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Independent semitotal domination in graphs

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Abstract

In this paper a slight different parameter, independent semitotal domination number- $\gamma_{it2}(G)$ is introduced. This parameter lies between $i(G)$ and $\beta_0(G)$ of a graph. We initiate a study on this parameter $\gamma_{it2}(G)$. We construct graphs for prescribed $\gamma_t(G)$ and $\gamma_{it2}(G)$. Also we determine bounds for γ_{it2} value of graphs.

Subject Classification: *(2010) 05C69.*

Keywords: *Total domination, Semitotal domination, Independent semitotal domination.*

1. Introduction

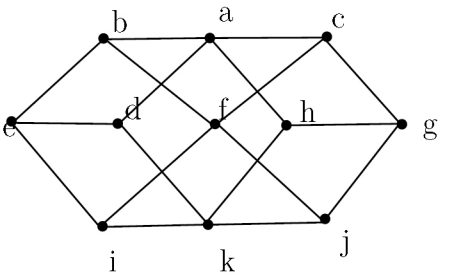
Here, we survey a parameter which is quite interesting, Independent semitotal domination number $\gamma_{it2}(G)$, that lies between $i(G)$ and $\beta_0(G)$ of a graph thereby extending the domination chain.

This initiates us to study this parameter $\gamma_{it2}(G)$.

Example 1.1: In herschel graph, $i(G) < \gamma_{it2}(G) < \beta_0(G)$.

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Herschel Graph

$$i(G) = |\{f, d, h\}| = 3.$$

$$\gamma_{it2}(G) = |\{f, a, k, e, g\}| = 5.$$

$$\beta_0(G) = |\{b, c, i, j, d, h\}| = 6$$

we directly use the definition of $\gamma(G)$, $i(G)$ and $\beta_0(G)$ as in [7].

Domination chain connects $i(G)$ and $\beta_0(G)$ by a chain of inequalites was rooted in [3] is given as

$$ir(G) \le \gamma(G) \le i(G) \le \beta_0(G) \le \Gamma(G) \le IR(G). \quad (1)$$

Expansion of this chain is one area for research.

This $\gamma_{it2}(G)$ and $\Gamma_{it2}(G)$ lies between $i(G)$ and $\beta_0(G)$ which in turn extends the chain (1) is shown in section (3)

A subset S of Vertices of G is an independent semitotal dominating set if S is a independent dominating set and to each u in S there is a v in S at a distance exactly two. *The minimum and maximum cardinality taken over all minimal independent semitotal dominating set are called the independent semitotal domination number* $\gamma_{it2}(G)$ *and upper independent semitotal domination number* $\Gamma_{it2}(G)$ *of G respectively*.

In every non-complete graph there is a $\beta_0(G)$-set as an *independent semitotal dominating set*. For complete graph $\beta_0(K_n) = 1$ and hence $\gamma_{it2}(K_n) = 1$. Thus γ_{it2} value is guaranteed for all graphs.

This Communication, initiates a study on $\gamma_{it2}(G)$.

Non-complete and connected graphs are considered throughout this paper. For disconnected graph we considered the components

We follow terms and definitions from Harary [6]

Theorem 1.1: [7] *"A dominating set D is a minimal dominating set iff for each vertex u in D, one of the following conditions holds. (i) u is an isolate of* $< D >$. *(ii) There exists a vertex v in V-D, for which* $N(v) \cap D = \{u\}$."

