Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 45 (2024), No. 3, pp. 717–733
DOI : 10.47974/JIOS-1402

Integrated assembled production decision support inventory system

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Abstract

In the effective supply chains development, the interaction between financial and inventory decisions is important for suppliers and manufacturers. In addition, implementing effective coordination between supply chain participants has become a related research issue. Considering a two-echelon model of multi-suppliers and one manufacturer in this study, coordinate their situations to maximize the total supply chain profits. One or more components required for the final product produced are provided by each supplier. The payments which are permissible delay is coordinated to order quantity between two echelons in the proposed inventory level model. This study is to establish an integrated inventory model to minimize the joint total expected inventory cost and develop a decision support system (DSS).

Subject Classification: *03C13, 62F07.*

Keywords: *Permissible delay in payment, Two-echelon model, Decision support system.*

Introduction

Supply chain includes various establishments involved in procurement or acquisition, production or storage of finished products, intermediate products or raw materials.

The reason why many companies aim to determine the business process they can efficiently carry out is that demand of customers in nowadays economy is difficult to satisfy due to technical and organizational skills haven't been fully and efficiently mastered. Information flow, products, or cash should be integrated to manage these facilities like one company.

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When suppliers receive order forms, they will send assembled items to the manufacturer in ATO systems. Due to the benefits of risk pooling Hillier [1]

has been indicated that the utilization of a reduced number of common components instead of specific components can lead to a decrease in safety stock levels. He developed a model under the environment of ATO, and the components were restocked using a (Q,r) policy of replenishment. Iravani et al. [2] present an ATO system, every customer order comprises a mixture of key and non-key items. Ervolina et al. [3] raised a new idea named Available-to-Sell (ATS) which is a novel availability management process that implements a better supply chain efficiency. When the latter are stock-out, the substitution of higher-class components for lower one was often applied. Nevertheless, the decision for substitution should be made beforehand. In recent times, Reiman and Wang [4] develop a multiple stages stochastic optimization problem that presents a minimum estimate of the average inventory cost over the long term. Additionally, this program incorporates a policy of replenishment specifically designed for such systems. Chang et al. [5] examined an imperfect processes system involving in a two-stage assembly. Under stochastic environment, a new optimization method is proposed by Danilovic et al. [6] to develop a multi-period, problem of inventory control.

Elhafsi et al. [7] performed research on an assembled model that addresses the requirements for both end products and individual components. To deal with the integrated inventory model, Huang & Yang [8] Develops a system that assists in making informed decisions, as the reduction of inventory cost in production management.

The consideration of permissible delay in payment is a novel topic. In contrast to traditional EOQ model, the buyer has a permitted delay payment period upon receiving the products. Also, the buyer can alleviate the burden on cash flow by a permitted immovable period from buyer. During that time, the buyer does not only pay to the vendor but also can earns extra interest from the sale and keep products. Jaber and Osman [9] presented a centralized supply chain model with two levels, minimizing individual costs by optimizing their orders. Under trade credit policy with three levels, Pal et al. [10] examined supplier's optimal lot size and manufacturer's optimal production rate. Chiu and Yang et al. [11] present a better one-vender and one-buyer inventory model, and it enable enterprises to enhance profitability and lower costs in a situation under imperfect production with ordering quantity. Furthermore, it can be the reference applied in business because of the model more realistic and considered the factors impact inventory costs. Das et al. [12] developed a multi-secondary warehouse and one primary warehouse with multi-item include worsening. Items were sold from the main market's primary warehouse. If the stock level exceeds capacity in the primary warehouse, the surplus items are stored in multi-secondary warehouse with limited capacity. Sarkar et al. [13] supposed there are deteriorate items in the production, and considered the model with the lead time and the order quantity. Chen and Teng [14] developed a dedicated EOQ model tailored to cater to the needs of a retailer when: (1) his or her product has a limited lifespan and decreases continuously, and (2) he or she can pay in a delay period permitted by supplier. Yang and Tseng [15] present a three-level inventory model that takes into account factors such as adjustable lead time, permissible delay in payment, and consideration of backorders. The objective was to identify a policy of optimal inventory that the supply chain would get a maxima profit. Yang et al. [16] incorporated imperfect production and repair rates into the proposed model and questioned the potential impact of these factors on profitability in the next year. Furthermore, while developing the integrated model with a permissible delay period depends on the price, they will take into account the ordering cost, transportation cost, and holding cost, considering the potential presence of defective products. This research focus on an ATO system involving multiple suppliers and one manufacturer. It takes into account the permissible trade credit and aims to coordinate the operations to minimize the total costs of supply chain. Song et al. [17] integrated the model base, knowledge base, database, and method base to form the decision support layer used to help management to make decision.

