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Information processes in stochastic models for strategic thinking operations

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Abstract

It constitutes a particularly valuable adoption that the family of infinitely divisible distributions is frequently employed as a very suitable choice for investigating theoretical and practical properties of structural probabilistic concepts. In particular, this family is valuable in construction and study of mathematical expressions, or equivalently stochastic models, with extensive practical applicability. The present paper focuses on the development and investigation of a stochastic model by making use of the extremely useful concept of infinitely divisibility. In addition, the paper offers an application of the introduced mathematical expressions in making realistic decisions in the extremely important area of various information processes.

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1. Introduction

Research activities in several important scientific disciplines make extensive use of various stochastic models for the description and treatment situations of significant interest [3, 4, 5]. Formulation, investigation and interpretation are generally accepted as the three structural components of any stochastic modeling procedure [1, 2]. It is quite obvious that the construction of a mathematical expression constitutes a leading factor of the study and the application of such an expression. Consequently, the development of a stochastic model is recognized as the most important component of stochastic modeling. In addition, the abilities of modelers to represent real situations as mathematical structures are also adopted as a very strong advantage for stochastic modeling. The present paper mainly concentrates on introducing fundamental properties of the family of infinitely divisible distributions in the three structural components of stochastic modeling [6, 8].

The provision of a comment on the main research purposes of this paper or equivalently the creation of a valuable presentation for every useful research result is established by the second and the third section of the paper. The selection of several random variables of very extensive and extremely valuable theoretical and practical applicability is the first research result of the paper. Furthermore, the use of these random variables in order to formulate a modified random sum, having very important applications in both theory and practice, is the second contribution of the paper. Moreover, the selection of two stochastic processes of different kind constitutes the third research result of the paper. The fourth research result of the paper is the correspondence of a stochastic integral of a given kind to one of the stochastic processes and the correspondence of a stochastic integral of different kind to the other stochastic process. Formulating a stochastic model by establishing an interconnection between these stochastic integrals is the fifth research result of the paper. The above mentioned stochastic integrals and the random sum are the three structural factors of the undertaken research activity.

2. Stochastic Integrals in Continuous Discounting

We introduce the stochastic process $\{X(t), t \geq 0\}$. Important theoretical results for the stochastic integral

$$\int_0^1 t^{1/\alpha} dX(t),\ a > 0 \tag{1}$$

are very well known [11, 12]. Moreover, we introduce the stochastic process $\{V(t), t \geq 0\}$. Particularly, useful theoretical and practical results for the stochastic integral

$$\int_0^\infty e^{-t/a} dV(t) \tag{2}$$

are also very well known [8, 10].

From a theoretical perspective, it can be said that the second section of this work establishes an interconnection between the stochastic integral of the form (1) and the stochastic integral of the form (2). It is of some significant theoretical interest the connection of a modified random sum with the stochastic integrals (1) and (2). The modified random sum's mathematical structure substantially extends the practical applicability of the stochastic model arising as an interconnection of the stochastic integrals (1) and (2). Consequently, the stochastic integral (1), the stochastic integral (2), and the modified random sum can be adopted as the valuable principal components in order to formulate a stochastic model supporting the development and implementation of strategic operations for an organization. More clearly, the formulation of such a stochastic model concentrates on making valuable in theory and practice of the structural properties of the introduced principal components. From a completely practical perspective, the formulation of the stochastic model, including the above stochastic integrals and the developed random sum as principal components, is readily recognized as a mathematical transformation for the three principal components, and the most valuable advantage of this transformation is its ability to use leading probabilistic concepts in treating strategic operations of organizations.