2. Preliminary Results

Proposition 2.1:

(i) $\gamma_{it2}(P_n) = \lceil \frac{2n}{5} \rceil$

(ii) $\Gamma_{it2}(P_n) = \lceil \frac{n}{2} \rceil$

(iii) $\gamma_{it2}(C_n) = \Gamma_{it2}(C_n) = \lceil \frac{2n}{5} \rceil$

(iv) $\gamma_{it2}(W_{n+1}) = \gamma(C_n) = \lfloor \frac{2n}{5} \rfloor = \lceil \frac{n}{3} \rceil$

(v) $\Gamma_{it2}(W_{n+1}) = \lfloor \frac{n}{2} \rfloor$

(vi) $\gamma_{it2}(K_{r_1, r_2 \ldots r_n}) = r_i$ where r_i is the minimum vertices of a partite set V_i (ie) $r_i = min(r_1, r_2, \ldots . r_n)$

(vii) $\Gamma_{it2}(K_{r_1, r_2 \ldots r_n}) = r_j$. where $r_j = max(r_1, r_2, \ldots . r_n)$

Proof:

(i) $\gamma_{it2}(P_3) = 2$ is clear. let $n \neq 3$ and $V(P_n) = \{v_1, v_2, \ldots v_n\}$

If $n \equiv 0, 4 \, mod \, 5$, then choose the vertices $S = \{v_i / i \equiv 2, 4 \, mod \, 5\}$. This forms an independent semitotal dominating set for P_n.

$$\therefore \gamma_{it2}(P_n) \leq \lceil \tfrac{2n}{5} \rceil.$$

No set with less than $\lceil \frac{2n}{5} \rceil$ forms a γ_{it2}-set. This set is a *minimum independent semitotal dominating set*.

$$\therefore \gamma_{it2}(P_n) = \lceil \tfrac{2n}{5} \rceil.$$

If $n \equiv 1, 2, 3 \, mod \, 5$ then partition the set of vertices of P_n into groups S_i of 5. (ie) $S_1 = \{v_1, v_2, \ldots v_5\}$ $S_2 = \{v_6, v_7, \ldots v_{10}\}$ ...

If S_i has five vertices choose v_j in S_i for $j \equiv 2, 4 \, mod \, 5$

If S_i has less than five vertices choose v_j in S_i for $j \equiv 1, 3 \, mod 5$.

Hence $\gamma_{it2}(P_n) = \lceil \frac{2n}{5} \rceil$.

(ii) The set $S = \{v_i / i = 1(2)n\}$ is a *minimal independent semitotal dominating set*.

Therefore $\Gamma_{it2}(P_n) \geq \lceil \frac{n}{2} \rceil$

Also no set with more than $\lceil \frac{n}{2} \rceil$ vertices forms an independent semitotal dominating set. Hence $\Gamma_{it2}(P_n) = \lceil \frac{n}{2} \rceil$.

(iii) In the same way we can prove that $\gamma_{it2}(C_n) = \lceil \frac{2n}{5} \rceil$ and for a cycle $\beta_0(C_n)$ serves as an upper independent semitotal dominating set. Also no set with more than $\beta_0(C_n)$ can serve as an upper independent semitotal dominating set. Hence $\Gamma_{it2}(C_n) = \beta_0(C_n) = \lceil \frac{2n}{5} \rceil$. Thus $\gamma_{it2}(C_n) = \Gamma_{it2}(C_n) = \lceil \frac{2n}{5} \rceil$

(iv) For wheel W_{n+1}, every dominating set of C_n serve as an independent semitotal dominating set for W_{n+1}. Therefore $\gamma_{it2}(W_{n+1}) \leq \gamma(C_n) = \lceil \frac{n}{3} \rceil$. No set with vertices less than $\lceil \frac{n}{3} \rceil$ can serve as an independent semitotal dominating set of W_{n+1}. Hence $\gamma_{it2}(W_{n+1}) = \lceil \frac{n}{3} \rceil$.

(v) Similarly every upper dominating set of C_n serves as upper independent semitotal dominating set for W_{n+1}. Therefore $\Gamma_{it2}(W_{n+1}) \geq \Gamma(C_n) = \lfloor \frac{n}{2} \rfloor$. Also no set with vertices more than $\lfloor \frac{n}{2} \rfloor$ vertices can serve as an upper independent semitotal dominating set of W_{n+1}. Hence $\Gamma_{it2}(W_{n+1}) = \lfloor \frac{n}{2} \rfloor$.

