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Production run time setting by considering time value of money for imperfect production process

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Abstract

Chung and Hou develop an imperfect manufacturing process for obtaining the production run. In this research, we consider the extended problem under time value of money for determining the production run. Taguchi's concept is adopted as the measurement of performance.

Subject Classification: *(2010) 60 Probability theory & Stochastic processes.*

Keywords: *Imperfect production, Production run, Taguchi's loss function.*

1. Introduction

Classical inventory control model addresses the perfect products. The defective product problem has been proposed by Porteus [1], Rosenblatt & Lee [2, 3], Kim & Hong [4], and Chung & Hou [5]. Recently, Chen et. al., [6] extended Chung & Hou's [7] researchl for getting the mean and manufacturing run.

Teran et. al., [8] transferred Taguchi's [9] quality performance measurement into the present worth per unit. Hence, Teran et al.'s study also considered time value of money per unit when customer holds it. Chou & Liang [10] and Chou & Chen [11] have presented the bivariate/

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multivariable quality loss with the time value of money. Their work addressed the sensitivity analyses of parameters for the effects of biases at production time and associated quality loss. Li et. al., [12] presented the optimal selection for propylene carbonate porcess.

In this article, the authors propose the extended Chung and Hou's model having time value of money in order to obtain the optimal manufacturing run.

2. Mathematical Model

2.1 *Notation*

d	demand quantity
p	production quantity
h	handing cost
r_1	interest rate
A	setup cost
k_1	quality loss coefficient
X	process shift time
$f(x)$	p.d.f. of X
α	probability of non-conforming units
t_0	production run
Y	normal characteristic of product
$f(y)$	p.d.f. of Y
μ_1	mean
σ_1^2	variability
LSL	lowest value of conforming product
USL	largest value of conforming product
$\Phi(\cdot)$	c.d.f. of standard n. r.v.
$\phi(\cdot)$	p.d.f. of standard n. r.v.
$E(N)$	expected defective numbers
$E_1(N)$	expected rework numbers
$E_2(N)$	expected number of scrap items
C_R	rework cost
C_j	scrap cost

T_1 target value

$E[Loss(Y)]$ expected quality cost

$ETC(t_0)$ expected total relevant cost.

2.2 *Mathematical Model*

According to Chen et. al., [6], the probability of defective is

$$\alpha = \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right) + 1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) \tag{1}$$

The expected defective number is

$$E(N) = p\left[\Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right) + 1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right)\right]\left\{t + \frac{\exp(-\lambda_1 t)-1}{\lambda_1}\right\} = p\alpha\left(t + \frac{\exp(-\lambda_1 t)-1}{\lambda_1}\right) \tag{2}$$

The expected number of defective units with time value of money is

$$\begin{aligned}\int_0^{t_0} E(N)e^{-rt}dt &= \int_0^{t_0} p\alpha\left(t + \frac{\exp(-\lambda_1 t)-1}{\lambda_1}\right)\exp(-\lambda_1 t)dt \\ &= p\alpha\left(\frac{1-\exp(-r_1 t_0)}{r_1^2} - \frac{t_0\exp(-\lambda_1 t_0)}{r_1} + \frac{1-\exp(-(\lambda_1+r_1)t_0)}{\lambda_1(\lambda_1+r_1)} - \frac{1-\exp(-\lambda_1 t)}{\lambda_1 r_1}\right)\end{aligned} \tag{3}$$

The expect rework cost with time value of money is

$$\begin{aligned}ETC_1 &= C_R E_1(N) \\ &= C_R p\left[1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right)\right] \\ &\left(\frac{1-\exp(-r_1 t_0)}{r_1^2} - \frac{t_0\exp(-r_1 t_0)}{r_1} + \frac{1-\exp(-(\lambda_1+r_1)t_0)}{\lambda_1(\lambda_1+r_1)} - \frac{1-\exp(-r_1 t_0)}{\lambda_1 r_1}\right)\end{aligned} \tag{4}$$