Sprague and Carson [18] propose that an all-inclusive framework for a decision support system should encompass the elements illustrated in Figure 1:

- Model Base: The decision analysis model and data analysis model are stored separately, considering them as independent sub-patterns.
- Data Base: The necessary data are store in the decision support system.
- Software System: It is composed of DBMSs, MBMSs, and DGMSs.

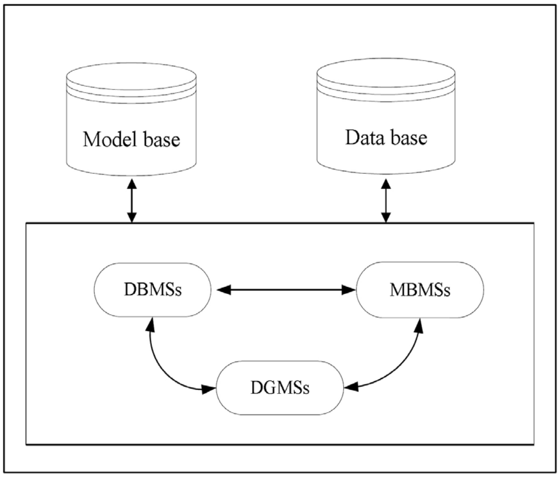

Figure 1
Framework of DSS System

The study aims to minimize the joint total expected inventory of integrated inventory model and devise a decision support system procedure, using the concepts to determine the optimal solution and assist supervision in making decisions.

1. Notation and assumptions

We split certain notations used in the expected joint annual stock model into two components: annual profit of multiple suppliers and a manufacturer. This division helps in developing a two-level inventory model that incorporates an assembly system and accounts for the permissible delay in payment.

The notations and assumptions of the two-level inventory model is presented in the following section:

Notation	Meaning
Q	Economic quantity of manufacturer, a decision variable.
n_s	Quantity of lots manufacturer received from the s^{th} supplier in production cycle, a natural number, a decision variable.

- Notations used on the model of supplier

Q_s = The quantity of economic delivery components per supplier, where $Q_s = Q * \Sigma_i u_{si}$.

D_s = the s^{th} supplier Average annual demand per unit.

P_s = The rate of s^{th} supplier's production.

F_{si} = Transportation cost per ordering of item i for supplier.

C_{si} = Purchasing cost per unit of item i for supplier.

A_s = Ordering cost for each order of supplier.

I_{sp} = The opportunity cost per dollar per year for supplier.

h_{si} = The unit of item i holding cost for supplier.

I_{se} = Supplier's interest earned per dollar for one year.

u_{si} = The quantity of units needed in the final product per unit the s^{th} supplier supplied.

m = suppliers' number. (s = 1, 2, …, m)

k_s = Number of the item supplier s supplied different types to manufacturer. (i = 1, 2, …, k_s)

k = M suppliers supplied the number of different types of items.

* Note that $k = \sum_{s=1}^{m} k_s$ and each supplier provides distinct items that it is specific to each supplier and differs from one supplier to another.

T_s = Supplier's cycle time.

- Notations used on the model of manufacturer

P_m = Production rate of manufacturer where equals to Assembling rate of manufacturer.

D = Average annual demand per unit time.

C_{mi} = The unit of item i purchasing cost for manufacturer.

C_p = The unit selling price cost for manufacturer.

A_m = Ordering cost of manufacturer for each order placed.

B = Assembling cost of manufacturer of one unit.

h_{mi} = The unit of item i holding cost for manufacturer.

h_m = The unit of finished product holding cost for manufacturer.

I_{me} = Manufacturer's interest earned per dollar for one year.

