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Analysing the efficiency of Indian non-life insurers : A two-stage fuzzy closed system DEA approach

K. Kapil Raj †
S. Srinivasan *
C. D. Nandakumar §
Department of Mathematics & Actuarial Science
B. S. Abdur Rahman Crescent Institute of Science & Technology
Chennai 600048
Tamil Nadu
India

Abstract

This paper adopts a two-stage fuzzy closed system Data Envelopment Analysis (DEA) to assess the efficiency of India's non-life insurance industry. The framework of marketability and profitability's sub-processes is evaluated in two stages, with the first stage considering three inputs and three outputs, and the second stage considers the outputs of first stage as inputs and assuming three different outputs as variables for 12 different non-life insurers (both public and private) which are a basis of secondary information from Insurance Regulatory and Development Authority of India for the period of FY 2015 to 2020. Our findings reveal the individual efficiency measure for marketability and profitability sub – processes of the selected non-life insurance firms functioning in India.

Subject Classification: *62P05, 97M40, 90Bxx, 90-XX, 03B52.*

Keywords: *Non-life insurer, Two-stage fuzzy DEA, GMIR defuzzification, Marketability, Profitability.*

1. Introduction

Data Envelopment Analysis is a fascinating application designed for measuring the performance efficiency of any company or organisation by the pitch of operations research. To access the efficiency of the Indian non-

† *E-mail:* **kapilraj.as@bhc.edu.in**
* *E-mail:* **drssrinivasan198@gmail.com (Corresponding Author)**
§ *E-mail:* **prof.cdnandakumar@gmail.com**

life insurance market in 2 distinct sub-processes, the fuzzy DEA non-parametric linear programming model is employed. This DEA approach assesses how effectively is each business, refers to as decision-making units (DMUs), performs in the market by outperforming the competition with the resources available [2,4]. Since the relative measure is the ratio of total outputs to total inputs, the performance score of DMUs varies from 0 to 1, which is either higher or smaller measure due to the varying inputs and outputs considered. [16,17]. Seiford, L.M., and Zhu, J., (1999) [21] investigated the performance of the top 55 US commercial banks using a two-stage production process that looked at the efficiency of the profitability and marketability sub-processes. The author of this study established a novel context of dependent performance metrics for profitability and marketability that employs DEA stratification and attractiveness measurement. The results showed that the context-dependent performance measure is better characterised than the original DEA measure for commercial banks' profitability and marketability. Joe Zhu., (2000) [10] used a two-stage DEA to develop a multi-aspect financial performance model that fundamentally separates trade-offs among various financial metrics by presenting a different perspective and description on the fortune 500 firm's operations. Having evaluated from a multidimensional perspective, the findings revealed that top-ranked firms by revenue do not have to be top-ranked by performance. In Major League Baseball, Sexton, T. R., et al. (2003) [22] applied DEA to model DMUs in two phases, with the result of the first phase becoming the input of the second stage, proving its benefits over the usual DEA technique. The suggested approach separates inefficiencies between phases, allowing decision-makers to target the inefficient segment in the production chain.

