neutrosophic

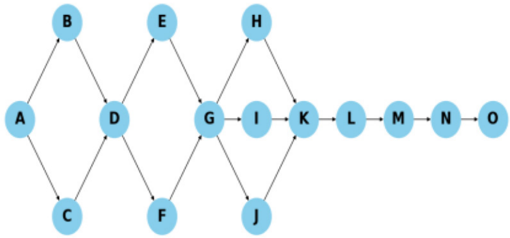

Figure 2

Project Network Diagram

activity time and (A, B), (B, C),.., (N, O) represents the connectivity positions. Using this network, the implementation was carried forward by involving two scenarios, example 5.1 solves with the trapezoidal neutrosophic number having $(\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi))$ parameters, and example 5.2 involves working up with the same parameters of trapezoidal and making $(\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi))$ as constants.

Example 5.1: Consider an uncertainty model i.e., the trapezoidal neutrosophic environment. As mentioned earlier, the study of real-life applications revolves around uncertain activity durations, due to which the completion time of the project increases. To reduce such cases, the T, I, and F parameters could be adjusted between the intervals [0,1]. Table 1 shows the considered activity time of neutrosophic trapezoidal.

Table 1

Trapezoidal Neutrosophic duration time

Act	Immediate Predecessors	Trapezoidal Neutrosophic Activity time	Act	Immediate Predecessors	Trapezoidal Neutrosophic Activity time
A	-	(1.8, 3.1, 3.9, 8.2;0.89, 0.32, 0.11)	I	G	(1.1, 1.6, 1.9, 3.9;0.6, 0.2, 0.1)
B	A	(1.2, 1.4, 1.6, 4.8;0.72, 0.11, 0.28)	J	G	(2.4, 2.6, 3.4, 3.6;0.99, 0.05, 0.01)
C	A	(1.7, 2.3, 3.7, 4.3;0.86, 0.13, 0.14)	K	H, I, J	(0.7, 1.3, 2.7, 3.3;0.88, 0.19, 0.1)
D	B, C	(1.4, 2.6, 3.4, 4.6;0.97, 0.33, 0.03)	L	K	(5, 5.5, 6.5, 7;0.97, 0.11, 0.02)
E	D	(0.8, 1.8, 2.2, 3.2;0.86, 0.17, 0.1)	M	L	(5.7, 6.7, 7.3, 8.3;0.99, 0.02, 0.01)

F	D	(1, 1.6, 2.4, 3;0.89, 0.34, 0.11)	N	M	(1, 1.6, 2.4, 3;0.66, 0.13, 0.13)
G	E, F	(1.9, 2.8, 3.2, 4.1;0.99, 0.11, 0.01)	O	N	(0.9, 1.5, 2.5, 3.1;0.95, 0.18, 0.05)
H	G	(2.8, 3.3, 4.7, 5.2;0.76, 0.03, 0.2)			

```
'A': (1.80,3.10,3.90,8.20); 0.89,0.32,0.11
'B': (3.00,4.50,5.50,13.00); 0.72,0.32,0.28
'C': (3.50,5.40,7.60,12.50); 0.86,0.32,0.14
'D': (4.90,8.00,11.00,17.10); 0.86,0.33,0.14
'E': (5.70,9.80,13.20,20.30); 0.86,0.33,0.14
'F': (5.90,9.60,13.40,20.10); 0.86,0.34,0.14
'G': (7.60,12.60,16.40,24.40); 0.86,0.33,0.14
'H': (10.40,15.90,21.10,29.60); 0.76,0.33,0.20
'I': (8.70,14.20,18.30,28.30); 0.60,0.33,0.14
'J': (10.00,15.20,19.80,28.00); 0.86,0.33,0.14
'k': (11.10,17.20,23.80,32.90); 0.76,0.33,0.20
'L': (16.10,22.70,30.30,39.90); 0.76,0.33,0.20
'M': (21.80,29.40,37.60,48.20); 0.76,0.33,0.20
'N': (22.80,31.00,40.00,51.20); 0.66,0.33,0.20
'O': (23.70,32.50,42.50,54.30); 0.66,0.33,0.20
```