The expect scrap cost of product with time value of money is

$$\begin{aligned}ETC_2 &= C_j E_2(N) \\ &= C_j p\Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\left(\frac{1-\exp(-r_1 t_0)}{r_1^2} - \frac{t_0\exp(-r_1 t_0)}{r_1} + \frac{1-\exp(-(\lambda_1+r_1)t_0)}{\lambda_1(\lambda_1+r_1)} - \frac{1-\exp(-r_1 t_0)}{\lambda_1 r_1}\right)\end{aligned} \tag{5}$$

The expected conformance quality cost is

$$\begin{aligned}&E[Loss(Y)] \\ &= \int_{LSL}^{USL} k_1(y-T_1)^2 f(y)dy \\ &= k_1\left\{\{(\mu_y - T_1)^2 + \sigma_1^2\}\left[\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) - \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\right.\end{aligned}$$

$$-\sigma_1\left[USL\phi\left(\frac{USL-\mu_1}{\sigma_1}\right)-LSL\phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\Bigg\} \tag{6}$$

The expected conformance quality cost with time value of money is

$$\begin{aligned} ETC_3 &= \int_0^{t_0}[pt-E(N)]\cdot E[Loss(Y)]e^{-r_1 t}dt = E[Loss(Y)]\int_0^{t_0}[pt-E(N)]e^{-r_1 t}dt \\ &= E[Loss(Y)]\Bigg\{p\left(\frac{1-\exp(-r_1 t_0)}{r_1^2}-\frac{t_0\exp(-r_1 t_0)}{r_1}\right)- \\ & p\alpha\left(\frac{1-\exp(-r_1 t_0)}{r_1^2}-\frac{t_0\exp(-r_1 t_0)}{r_1}+\frac{1-\exp(-(\lambda_1+r_1)t_0)}{\lambda_1(\lambda_1+r_1)}-\frac{1-\exp(-r_1 t_0)}{\lambda_1 r_1}\right)\Bigg\} \end{aligned} \tag{7}$$

The holding cost with time value of money is

$$\begin{aligned} ETC_4 &= H\left[\int_0^{t_1}(p-d)t\exp(-r_1 t)dt+\int_{t_1}^{t_0}d(t_0-t)\exp(-r_1 t)dt\right] \\ &= \frac{H}{r_1^2}\left\{p(1-\exp(-r_1(d/p)t_0)-d(1-\exp(-r_1 t_0))\right\} \end{aligned} \tag{8}$$

Hence, the total expected cost is

$$ETC(t_0)=\frac{ETC_1+ETC_2+K+ETC_3+ETC_4}{t_0} \tag{9}$$

Eq. (9) needs to obtain t_0 minimizing the expected total cost. One consider numerical analysis method for Eq. (9).

3. Numerical Example

Consider some parameters: $\mu_y = y_0 = 5$, $\sigma_y = 0.4$, $LSL = 3$, $p = 30$, $d = 10$, A=100, $k = 20$, $h = 80$, $\lambda_1 = 0.05$, $r_1 = 0.03$, $C_R = 100$, and $C_j = 10$. By solving Eq. (9), one obtains $t_0 = 0.58$ with $ETC(t_0) = 353.39$.

Table 1 shows $\pm 30\%$ change of parameter value for the solution. From Table 1, we have the result: the d, p, σ_y, and h have a significant effect for this problem.

4. Conclusions

In this research, we have considered the modified one with time value of money and without allowable shortage. The extension should consider the economic model with sampling inspection.

Table 1

Sensitivity results

H	t_0	$ETC(t_0)$
56	0.65	304.87
104	0.50	395.68
K	t_0	$ETC(t_0)$
14	0.58	345.14
26	0.56	361.46
C_R	t_0	$ETC(t_0)$
70	0.58	353.39
130	0.58	353.39
C_j	t_0	$ETC(t_0)$
7	0.58	353.39
13	0.58	353.39
λ	t_0	$ETC(t_0)$
0.035	0.58	353.39
0.065	0.58	353.39
σ_y	t_0	$ETC(t_0)$
0.28	0.59	339.28
0.52	0.54	371.41
r_1	t_0	$ETC(t_0)$
0.021	0.55	353.89
0.039	0.56	352.86
p	t_0	$ETC(t_0)$
21	0.66	310.37
39	0.53	377.95
d	t_0	$ETC(t_0)$
7	0.63	322.62
13	0.54	368.74

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