For the selected insurers, Z, Yang., (2006) [28] combines the production and investment performance evaluations independently and comprehensively. This study also discovered scale efficiency in the Canadian life and health insurance sector. In Taiwanese non-life insurance companies, Kao, C., and Hwang, S.N. (2008) [12] revised the traditional DEA model by considering internal sub-processes to analyse efficiency for the whole as two segments and concluded that this impression of disintegration can be extended to systems with multiple stages associated in sequence. Cook et al (2010) [5] formulated an assessment and future outlook for the performance measurement of DEA's two-stage network architecture. Eling, M., and Luhnen, M., (2010) [6] conducted a cross-country analysis of 6462 insurance businesses from 36 countries to determine the efficiency of the worldwide insurance sector. The study examines the technological and cost-effective expansion of international insurance markets from 2002 to 2006, revealing a significant variation in efficiency among the countries studied. W. Huang and M. Eling (2013) [8] compared the efficiency of the non-life insurance business in the four world's fastest expanding markets: Brazil, Russia, India, and China, and concluded that the environment has a major influence on the efficiency of the non-life insurance market in BRIC nations. S. Lim and J. Zhu (2016) [13] revealed the duality in the conventional DEA model, which drifts to two-stage network DEA, and formulas are also evolved to get frontier projections and divisional effectiveness ratings using a DEA model and its dual solutions. The two-stage network DEA model proposed by Li, H., et al. (2018) [14] requires the leader- follower assumption in the decision-making process, as well as real-time accusations in the performance evaluation of the selected DMUs. A. Ashrafi, M. Mansouri Kaleibar., (2017) [1] proposed a generalised framework for calculating Cost, Revenue, and Profit efficiency with fuzzy data, when the usual technique is ineffective since the inputs and outputs are in the form of ranges. Furthermore, the fuzzy DEA model evolved as a new class of DEA models to interpret for DMUs with ambiguous inputs and outputs. Ying Liu & Su Wu., (2018) [27] used the DEA – BCC technique with Variable Return to Scale to examine the innovation effectiveness of cultural and creative enterprises in various Chinese regions from 2010 to 2013. The innovation efficiency of the selected companies was evaluated and decomposed, followed by a comparative examination of the differences in the innovation efficiencies. The development balance of industries in different regions of China in terms of innovation was recommended by this study. RP. Sinha., (2019) [17] adds to the existing literature by estimating the performance efficiency of non-life insurers using a network system of DEA models that enhances efficiency in various sub-stages, as well as investigating the impact of ROE and SR in the efficiency performance of the chosen insurance companies. M. J. Christinal and Dr. P. Mariappan., (2021) [15] have used a two-stage data envelopment analysis approach to investigate the efficiency of public sector banks in India. The comparative valuation and efficiency ratings among the chosen banks in India are validated in this study.

This paper differs from previous studies by using a two-stage fuzzy closed system data envelopment analysis to assess the efficiency of the Indian non-life insurance industry's marketing and profit-making sub-processes spanning the period of financial years 2015 to 2020.

2. Methodology

2.1 *Fuzzy DEA Approach*

In practise, the experimental concepts are frequently amorphous or unfocused. Such information will not provide us with conclusive or sufficient proof. As a result, bounded intervals or fuzzy integers are frequently used to express ambiguous data. Fuzzy DEA models are important in circumstances when part of the input and output variables are inexact or unclear, according to new study. Fuzzy DEA models are a type of DEA model that is used to justify the inputs and outputs of DMUs that are difficult to predict.

2.2 *Fuzzy Constant Return to Scale Model*

Symbols & Notations

$\tilde{E}_n$ = Efficiency of the n^{th} DMU in stage 1

$\tilde{Q}_{jn}$ = j^{th} fuzzy output value of n^{th} DMU in stage 1

S_{jn} = j^{th} output variable of n^{th} DMU in stage 1 & stage 2

$\tilde{P}_{in}$ = i^{th} fuzzy input value of n^{th} DMU in stage 1

R_{in} = i^{th} input variable of n^{th} DMU in stage 1 & stage 2

$\tilde{E}_n^*$ = Efficiency of the n^{th} DMU in stage 2

$\tilde{T}_{jn}$ = j^{th} fuzzy output value of n^{th} DMU in stage 2

$\tilde{Q}_{in}$ = i^{th} fuzzy input value of n^{th} DMU in stage 2

θ_n = free variable

2.2.1 General form of fuzzy Constant Return to Scale Model (Stage – 1)

To make n^{th} DMU as efficient as possible out of m number of DMU's. The fuzzy fractional DEA problem for stage 1 is given by,

$$\text{Max } \tilde{E}_n = \frac{\sum_{j=1}^{K} \tilde{Q}_{jn} \; S_{jn}}{\sum_{i=1}^{L} \tilde{P}_{in} R_{in}}$$