Figure 3

Forward pass calculation under Python Environment

```
'O': ('23.70, 32.50, 42.50, 54.30); 0.66, 0.33, 0.20
'N': ('22.80, 31.00, 40.00, 51.20); 0.66, 0.33, 0.20
'M': ('21.80, 29.40, 37.60, 48.20); 0.66, 0.33, 0.20
'L': ('16.10, 22.70, 30.30, 39.90); 0.66, 0.33, 0.20
'K': ('11.10, 17.20, 23.80, 32.90); 0.66, 0.33, 0.20
'J': ('10.40, 15.90, 21.10, 29.60); 0.66, 0.33, 0.20
'I': ('10.40, 15.90, 21.10, 29.60); 0.66, 0.33, 0.20
'H': ('10.40, 15.90, 21.10, 29.60); 0.66, 0.33, 0.20
'G': ('7.60, 12.60, 16.40, 24.40); 0.66, 0.33, 0.20
'F': ('5.70, 9.80, 13.20, 20.30); 0.66, 0.33, 0.20
'E': ('5.70, 9.80, 13.20, 20.30); 0.66, 0.33, 0.20
'D': ('4.90, 8.00, 11.00, 17.10); 0.66, 0.33, 0.20
'C': ('3.50, 5.40, 7.60, 12.50); 0.66, 0.33, 0.20
'B': ('3.50, 5.40, 7.60, 12.50); 0.66, 0.33, 0.20
'A': ('1.80, 3.10, 3.90, 8.20); 0.66, 0.33, 0.20
```

Figure 4

Backward pass calculation under Python Environment

Using the study of Table 1, the implementation is solved using the proposed method of CPM from section 4 and the observed outcomes are generated from the Python environment calculating the forward and backward pass methods are mentioned in Figure 3 and Figure 4. The results showcase the neutrosophic trapezoidal parameters, as the basic information; forward pass of node A having $\left\langle \begin{array}{l}(tp_1,tp_2,tp_3,tp_4);\\ \psi_{(n_s)^N}(\xi),\pi_{(n_s)^N}(\xi),\rho_{(n_s)^N}(\xi)\end{array}\right\rangle$ as $\left\langle \begin{array}{l}(1.80,3.10,3.90,8.20);\\ 0.89,0.32,0.11\end{array}\right\rangle$ and using the definition 2.6 lead to the crisp value of the activity time. The analysis offers insights into task scheduling and performance highlighting factors of project control and optimizing timeline.

Incorporating the Python environment, by calculating the total float (slack) formulation, it considers the activity delay by not affecting the project completion time, the results are showcased in Figure 5 and the possible obtained neutrosophic critical path (NCP) are shown in Figure 6 and Figure 7 respectively.

```
'A': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'B': (0.50,0.90,2.10,-0.50); 0.66,0.33,0.28 - 1
'C': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'D': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'E': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'F': (-0.20,0.20,-0.20,0.20); 0.66,0.34,0.20 - 0
'G': (0.00,0.20,0.00,0.00); 0.66,0.33,0.20 - 0
'H': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'I': (1.70,1.70,2.80,1.30); 0.60,0.33,0.20 - 1
'J': (0.40,0.50,1.30,1.60); 0.66,0.33,0.20 - 1
'K': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'L': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'M': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'N': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
'O': (0.00,0.00,0.00,0.00); 0.66,0.33,0.20 - 0
```

Figure 5

Total float (Slack) Formulation under Python Environment

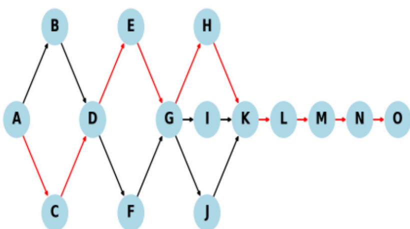

Figure 6

Possible Critical Path Network-1

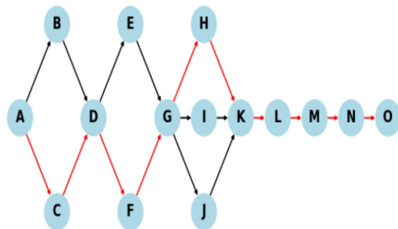

Figure 7

Possible Critical Path Network-2

From the obtained possible outcomes in finding the NCP (Figure 6 and 7). By incorporating parameters of truthfulness, indeterminacy, and falsity into the take of uncertainty. The NCP for Figure 6 provides the neutrosophic critical path (NCP) as $A \to C \to D \to E \to G \to H \to K \to L \to M \to N \to O$, and its neutrosophic critical length (NCL) as