I_{mp} = Manufacturer's opportunity cost per dollar for one year.

X = Allowable payment delay period for manufacturer.

n_m = Quantity of lots delivered from the manufacturer to a retailer per production cycle, a natural number.

TP_{sj} = The annual total profit of supplier in case j.

CTP_{sj} = All supplier's collective annual total profit in case j.

TP_{mj} = Annual total profit of manufacturer in case j.

$EJTP_j$ = Annual expected joint total profit in case j.

"j" indicates two separate situations concerning the connection between manufacturer's permissible payment period and the cycle time of the supplier (j = 1, 2). In the upcoming section, we will delve into specific particulars.

There are 7 assumptions we assumed as below:

(1) The supply chain system involves multiple suppliers and a manufacturer.

(2) Demand is constant over time and deterministic.

(3) The finished product requires k items.

(4) Shortages are not allowed.

(5) The economic order quantity (EOQ) is determined by multiplying the quantity of economical delivery by quantity of deliveries per production run.

(6) The time horizon is infinite.

(7) The purchasing cost must lower than the sale price at every echelon.

$$C_p > \sum_s \sum_i C_{mi} * u_{si} > \sum_s \sum_i C_{si} * u_{si}$$

2. Model formulation

We discuss and combine supplier and manufacture's model into an integrated joint inventory model in this section.

A. Annual total profit of the supplier

In this model, manufacturer receive i^{th} item from the supplier and each supplier provide one or more unique item. The supplier's model is divided into a few parts including ordering cost, purchasing cost, transportation cost, holding cost, sales revenue, interest income and opportunity cost and. The total annual profit of supplier includes consists as follow:

(i) Sales revenue $= \mathrm{D} * \sum_i C_{mi} * u_{si}$

(ii) Ordering cost $= \frac{A_s D}{Q}$

(iii) Purchasing cost $= \mathrm{D} * \sum_i C_{si} * u_{si}$

(iv) Holding cost $= \sum_i \left(h_{si} * \frac{D_s Q_s}{2P_s} \right)$

(v) Transportation cost $= \sum_i \frac{n_s F_{si} * D}{Q}$

The grey area in Figure 2 calculates the inventory level of supplier. We extend the function as follow:

$$\text{Area}_s = \frac{1}{2} * Q_s * \frac{Q_s}{P_s} = \frac{Q_s^{\,2}}{2P_s}$$

$$H_s = \Sigma_i h_{si} * \frac{D_s}{Q_s} * \text{Area}_s = \Sigma_i \left(h_{si} * \frac{D_s Q_s}{2P_s} \right)$$

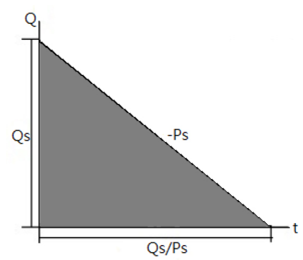

Figure 2

inventory level of the supplier

We need to investigate two cases in the context of permissible delay in payments.

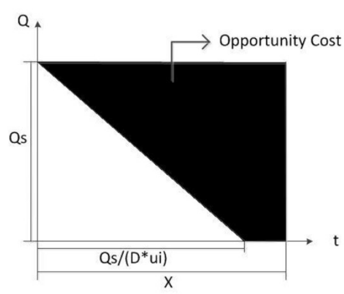

Figure 3

Q/D>X

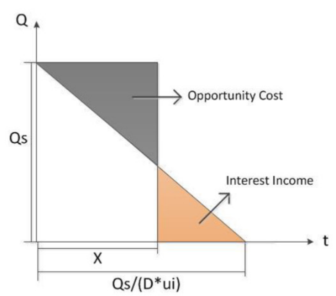

Figure 4

Q/D<X

In Figure 3, when the cycle time $T_s(Q/D)$ shorter than the payment time X, manufacturer will have additional interest income paid by the supplier. In contrast (Figure 4), the supplier would have fewer opportunity cost and earn interest income suppose that the cycle time $T_s(Q/D)$ was longer than payment time X, and bring fewer interest income and additional opportunity cost to manufacturer. The profit function of the supplier can be categorized into two cases depending on the relationship between allowable payment delay period X and the cycle time $T_s(Q/D)$, where the cycle time is longer than the payment time. These cases involve two different parts, as mention earlier.