Subject to constraints,

$$0 \leq \frac{\sum_{j=1}^{K} \tilde{Q}_{jn} S_{jn}}{\sum_{i=1}^{L} \tilde{P}_{in} R_{in}} \leq 1; \; n = 1, 2, 3 \ldots m$$

$$\tilde{Q}_{jn}, \; S_{jn}, \tilde{P}_{in}, \; R_{in} \geq 0 \quad \forall \quad j = 1, 2, 3 \ldots .K \; ; \; i = 1, 2, 3 \ldots .L$$

The fuzzy fractional programming DEA model described above may be transformed to a fuzzy linear model using the following formula,

$$\text{Max } \tilde{E}_n = \sum_{j=1}^{K} \tilde{Q}_{jn} . S_{jn}$$

Subject to constraints,

$$\sum_{i=1}^{L} \tilde{P}_{in} . R_{in} = 1$$

$$\sum_{j=1}^{K} \tilde{Q}_{jn} . S_{jn} - \sum_{i=1}^{L} \tilde{P}_{in} . R_{in} \leq 0, \; n = 1,2,3,\ldots.m$$

$$\tilde{Q}_{jn},\; S_{jn}, \tilde{P}_{in},\; R_{in} \geq 0 \quad \forall \quad j = 1,2,3\ldots.K \;;\; i = 1,2,3\ldots.L$$

2.2.2 General form of fuzzy Constant Return to Scale Model (Stage – 2)

To maximize the efficiency of n^{th} DMU out of m number of DMU's then the fuzzy fractional DEA problem for stage 2 is given by,

$$\text{Max} \tilde{E}_n^* = \frac{\sum_{j=1}^{K} \tilde{T}_{jn} S_{jn}}{\sum_{i=1}^{L} \tilde{Q}_{in} R_{in}}$$

Subject to constraints,

$$0 \leq \frac{\sum_{j=1}^{K} \tilde{T}_{jn} S_{jn}}{\sum_{i=1}^{L} \tilde{Q}_{in} R_{in}} \leq 1; \; n = 1,2,3\ldots m$$

$$\tilde{T}_{jn},\; S_{jn}, \tilde{Q}_{in},\; R_{in} \geq 0 \quad \forall \quad j = 1,2,3\ldots.\text{K}; \; i = 1,2,3\ldots.\text{L}$$

The above fuzzy fractional programming second stage DEA model can be converted into equivalent fuzzy linear model by,

$$\text{Max } \; \tilde{E}_n^* = \sum_{j=1}^{K} \tilde{T}_{jn} . S_{jn}$$

Subject to constraints,

$$\sum_{i=1}^{L} \tilde{Q}_{in} . R_{in} = 1$$

$$\sum_{j=1}^{K} \tilde{T}_{jn} . S_{jn} - \sum_{i=1}^{L} \tilde{Q}_{in} . R_{in} \leq 0, \; n = 1,2,3,\ldots.m$$

$$\tilde{T}_{jn},\; S_{jn}, \tilde{Q}_{in},\; R_{in} \geq 0 \quad \forall \quad j = 1,2,3\ldots.K; \; i = 1,2,3\ldots.L$$

2.3 *Fuzzy Variable Return to Scale Model*

2.3.1 General form of fuzzy Variable Return to Scale Model (Stage 1)

The output-oriented DEA model under fuzzy variable return to scale in stage 1 is coined by,

$$\text{Max}\,\theta_n$$

Subject to constraints,

$$\tilde{Q}x_i \geq \tilde{Q}_n$$

$$Px_i \leq \theta_n(P_n)$$

$$\textstyle\sum_{i=1}^{m} x_i = 1$$

$$x_i \geq 0$$

2.3.2 General form of fuzzy Variable Return to Scale Model (Stage 2)

The output-oriented DEA model under fuzzy Variable return to scale in stage 2 is coined by,

$$\text{Max}\,\theta_n$$

Subject to constraints,

$$\tilde{T}x_i \geq \tilde{T}_n$$

$$Qx_i \leq \theta_n(Q_n)$$

$$\textstyle\sum_{i=1}^{m} x_i = 1$$

$$x_i \geq 0$$

2.4 *Fuzzy Number and Its Types*

Let us consider X which remains a non-empty set. Then a fuzzy set A in X is represented by a membership function as $\mu_A : X \to [0,1]$ (i.e)