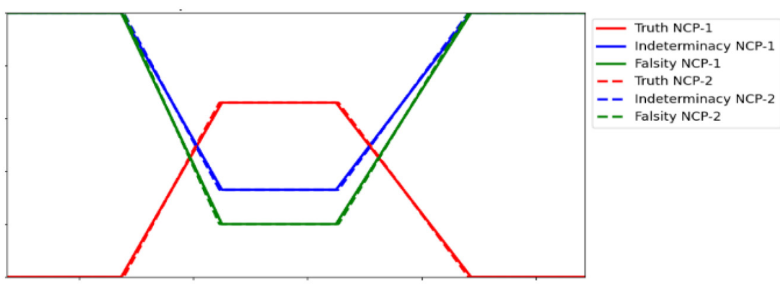

Figure 8

Comparison of NCPL-1 and NCPL-2

$\langle(23.70,\ 32.50,\ 42.50,\ 54.30);\ 0.66,\ 0.33,\ 0.20\rangle$, whereas from Figure 7, $A \to C \to D \to F \to G \to H \to K \to L \to M \to N \to O$ provides the NCP, and its NCL is obtained as $\langle(23.90,\ 32.30,\ 42.70,\ 54.10);\ 0.66,\ 0.33,\ 0.20\rangle$, from both the studies the optimality crisp solution is defined from the definition 2.6 is 26.98 weeks.

From the obtained NCPL, Figure 6 and Figure 7 have the same criticality outcome and the representations for NCPL-1 are the solid style and NCPL-2 is of dashed style has been depicted in Figure 8.

Example 5.2: Implementing the same network from Figure 1 and adjusting the values of $(\psi_{(n_s)^N}(\xi),\ \pi_{(n_s)^N}(\xi),\ \rho_{(n_s)^N}(\xi))$ MF to (0.89, 0.32, 0.11) and setting the trapezoidal parameters $(tp_1,\ tp_2,\ tp_3,\ tp_4)$ as constants is mentioned in Table 2, example 5.2 study aims to explore how levels of uncertainty may impact project completion time without altering the critical path. This approach allows for an analysis of how various scenarios involving confidence, ambiguity, and risk influence the timeline of a project.

Table 2

Trapezoidal Neutrosophic duration time with change in (T, I, F)

Act	Immediate Predecessors	Trapezoidal Neutrosophic Activity time
A	-	(1.8,3.1,3.9,8.2; 0.89, 0.32, 0.11)
B	A	(1.2,1.4,1.6,4.8; 0.89, 0.32, 0.11)
C	A	(1.7,2.3,3.7,4.3; 0.89, 0.32, 0.11)
D	B, C	(1.4,2.6,3.4,4.6; 0.89, 0.32, 0.11)
E	D	(0.8,1.8,2.2,3.2; 0.89, 0.32, 0.11)
F	D	(1,1.6,2.4,3; 0.89, 0.32, 0.11)
G	E, F	(1.9,2.8,3.2,4.1; 0.89, 0.32, 0.11)
H	G	(2.8,3.3,4.7,5.2; 0.89, 0.32, 0.11)
I	G	(1.1,1.6,1.9,3.9; 0.89, 0.32, 0.11)
J	G	(2.4,2.6,3.4,3.6; 0.89, 0.32, 0.11)
K	H, I, J	(0.7,1.3,2.7,3.3; 0.89, 0.32, 0.11)
L	K	(5,5.5,6.5,7; 0.89, 0.32, 0.11)
M	L	(5.7,6.7,7.3,8.3; 0.89, 0.32, 0.11)
N	M	(1,1.6,2.4,3; 0.89, 0.32, 0.11)
O	N	(0.9,1.5,2.5,3.1; 0.89, 0.32, 0.11)