Case 1 $(T_s < X)$	Case 2 $(T_s \geq X)$
(i) Opportunity cost $= \Sigma_i C_{si} * I_{sp} * \left(D_s X - \frac{D_s}{2}\right)$	(i) Opportunity cost $= \Sigma_i C_{si} * I_{sp} * \frac{(D_s * X)^2}{2Q_s}$ (ii) Interest income $= \Sigma_i C_{mi} * I_{se} * \frac{(Q_s - D_s * X)^2}{2Q_s}$

The expression for collective total annual cost for m suppliers in case 1 is as follows:

$$TP_{s1} = \Sigma_s \left[D * \Sigma_i (C_{mi} - C_{si}) * u_{si} - \frac{A_s D}{Q} - \Sigma_i \frac{n_s F_{si} * D}{Q} - \Sigma_i \left(h_{si} * \frac{D_s Q_s}{2P_s} \right) - \Sigma_i C_{si} * I_{sp} * \left(D_s X - \frac{D_s}{2} \right) \right] \quad (1)$$

The expression for collective total annual cost for m suppliers in case 2 is as follows:

$$TP_{s2} = \Sigma_s \left[D * \Sigma_i (C_{mi} - C_{si}) * u_{si} - \frac{A_s D}{Q} - \Sigma_i \frac{n_s F_{si} * D}{Q} - \Sigma_i \left(h_{si} * \frac{D_S Q_S}{2P_S} \right) - \Sigma_i C_{si} * I_{sp} * \frac{(D_s * X)^2}{2Q_s} + \Sigma_i C_{mi} * I_{se} * \frac{(Q_s - D_s * X)^2}{2Q_s} \right] \quad (2)$$

B. *Annual total profit of the manufacturer*

We divide several parts of the manufacturer's model in each production run which includes sales revenue, holding cost for finished products, holding cost, ordering cost, purchasing cost, assembling cost for items, interest income and opportunity cost. The elements of manufacturer's total annual profit is as follow:

(i) Sales revenue $= D * C_p$

(ii) Items's holding cost $= \Sigma_s \Sigma_i h_{mi} * \frac{D_s Q_s}{n_s}\left(\frac{2-n_s}{2p_s} + \frac{n_s - 1}{2D_s}\right)$

(iii) Ordering cost $= \frac{A_m D}{Q}$

(iv) Purchasing cost $= D * \Sigma_s \Sigma_i (C_{mi} * u_{si})$

(v) Assembling cost $= BP_m$

In Figure 5, the gray area indicates inventory of the manufacturer in one period, calculating by the following function:

$$Area_m = Q_s\left[\frac{Q_s}{n_s P_s} + \frac{(n_s-1)Q_s}{n_s D_s}\right] - \frac{1}{2} * Q_s * \frac{Q_s}{P_s} - \frac{Q_s}{n_s} * \frac{Q_s}{n_s D_s} * \frac{(n_s-1)n_s}{2} = \frac{Q_s{}^2}{n_s}\left(\frac{2-n_s}{2P_s} + \frac{n_s-1}{2D_s}\right)$$

$$H_m = h_{mi} * \frac{D_s}{Q_s} * Area_m = h_{mi} * \frac{D_s Q_s}{n_s}\left(\frac{2-n_s}{2p_s} + \frac{n_s-1}{2D_s}\right)$$

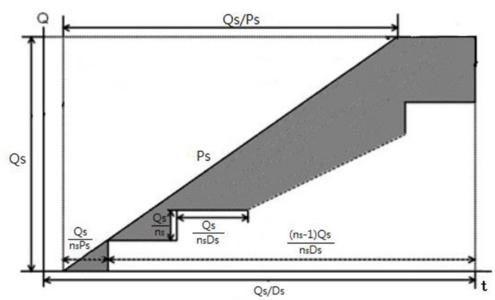

Figure 5

Inventory level of the manufacturer

The manufacturer will start to assemble the products when the manufacturer has received the items from multi-suppliers. The finished products' holding cost can be express in the following:

(i) The finished products' holding cost $= h_m * \frac{DQ}{n_m}\left(\frac{2-n_m}{2p_m} + \frac{n_m-1}{2D}\right)$

We discuss the relationship between the cycle time and the allowable payment time similar to the last section. The manufacturer's model also encompasses two cases that we must be investigated, and the models of the supplier and the manufacturer share similarities. Due to the two cases arise from the profit function of the manufacturer, the two different parts between two possible are determined by payment time X and cycle time T_s's length are as follow.