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } x \notin X \\ (0,1), & \text{if } x \text{ is partly in } X \end{cases}$$

In this paper we used pentagonal membership since the efficiency measure was calculated for five years.

2.4.1 Pentagonal fuzzy Membership Function

A fuzzy number which is signified by five points such that A = $(a_1, a_2, a_3, a_{4,} a_5)$ where $a_1, a_2, a_3, a_4, a_5 \in R$ is a pentagonal fuzzy number with its membership function represented as $\mu_A(x)$

$$\mu_A(x) = \begin{cases} 0 & x \leq a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & a_1 < x \leq a_2 \\ \frac{(x-a_2)}{(a_3-a_2)} & a_2 \leq x \leq a_3 \\ 1 & x = a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} & a_3 \leq x < a_4 \\ \frac{(a_5-x)}{(a_5-a_4)} & a_4 \leq x < a_5 \\ 0 & x \geq a_4 \end{cases}$$

2.4.2 Defuzzification & Its Types

Fuzzy fallouts are typically be transformed into crisp yield since fuzzified assessments cannot be used in many real-life practices. The method of altering a fuzzified turnout into a single crisp or deterministic value with respect to a fuzzy set is accepted as defuzzification. Since the efficiency scores of the decision-making units are crisp rather than one or more fuzzy value which is the performance score of the operating non-life companies functioning in India with respect to marketability and profitability sub - processes. We used Graded Mean Integration Representation technique to retrieve the crisp efficiency score for all the five years.

If A = $(a_1, a_2, a_3, a_4, a_5)$ is a pentagonal fuzzy number, then the Graded Mean Integration Representation method of defuzzifying the efficiency score is given by,

$$R(A) = \frac{a_1 + 3a_2 + 4a_3 + 3a_4 + a_5}{12}$$

2.5 *Selection of Input and Output variables*

In this work, Figure 1, communicates Commissions, Operating expenses and the Net claims incurred are considered as an input for stage 1 and Gross premium income, Net premium earned and Investments are Outputs because of stage 1 being the marketability sub – process. This is a closed system DEA; the outputs of stage 1 are same as the inputs for stage 2. Share capital of the company, Investment profit and Reserves & Surplus being the outputs for stage 2.

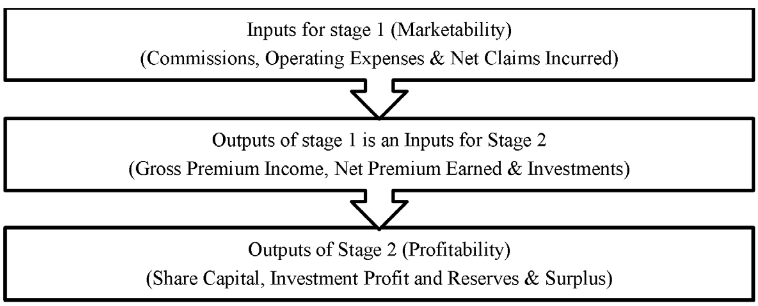

Figure 1

Selected Input & Output variables for both stages

3. Problem Formulation

Fuzzy Constant Return to Scale Model (Output Maximization)

National Insurance Company

$$\text{Max}\,\tilde{E}_{\text{National Insurance Company}} = \frac{12018.98x_1 + 10,791.38x_2 + 21759.77x_3}{619.86x_4 + 3,509.01x_5 + 10,282.40x_6}$$