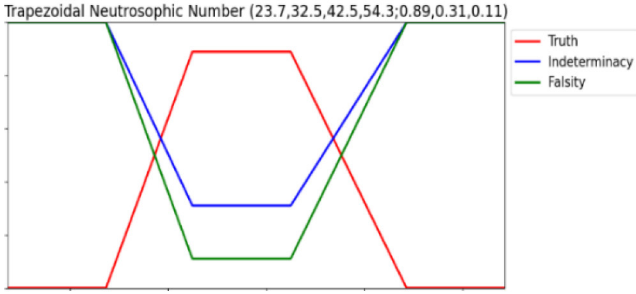

Figure 9

Trapezoidal Neutrosophic change in (T, I, F)

The result outcome of for example 5.2 having the NCL $\langle(23.70, 32.50, 42.50, 54.30); 0.89, 0.32, 0.11\rangle$ is presented in Figure 9 with the outcome of project completion crisp time having 31.18 weeks, modifying the values of $(\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi))$ values varied, but the critical path of the project network remained consistent. and maintaining the parameters at (23.70, 32.50, 42.50, 54.30).

Despite these variations, the alteration (T, I, F) of $(\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi))$ values led to a sudden shift, in projected completion time from 26.98 to 31.18 weeks of measurement is observed. This increase can be attributed to a lower falsity value of 0.29 to 0.11 indicating a decreased perception of risk and challenges in managing the project's lifecycle while also experiencing a marginal constant in uncertainty due to indeterminacy parameter study. Although there was also an increase in truth value suggesting confidence, in execution strategies; however, the lower falsity value lowered the perception of risk, leading to a more aggressive project

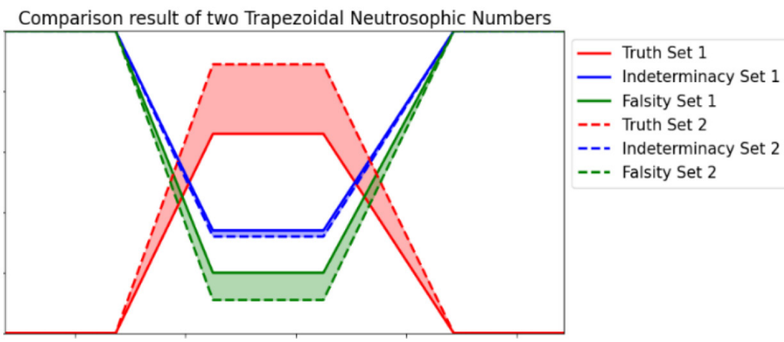

Figure 10

Comparison result of two Trapezoidal Neutrosophic Numbers

timeline, resulting in a longer completion time that has led to 31.16 weeks. Such changes highlight the study perception effects in project timelines even when the core structure remains constant. From the outcome of examples 5.1 and 5.2, the illustration purposes as shown in Figure 10 demonstrate the comparison results and that occurred within the environment.

6. Conclusion

An essential part of handling complicated tasks is scheduling, planning, and managing. Using management techniques is crucial to complete a project within time. Here, the empirical network study evaluated CPM from the classical environment as an example. Further, the proposed study was then taken forward using a linear trapezoidal shape for example 5.1 summarized by having NCPL-1 and NCPL-2 both represented as $\langle(23.70, 32.50, 42.50, 54.30); 0.66, 0.33, 0.20\rangle$ and the $\langle(23.90, 32.30, 42.70, 54.10); 0.66, 0.33, 0.20\rangle$ obtained result for both the critical path of the crisp value is defined by 26.98 weeks, implementing a better optimum result considering the uncertainty study. In further example 5.2, by changing the $(\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi))$ parameters while keeping the trapezoidal numbers (tp_1, tp_2, tp_3, tp_4) as constants, the CPL is formed as $\langle 23.70, 32.50, 42.50, 54.30; 0.89, 0.32, 0.11\rangle$ the estimated duration of completing the project is increased to 31.16 weeks. To find out the study results accurately, the implementation was further done by the Python environment to accurately predict the outcomes of the real-case data. The results indicate that slight change in $(\psi_{(n_s)^N}(\xi), \pi_{(n_s)^N}(\xi), \rho_{(n_s)^N}(\xi))$ parameters can significantly affect the project completion estimate. Future research might focus on applying these findings to a variety of fields while investigating through uncertainty effect in project prediction.

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