Case 1	Case 2
(i) Interest income $=\Sigma_s\Sigma_i\left[C_p * \frac{u_{si}}{\Sigma_s\Sigma_i u_{si}} * I_{me}\left(D_s X-\frac{D_s}{2}\right)\right]$	(i) Opportunity cost $=$ $\Sigma_s\Sigma_i\left[C_{mi} * I_{mp} * \frac{(Q_s-D_s * X)^2}{2Q_s}\right]$ (ii) Interest income $=$ $\Sigma_s\Sigma_i\left[C_p * \frac{u_{si}}{\Sigma_s\Sigma_i u_{si}} * I_{me} * \frac{(D_s * X)^2}{2Q_s}\right]$

As a result, TP_{m1} and TP_{m2} are given by:

$$TP_{m1}=D *[C_p-\Sigma_s\Sigma_i(C_{mi} * u_{si})]-\frac{A_m D}{Q}-BP_m-\Sigma_s\Sigma_i h_{mi} * \frac{D_s Q_s}{n_s}\left(\frac{2-n_s}{2p_s}+\frac{n_s-1}{2D_s}\right)$$
$$-h_m * \frac{DQ}{n_m}\left(\frac{2-n_m}{2p_m}+\frac{n_m-1}{2D}\right)+\Sigma_s\Sigma_i\left[C_p * \frac{u_{si}}{\Sigma_s\Sigma_i u_{si}} * I_{me}\left(D_s X-\frac{D_s}{2}\right)\right] \quad (3)$$

$$TP_{m2}=D *[C_p-\Sigma_s\Sigma_i(C_{mi} * u_{si})]-\frac{A_m D}{Q}-BP_m-\Sigma_s\Sigma_i h_{mi} * \frac{D_s Q_s}{n_s}\left(\frac{2-n_s}{2p_s}+\frac{n_s-1}{2D_s}\right)-h_m$$
$$* \frac{DQ}{n_m}\left(\frac{2-n_m}{2p_m}+\frac{n_m-1}{2D}\right)-\Sigma_s\Sigma_i\left[C_{mi} * I_{mp} * \frac{(Q_s-D_s * X)^2}{2Q_s}\right]+\Sigma_s\Sigma_i\left[C_p * \frac{u_{si}}{\Sigma_s\Sigma_i u_{si}} * I_{me} * \frac{(D_s * X)^2}{2Q_s}\right] \quad (4)$$

C. *The expected joint annual total profit (EJTP):*

Following function presents the EJTP according to the supplier and manufacturer's annual total profit:

$$EJPT(n_s,Q_s)=\begin{cases} EJTP_1(n_s,Q)=CTP_{s1}+TP_{m1} \\ EJTP_2(n_s,Q)=CTP_{s2}+TP_{m2} \end{cases},$$

where:

$$EJTP_1(n_s,Q)=D *[C_p-\Sigma_s\Sigma_i(C_{si} * u_{si})]-\frac{D}{Q}(\Sigma_s A_s+A_m+\Sigma_s\Sigma_i n_s F_{si})$$
$$-BP_m-\Sigma_s\Sigma_i\left(h_{si} * \frac{D_s Q_s}{2P_s}\right)-\Sigma_s\Sigma_i h_{mi} * \frac{D_s Q_s}{n_s}\left(\frac{2-n_s}{2p_s}+\frac{n_s-1}{2D_s}\right)-h_m * \frac{DQ}{n_m}\left(\frac{2-n_m}{2p_m}+\frac{n_m-1}{2D}\right)$$
$$+\left(D_s X-\frac{D_s}{2}\right)\Sigma_s\Sigma_i\left[C_p * \frac{u_{si}}{\Sigma_s\Sigma_i u_{si}} * I_{me}-C_{si} * I_{sp}\right] \quad (5)$$