Subject to Constraints,

$$\frac{17763.31x_1 + 14,959.83x_2 + 44545.72x_3}{1,403.75x_4 + 3,515.28x_5 + 13,141.19x_6} \leq 1$$

$$\frac{8611.59x_1 + 7,023.90x_2 + 19599.28x_3}{463.10x_4 + 2,580.09x_5 + 5,879.59x_6} \leq 1$$

$$\frac{12250.36x_1 + 10,022.87x_2 + 23238.24x_3}{513.35x_4 + 2,924.07x_5 + 8,801.09x_6} \leq 1$$

$$\frac{5832.15x_1 + 4,223.65x_2 + 8,934.64x_3}{93.93x_4 + 1,140.71x_5 + 0.03x_6} \le 1$$

$$\frac{1274.41x_1 + 1,158.22x_2 + 2,841.20x_3}{32.69x_4 + 489.35x_5 + 0.01x_6} \le 1$$

$$\frac{2452.00x_1 + 1,690.80x_2 + 3,860.72x_3}{33.96x_4 + 542.04x_5 + 0.01x_6} \le 1$$

$$\frac{3.71x_1 + 0.06x_2 + 114.43x_3}{0.21x_4 + 15.87x_5 + 0.01x_6} \le 1$$

$$\frac{408.72x_1 + 296.30x_2 + 503.35x_3}{10.31x_4 + 227.29x_5 + 0.01x_6} \le 1$$

$$\frac{28.76x_1 + 21.49x_2 + 255.67x_3}{4.47x_4 + 17.41x_5 + 0.01x_6} \le 1$$

$$\frac{1694.12x_1 + 1,390.02x_2 + 2,715.60x_3}{56.45x_4 + 454.51x_5 + 0.01x_6} \le 1$$

$$\frac{1712.27x_1 + 1,481.06x_2 + 6,019.71x_3}{29.83x_4 + 144.86x_5 + 0.01x_6} \le 1$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

The Corresponding LPP Structure for the above problem can be framed as follows,

$$\text{Max}\, \tilde{E}_{\text{National Insurance Company}} = 12018.98x_1 + 10,791.38x_2 + 21759.77x_3$$

Subject to Constraints,

$$619.86x_4 + 3,509.01x_5 + 10,282.40x_6 = 1$$

$$17763.31x_1 + 14,959.83x_2 + 44545.72x_3 - 1,403.75x_4 - 3,515.28x_5 - 13,141.19x_6 \le 0$$

$$8611.59x_1 + 7,023.90x_2 + 19599.28x_3 - 463.10x_4 - 2,580.09x_5 - 5,879.59x_6 \le 0$$

$$12250.36x_1 + 10,022.87x_2 + 23238.24x_3 - 513.35x_4 - 2,924.07x_5 - 8,801.09x_6 \le 0$$

$$5832.15x_1 + 4,223.65x_2 + 8,934.64x_3 - 93.93x_4 - 1,140.71x_5 - 0.03x_6 \le 0$$

$$1274.41x_1 + 1,158.22x_2 + 2,841.20x_3 - 32.69x_4 - 489.35x_5 - 0.01x_6 \le 0$$

$$2452.00x_1 + 1,690.80x_2 + 3,860.72x_3 - 33.96x_4 - 542.04x_5 - 0.01x_6 \le 0$$

$$3.71x_1 + 0.06x_2 + 114.43x_3 - 0.21x_4 - 15.87x_5 - 0.01x_6 \leq 0$$

$$408.72x_1 + 296.30x_2 + 503.35x_3 - 10.31x_4 - 227.29x_5 - 0.01x_6 \leq 0$$

$$28.76x_1 + 21.49x_2 + 255.67x_3 - 4.47x_4 - 17.41x_5 - 0.01x_6 \leq 0$$

$$1694.12x_1 + 1,390.02x_2 + 2,715.60x_3 - 56.45x_4 - 454.51x_5 - 0.01x_6 \leq 0$$