Theorem: *We consider the stochastic process* $\{X(t), t \geq 0\}$, *the positive random variable* $L = X(t+1) - X(t)$, *the characteristic function* $\varphi_L(u)$, *the sequence of discrete independent random variables* $\{Y_n, n = 1, 2, \ldots\}$ *distributed as the random variable Y with p.g.f.* $P_Y(z)$, *the discrete random variable N with p.g.f.* $P_N(z)$, *the random sum* $C = Y_1 + Y_2 + \ldots + Y_N$ *with p.g.f.* $P_C(z) = P_N(P_Y(z))$, *the sequence of positive, independent random variables* $\{S_c, c = 1, 2, \ldots\}$ *distributed as the random variable S with characteristic function* $\varphi_S(u)$, *the random sum*

$$H = S_1 + S_2 + \ldots + S_C \tag{3}$$

with characteristic function $\varphi_{\mathrm{H}}(u)=P_N(P_Y(\varphi_S(u)))$, the stochastic process $\{V(t), t\geq 0\}$, the positive random variable $\Pi = V(t+1)-V(t)$, the characteristic function $\varphi_{\Pi}(u) = \varphi_H(u)$, the stochastic integral

$$\int_0^1 t^{1/a}dX(t)$$

independent of the random sum H, and the stochastic integral

$$\int_0^\infty e^{-t/a}dV(t),$$

then

$$L\overset{d}{=}H+\int_0^\infty e^{-t/a}dV(t) \tag{4}$$

if and only if,

$$L\overset{d}{=}H+\int_0^1 t^{1/a}dX(t) \tag{5}$$

where $\overset{d}{=}$ denotes equality in distribution.

Proof: If we introduce characteristic functions in (5) we obtain the integral equation

$$\varphi_L(u)=P_N(P_Y(\varphi_S(u)))\exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_L(w)w^{a-1}dw\right\}. \tag{6}$$

It is readily seen that (6) implies that

$$\log\varphi_L(u)=\log P_N(P_Y(\varphi_S(u)))+\frac{a}{u^a}\int_0^u \log\varphi_L(w)w^{a-1}dw. \tag{7}$$

From (7) we get that

$$u^a\log\varphi_L(u)=u^a\log P_N(P_Y(\varphi_S(u)))+a\int_0^u \log\varphi_L(w)w^{a-1}dw \tag{8}$$

It is easily shown that the differentiation of (8) implies that

$$\alpha u^{a-1}\log\varphi_L(u)+u^a\frac{d}{du}\log\varphi_L(u)=$$
$$au^{a-1}\log P_N(P_Y(\varphi_S(u)))+u^a\frac{d}{du}\log P_N(P_Y(\varphi_S(u)))+au^{a-1}\log\varphi_L(u) \tag{9}$$

In addition, it is obvious that the differential equation (9) implies that

$$a\log\varphi_L(u)+u\frac{d}{du}\log\varphi_L(u)=$$
$$a\log P_N(P_Y(\varphi_S(u)))+u\frac{d}{du}\log P_N(P_Y(\varphi_S(u)))+a\log\varphi_L(u) \tag{10}$$

for $u \neq 0$. Moreover, from (10) we get that

$$u \frac{d}{du} \log \varphi_L(u) = a \log P_N(P_Y(\varphi_S(u))) + u \frac{d}{du} \log P_N(P_Y(\varphi_S(u)))$$

or equivalently (10) implies that

$$\frac{d}{du} \log \varphi_L(u) = a \frac{\log P_N(P_Y(\varphi_S(u)))}{u} + \frac{d}{du} \log P_N(P_Y(\varphi_S(u))) \tag{11}$$

Integrating in (11) we get that

$$\varphi_L(u) = P_N(P_Y(\varphi_S(u))) \exp\{a \int_0^u \frac{\log P_N(P_Y(\varphi_S(w)))}{w} dw\} \tag{12}$$

If we use random variables in (12) we get the equality in distribution

$$L \stackrel{d}{=} H + \int_0^\infty e^{-t/a} dV(t) \tag{13}$$

The formulation of the stochastic model (13) by the incorporation of the stochastic model (5) is the theoretical contribution of the present section. The random sum H is a principal component of the stochastic models (4) and (5). Consequently, the random sum H is readily recognized as particular important probabilistic concept for investigating the structure and behavior of the stochastic models (4) and (5). It is known that a direct or indirect practical interpretation of the stochastic integral (1) has not been established [11]. In addition, a direct or indirect practical interpretation of the stochastic model (5) has also not been established [8]. This section's main theoretical contribution concentrates on incorporating the analytical interconnection between the stochastic models (4) and (5) for the practical interpretations of these stochastic models in various strategic operations.