$$EJTP_2(n_s,Q) = D*[C_p - \Sigma_s\Sigma_i(C_{si}*u_{si})] - \frac{D}{Q}(\Sigma_s A_s + A_m + \Sigma_s\Sigma_i n_s F_{si}) - BP_m$$
$$-\Sigma_s\Sigma_i\left(h_{si}*\frac{D_sQ_s}{2P_s}\right) - \Sigma_s\Sigma_i h_{mi}*\frac{D_sQ_s}{n_s}\left(\frac{2-n_s}{2p_s}+\frac{n_s-1}{2D_s}\right) - h_m*\frac{DQ}{n_m}\left(\frac{2-n_m}{2p_m}+\frac{n_m-1}{2D}\right) + \Sigma_s\Sigma_i$$
$$\frac{(D_s*X)^2}{2Q_s}\left[C_p*\frac{u_{si}}{\Sigma_s\Sigma_i u_{si}}*I_{me} - C_{si}*I_{sp}\right] + \Sigma_s\Sigma_i\frac{(Q_s-D_s*X)^2}{2Q_s}[C_{mi}*I_{se} - C_{mi}*I_{mp}] \quad (6)$$

3. Solution procedure

Multiple decision makers are involved in this decentralized decision-making process, each of whom is inclined to optimize individual performance with an aim of maximizing the EJTP. Therefore, so as to maximize $EJPT_j(n_s,Q)$, we set $[\partial EJTP_1(n_s,Q)/\partial Q]=0$ and obtain the economic value of $Q=Q^*_{EJPT1}$, Q^*_{EJPT2}. We set some notations as follows to simplify the equations:

$$H_{si} = \Sigma_s\Sigma_i\left(h_{si}*\frac{D_s*u_{si}}{2P_s}\right),\ H_m = h_m*\frac{D}{n_m}\left(\frac{2-n_m}{2p_m}+\frac{n_m-1}{2D}\right)$$
$$H_{mi} = \Sigma_s\Sigma_i h_{mi}*\frac{D_s*u_{si}}{n_s}\left(\frac{2-n_s}{2p_s}+\frac{n_s-1}{2D_s}\right)$$

After calculating, we can know that:

$$Q^*_{EJTP1} = \sqrt{\frac{D(\Sigma_s A_s + A_m + \Sigma_s\Sigma_i n_s F_{si})}{H_{si}+H_{mi}+H_m}} \quad (7)$$

$$Q^*_{EJTP2} = \sqrt{\frac{\Sigma_i u_{si}\left[2D\Sigma_i u_{si}(\Sigma_s A_s + A_m + \Sigma_s\Sigma_i n_s F_{si}) - \Sigma_s\Sigma_i\left(C_p*\frac{u_{si}}{\Sigma_s\Sigma_i u_{si}}*I_{me} - C_{si}*I_{sp}\right)\cdot(D_s*X)^2 - \Sigma_s\Sigma_i(C_{mi}*I_{se} - C_{mi}*I_{mp})\cdot(D_s*X)^2\right]}{2\Sigma_i u_{si}(H_{si}+H_{mi}+H_m) - \Sigma_i u_{si}{}^2(C_{mi}*I_{se} - C_{mi}*I_{mp})}} \quad (8)$$

Algorithm

We have to follow these steps in order to receive the optimal values of $EJPT_j(n_s,Q)$:

Step 1. Select *s* supplier (s=1, 2, 3, …, m.)

Step 2. To obtain Q_{EJPT1} and Q_{EJPT2}, set $n=n_{sj}=1$ where j = 1, 2 and substitute into (7) and (8).

Step 3. By substituting n_{sj} and Q_{EJPTj}, where j=1,2, we can find $EJTP_j$, into (5) and (6).

Step 4. Let $n_i = n_i + 1$. Until $EJTP_j(n_{sj}) > EJTP_j(n_{sj}+1)$, repeat step 1 to step 3.