$$1712.27x_1 + 1,481.06x_2 + 6,019.71x_3 - 29.83x_4 - 144.86x_5 - 0.01x_6 \leq 0$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Fuzzy Variable Return to Scale Model (Output Maximization)
National Insurance Company

$$\text{Min}\, x_{13} - x_{14}$$

Subject to Constraints,

$$12018.98x_1 + 17763.31x_2 + 8611.59x_3 + 12250.36x_4 + 5832.15x_5 + 1274.41x_6 +$$
$$2452.00x_7 + 3.71x_8 + 408.72x_9 + 28.76x_{10} + 1694.12x_{11} + 1712.27x_{12} \geq 12018.98$$

$$10,791.38x_1 + 14,959.83x_2 + 7,023.90x_3 + 10,022.87x_4 + 4,223.65x_5 +$$
$$1,158.22 + 1,690.80x_7 + 0.06x_8 + 296.30x_9 + 21.49x_{10} + 1,390.02x_{11} +$$
$$1,481.06x_{12} \geq 10791.38$$

$$21759.77x_1 + 44545.72x_2 + 19599.28x_3 + 23238.24x_4 + 8,934.64x_5 +$$
$$2,841.20x_6 + 3,860.72x_7 + 114.43x_8 + 503.35x_9 + 255.67x_{10} + 2,715.60x_{11} +$$
$$6,019.71x_{12} \geq 21759.77$$

$$19.86x_1 + 1,403.75x_2 + 463.10x_3 + 513.35x_4 + 93.93x_5 + 32.69x_6 + 33.96x_7 +$$
$$0.21x_8 + 10.31x_9 + 4.47x_{10} + 56.45x_{11} + 29.83x_{12} + 19.86x_{13} - 19.86x_{14} \leq 0$$

$$3,509.01x_1 + 3,515.28x_2 + 2,580.09x_3 + 2,924.07x_4 + 1,140.71x_5 + 489.35x_6 +$$
$$542.04x_7 + 15.87x_8 + 227.29x_9 + 17.41x_{10} + 454.51x_{11} + 144.86x_{12} +$$
$$3509.01x_{13} - 3509.01x_{14} \leq 0$$

$$10,282.40x_1 + 13,141.19x_2 + 5,879.59x_3 + 8,801.09x_4 + 0.03x_5 + 0.01x_6 +$$
$$0.01x_7 + 0.01x_8 + 0.01x_9 + 0.01x_{10} + 0.01x_{11} + 0.01x_{12} + 10282.40x_{13} -$$
$$10282.40x_{14} \leq 0$$

$$\sum_{i=1}^{14} x_i = 1$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \geq 0$$

4. Experimental Results

Stage 1: Fuzzy - CRS Results

In stage 1, Table 1 shows the efficiency findings of the marketability sub-process based on the output maximisation technical efficiency using the fuzzy Constant Return to Scale model. This table displays those seven non-life insurance businesses in India achieving the highest efficiency score of 1 in marketability, out of 12 companies chosen from the private and state sectors.

Table 1

Output Oriented Technical efficiency (Fuzzy - CRS) in Stage 1

Stage 1 Output Oriented Fuzzy - CRS Results			
DMUs	**Efficiency Score for the FY 2015 – 2016 to 2019 – 2020**	**Defuzzified Efficiency Score (DES)**	**Rank of (DES)**
National Insurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
The New India Assurance	(1.000,1.000.1.000,1.000,1.000)	1.000	1
Oriental Insurance	(0.824,0.977,0.944,1.000,1.000)	0.961	8
United India Insurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Bajaj Allianz	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Bharti AXA	(0.778,0.795,0.864,1.000,0.703)	0.860	10
Cholamandalam	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Kotak Mahindra	(1.000,0.790,0.865,0.928,0.912)	0.877	9
Liberty General	(1.000,1.000,0.551,0.747,1.000)	0.787	11
Raheja QBE	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Royal Sundaram	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Shriram	(1.000,1.000,0.838,0.470,0.573)	0.778	12

Stage 1: Fuzzy - VRS Results

Table 2 shows the efficiency findings of the marketability sub-process in stage 1 using the output maximisation technical efficiency under the fuzzy Variable Return to Scale model. This table shows that seven businesses achieve the highest efficiency score 1 in marketability among the 12 selected private and public sector non-life insurance markets in India.