3. Practical Interpretations

We assume that N represents the number of groups of cash flows at the time point 0 and Y_n represents the number of cash flows of the nth group. In consequence, the random sum

$$C = Y_1 + Y_2 + \ldots + Y_N$$

represents the number of cash flows at the moment 0. In addition, we assume that the positive random variable S_c denotes the size of the cth case flow at the moment 0. Hence the random sum

$$H = S_1 + S_2 + \ldots + S_C$$

denotes the size of C cash flows at the moment 0. Equivalently, the random sum H is a strategic tool for stochastic models denoting the present values of C random cash flows arising at the time point 0. Furthermore, the stochastic integral

$$\int_0^\infty e^{-t/a} dV(t)$$

denotes the present value, at the moment 0, of the income produced by an asset during its infinite lifetime, where $1/a$ is the continuous rate of interest [5]. As a result, it is obvious that

$$L = H + \int_0^\infty e^{-t/a} dV(t)$$

is a stochastic discounting model. Specifically, this paper's theoretical contribution consists of the formulation of the stochastic discounting model (4) by the incorporation of the stochastic model (5). It appears to be particularly significant the consideration of the structural properties of the stochastic discounting model (4). It is obvious that such a consideration is mostly based on the investigation and interconnection of the primary components of the stochastic discounting model (4). It is easily seen that the random sum (3) and the stochastic integral (2) are the principal components of the stochastic discounting model (4). The theoretical framework and an application of the random sum (3) reveal how suitable such a random sum is for use in describing and carrying out structural strategic activities. In addition, the stochastic discounting model (4) very strongly supports the incorporation of this stochastic discounting model for thinking, managing, and decision making in strategic way [7, 9]. It is of theoretical and practical significance to provide an introduction on the presence of the discrete random variable N in the formulation of the stochastic model (5) and of the stochastic discounting model (4). It is quite obvious that the characteristic functions (6) and (12) are directly affected by the form of the probability generating function of the discrete random variable N. More clearly, the selection of the discrete random variable N constitutes an extremely important activity for establishing the paper's theoretical results. Consideration of certain, very important cases of the random variable N strongly supports the theoretical and practical applications of the stochastic discounting model (4). The recognition of infinite divisibility as a probabilistic property is very useful in formulating, investigating and interpreting stochastic modeling operations is a very solid justification for focusing on the study of the stochastic model (5) and the stochastic discounting model (4), under the assumption that the discrete random variable N has the property of infinite divisibility [10, 13, 14]. In consequence, it is easily seen that the presence of geometric random variable N in the random sum $C = Y_1 + Y_2 + \ldots + Y_N$ offers a significant reason for interpreting the stochastic model (4) as a valuable mathematical tool for treatment of significant practical situations arising in strategic thinking, strategic decision making, and strategic management.

It is thoroughly accepted that the information processes constitute extremely powerful structural factors in developing and applying of useful strategic methodologies. The stochastic integrals (1), (2) and the modified random sum (3) are easily adopted as very strong probabilistic tools significantly facilitating the proactive information extraction, analysis and transfer. In consequence, these stochastic integrals and the modified random sum are important analytical tools for developing valuable information processes for treating theoretical and practical situations. The structure of the three principal components of the stochastic model, introduced by this paper, makes quite clear the particular significance of this model in introducing, investigating, and interpreting information processes in the discipline of strategic thinking.

4. Conclusion

Stochastic modelling activities containing random sums and stochastic integrals constitute the research purpose of this work. The paper continues by establishing the presumed conditions that will be used to evaluate the proposed stochastic model's characteristic function. Practical interpretations of the proposed stochastic discounting model are also provided. Finally, the contribution of the paper consists of the establishment of a new method for the construction of new stochastic discounting integrals of great significance in information processes and strategic thinking activities.

In conclusion, it can be said that the contribution of the paper constitutes of two successive research components. The first research component concentrates on the development and implementation of suitable and valuable information processes. The second research component demonstrates how the information processes resulting from the first research component are appropriate for the field of strategic thinking.

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