The optimal $n_{sj}^* = n_{sj}$, where j=1, 2

Step 5. Repeat step1 to step4 until finding all n_{sj}^*, because there are multi-suppliers;

$$Q_j^* = Q(\text{all}\, n_{sj}^*) \text{ where s=1, 2, 3, …, m and j=1,2.}$$

Step 6. Analyze different payment periods and perform a sensitivity analysis, observing the impact on economic ordering policies across various X values.

4. Numerical example

The proposed models are illustrated through the use of a numerical example in this section. According to the two-level model, we set a situation with four items, three suppliers, single manufacturer. The following Table 1 is the input parameters of suppliers(s=1,2,3):

Table 1
The data of each cost

Suppliers(*s*)	P_s	A_s	D_s	I_{sp}	I_{se}
1	1300	50	2000	0.035	0.03
2	3000	40	5000	0.03	0.025
3	1400	55	3000	0.04	0.03
Manufacturer	P_m	A_m	D	I_{mp}	I_{me}
	1,200	70	1,000	0.04	0.035

With the following Table 2 input parameters, per unit (*i* = 1, 2, 3, 4) finished product requires 4 items:

Table 2

The data of items

s	Items(i)	u_{si}	C_{si}	C_{mi}	F_{si}	h_{si}	h_{mi}
1	**1**	2	8	20	50	3	3
2	**2**	1	5	15	50	5	5
	3	4	2	10	40	1	2
3	**4**	3	10	20	60	3	4

Other provided notations are as follows: $X = 0.205479$ (i.e 75 days), $B = 50$, $C_p = 300$, $n_m = 3$.

in this paper, it is already given the Equation and algorithm already, Table 3 summarized the economic ordering policy as follows:

Table 3

The economic ordering policy

	Case 1	Case 2
n_1^*	1	1
n_2^*	2	1
n_3^*	2	1
Q^*	124	14
$EJTP_j^*$	158,131.2	201,117.5*

Finally, under different values of X, sensitivity analysis which calculates the $EJTP_j$ is shown in Table 4.

Table 4

The inventory policy with different X

X (days) →		65	75	85
Case 1	n_1^*	1	1	1
	n_2^*	2	2	2
	n_3^*	2	2	2
	Q^*	124	124	124
	$EJTP_1^*$	157,115	158,131.2	159,147.4
Case 2	n_1^*	1	1	1
	n_2^*	1	1	1
	n_3^*	1	1	1
	Q^*	12	14	17
	$EJTP_2^*$	190,908.5	201,117.5	209,896.2

5. Conclusions

There is a scarcity of literature on two-echelon models incorporating permissible delay in payment and ATO system. Furthermore, the existing works often focus on a single situation. Thus, this article contributes to the existing research by introducing a novel two-echelon integrated inventory model that involves a manufacturer and multiple suppliers. We can get the following statement according to Table 3. and Table 4.:

(1) Case 2 $(T_s \geq X)$ earn more profit compare to case 1 $(T_s < X)$. It indicates that the credit period (X) should be shorter than the cycle time of suppliers (T_s) in this model.

(2) The quantity of ordering in case 1 is much more than case 2.

(3) Expected joint total profit marginal increase as the credit period (X) increases.

(4) In case 1, the value of ordering quantity (Q) unchanged, when the credit period (X) increases.

(5) There is almost no relation between $n_s$and the credit period (X).

In the light of the above points we proposed, certain arguments are clarified. First, in spite of the provision of credit period to manufacturers to bring additional costs to suppliers, it helps lessen the cost burden on the manufacturer.The supply chain can significantly improve by proficient management of the manufacturer's sales revenue .

Second, from managerial perspective, running case 2 is more profitable to case 1. However, there is a discrepancy in the order quantity, as Case 1 involves a much larger quantity compared to Case 2. This implies a higher number of orders and transportation requirements. Nevertheless, it should be noted that if there is a sudden increase in oil prices due to economic bubbles or an economic downturn, the discussed decision may undergo alterations.

Finally, moving ahead, this model has potential for extension in various directions, such as its application to systems involving multiple retailers or addressing situations involving defective products. We anticipate that the optimal policy in this paper, despite potentially increasing in complexity, will maintain the same underlying structure. Another possible extension of the model would consider situations where the demand for the finished product may result in backorders, rather than cash of single item of cash may also be backordered.

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Received August, 2022

Revised January, 2023