Table 2

Output Oriented Technical efficiency (Fuzzy - VRS) in Stage 1

Stage 1 Output Oriented F - VRS Results			
DMUs	**Efficiency Score for the FY 2015 – 2016 to 2019 – 2020**	**Defuzzified Efficiency Score (DES)**	**Rank (DES)**
National Insurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
The New India Assurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Oriental Insurance	(0.825,1.000,1.000,1.000,1.000)	0.985	8
United India Insurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Bajaj Allianz	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Bharti AXA	(0.816,0.825,0.949,1.000,0.757)	0.904	10
Cholamandalam	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Kotak Mahindra	(1.000,1.000,0.928,1.000,1.000)	0.976	9
Liberty General	(1.000,1.000,0.575,0.905,1.000)	0.835	12
Raheja QBE	(1.000, 1.000,1.000,1.000,1.000)	1.000	1
Royal Sundaram	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Shriram	(1.000,1.000,0.905,0.717,0.703)	0.873	11

Overall Mean Efficiency of Stage 1

Table 3 shows that National Insurance, The New India Assurance, United India Insurance, Bajaj Allianz, Cholamandalam, Raheja QBE, and Royal Sundaram are the seven businesses whose efficiency score is maximum in both constant and variable return to scale models. Three of the seven businesses are government-owned, while the other four are privately held. As a result, when it comes to marketing of non-life insurance products in India, these companies are extremely efficient.

Table 3

Overall Mean Efficiency Score of Stage 1 (Marketability Sub – Process)

Stage 1 - Mean Efficiency Score			
DMUs	**CRS ES for Stage 1**	**VRS ES for Stage 1**	**Mean ES**
National Insurance	1.000	1.000	1.000
The New India Assurance	1.000	1.000	1.000

Contd...

Oriental Insurance	0.961	0.985	0.973
United India Insurance	1.000	1.000	1.000
Bajaj Allianz	1.000	1.000	1.000
Bharti AXA	0.860	0.904	0.882
Cholamandalam	1.000	1.000	1.000
Kotak Mahindra	0.877	0.976	0.927
Liberty General	0.787	0.835	0.811
Raheja QBE	1.000	1.000	1.000
Royal Sundaram	1.000	1.000	1.000
Shriram	0.778	0.873	0.825

Stage 2: Fuzzy - CRS Results

Table 4 validates the efficiency results of profitability sub – process (stage 2) based on the output maximisation technical efficiency under fuzzy Constant Return to Scale model. This table discloses that among the 12 selected private and public sector non-life insurance firms in India, three companies attain the maximum efficiency score 1 in profitability sub – process.

Table 4

Output Oriented Technical efficiency (Fuzzy - CRS) in Stage 2

Stage 2 Output Oriented F - CRS Results			
DMUs	**Efficiency Score for the FY 2015 – 2016 to 2019 – 2020**	**Defuzzified Efficiency Score (DES)**	**Rank (DES)**
National Insurance	(0.625,1.000,1.000,1.000,1.000)	0.969	8
The New India Assurance	(1.000,0.140,0.194,0.638,0.246)	0.363	12
Oriental Insurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
United India Insurance	(1.000,1.000,0.467,0.391,0.222)	0.605	11
Bajaj Allianz	(1.000,1.000,0.558,0.321,0.378)	0.631	10
Bharti AXA	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Cholamandalam	(0.840,1.000,1.000,1.000,1.000)	0.987	6
Kotak Mahindra	(1.000,1.000,0.722,0.660,0.792)	0.805	9
Liberty General	(1.000,0.948,1.000,1.000,1.000)	0.987	5
Raheja QBE	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Royal Sundaram	(0.662,1.000,1.000,1.000,1.000)	0.972	7
Shriram	(0.977,1.000,1.000,1.000,0.881)	0.988	4

Stage 2: Fuzzy - VRS Results

Table 5 validates the profitability sub-process (stage 2) efficiency outcomes under the fuzzy variable return to scale model based on the output maximisation technological efficiency. This table reports that seven businesses in India's non-life insurance industry which includes both private and public, achieve a maximum efficiency score of 1 in the profitability sub-process.

Table 5

Output Oriented Technical efficiency (Fuzzy - VRS) in Stage 2

Stage 2 Output Oriented F - VRS Results			
DMUs	**Efficiency Score for the FY 2015 – 2016 to 2019 – 2020**	**Defuzzified Efficiency Score (DES)**	**Rank (DES)**
National Insurance	(0.639,1.000,1.000,1.000,1.000)	0.970	8
The New India Assurance	(1.000,0.165,0.207,1.000,0.255)	0.465	12
Oriental Insurance	(1.000,1.000,1.000,1.000,1.000)	1.000	1
United India Insurance	(1.000,1.000,0.481,0.466,0.330)	0.638	11
Bajaj Allianz	(1.000,1.000,1.000,0.377,0.380)	0.793	10
Bharti AXA	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Cholamandalam	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Kotak Mahindra	(1.000,1.000,1.000,0.921,0.810)	0.964	9
Liberty General	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Raheja QBE	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Royal Sundaram	(1.000,1.000,1.000,1.000,1.000)	1.000	1
Shriram	(1.000,1.000,1.000,1.000,1.000)	1.000	1

Overall Mean Efficiency of Stage 2

Table 6 shows that Oriental Insurance, Bharti AXA, and Raheja QBE are the three firms with the highest efficiency score in both constant and variable return to scale, as well as the highest total mean efficiency. Only one state-owned firm and two private companies are efficient among these three DMUs. As a result, in terms of profitability of non-life insurance products in India, these are extremely profit efficient businesses.

Table 6

Overall Mean Efficiency Score of Stage 2 (Profitability Sub – Process)

Stage 2 - Mean Efficiency Score			
DMUs	**CRS ES for Stage 2**	**VRS ES for Stage 2**	**Mean ES**
National Insurance	0.969	0.970	0.969
The New India Assurance	0.363	0.465	0.414
Oriental Insurance	1.000	1.000	1.000
United India Insurance	0.605	0.638	0.621
Bajaj Allianz	0.631	0.793	0.712
Bharti AXA	1.000	1.000	1.000
Cholamandalam	0.987	1.000	0.993
Kotak Mahindra	0.805	0.964	0.885
Liberty General	0.987	1.000	0.994
Raheja QBE	1.000	1.000	1.000
Royal Sundaram	0.972	1.000	0.986
Shriram	0.988	1.000	0.994

4. Conclusion

The performance efficiency measure based on two stage fuzzy closed system data envelopment analysis helps us to understand each phase of the insurance business better in terms of its efficiency by using the fuzzified value of inputs and outputs. The overall mean efficiency from Stage 1 communicates that National Insurance, The New India Assurance, United India Insurance, Bajaj Allianz, Cholamandalam, Raheja QBE and Royal Sundaram were efficient in Marketability. Oriental Insurance, Bharti AXA and Raheja QBE are companies which attains the maximum overall mean efficiency in stage 2 which is profitability. Other businesses that do not attain maximum efficiency score in either phase must focus on the marketing strategies in order to accomplish the greatest performance profitability score. This two-stage fuzzy closed system DEA allows us to look at efficiency in two separate segments and realize the company's strengths and paleness. This research may be extended by adding three or more sub-processes to have a better understanding of the expansion of non-life insurance industry in India. This research may also be extended by using the model to investigate the health insurance market or by using the open system fuzzy DEA to the same non-life insurance industries operating in India.

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