(vi) For multipartite graph $K_{r_1, r_2, \ldots r_n}$, it is clear that $\gamma_{it2}(K_{r_1, r_2 \ldots r_n}) = r_i$. where $r_i = min(r_1, r_2, \ldots . r_n)$

(vii) $\Gamma_{it2}(K_{r_1, r_2 \ldots r_n}) = r_j$. where $r_j = max(r_1, r_2, \ldots . r_n)$

3. Extended Domination Chain

Theorem 3.1: *For a graph G, S is a minimal independent semitotal dominating set iff each v in S is a private neighour of itself and satisfies at most all the below conditions*

(i) $epn(v, S) \neq \phi$

(ii) $ipn_{=2}(v, S) \neq \phi$

Proof: Let S be a *minimal independent semitotal dominating set*. Let v in S. Since S is independent $< S >$ has no edges.

$\therefore$ each v in S is a *private neighbour of itself* with respect to S.

$$\text{“}pn_{=2}(v, S; G) = \{w \in V / N_2(w, G) \cap S = \{v\}\}.$$

$$ipn_{=2}(v, S; G) = pn_{=2}(v, S; G) \cap S\text{”}$$

If epn(v,S) = ϕ and $ipn_{=2}(v, S) = \phi$, we are done.
By definition, S is a dominating set.

$\therefore$ each v in S has an external private neigbour or is a *private neigbour of itself* with respect to S

$\Rightarrow$ epn(v, S) $\neq \phi$

S is independent semitotal dominating set. By definition, each $v \in S$ has at least one $u \in S$ at distance exactly two.

If v is an unique element in S at distance two to u. Then u is an $ipn_{=2}(v,S)$ element.

$$\therefore ipn_{=2}(v,S) \neq \phi.$$

If not, then $ipn_{=2}(v,S) = \phi$.

Conversely, each $v \in S$ is its *own private neighbour*. $\Rightarrow$ S is independent and S is a dominating set.

If epn(v,S) $\neq \phi$ and $ipn_{=2}(v,S) = \phi$.

By theorem 1.1, S is dominating set implies S is minimal independent dominating set.

If epn(v,S) = ϕ and $ipn_{=2}(v,S) \neq \phi$.

S is minimal independent dominating set. Also S is semitotal. $\Rightarrow$ S is a *minimal independent semitotal dominating set*.

If epn(v,S) = ϕ and $ipn_{=2}(v,S) = \phi$, we have epn(v, S) = ϕ implies each $v \in$ V-S is adjacent to two vertices of S. $ipn_{=2}(v,S) = \phi$ implies there is no $v \in$ S that is at a distance two to u alone. S is an independent semitotal dominating set have each $v \in S$ as a private neighour of itself implies S is a minimal independent semitotal dominating set.

Proposition 3.2: Every minimal independent semitotal dominating set is a minimal independent dominating set.

Proof: Let S be a *minimal* independent semitotal dominating set of G. Then each $v \in S$ is a private neighbour of itself. By *theorem* 1.1, S is a *minimal* independent dominating set.

Corollary 3.3: $i(G) \leq \gamma_{it2}(G)$ *is true for all graphs.*

Proposition 3.4: Every minimal independent semitotal dominating set is a maximal independent set.

Proof: Let S be a minimal independent semitotal dominating set of G. Since S is independent each v in S is isolate. Also S is a dominating set. Since "Independent set is maximal independent iff it is dominating".

$\therefore$ S is maximal independent set.

Corollary 3.5: $\Gamma_{it2}(G) \leq \beta_0(G)$, *for any graphs.*

Corollary 3.6: $i(G) \leq \gamma_{it2}(G) \leq \Gamma_{it2}(G) \leq \beta_0(G)$, *for any graphs.*

Proof: Corollaries 3.3 and 3.5 establishes the above corollary.

Proposition 3.7: $ir(G) \leq \gamma(G) \leq i(G) \leq \gamma_{it2}(G) \leq \Gamma_{it2}(G) \leq \beta_0(G) \leq \Gamma(G) \leq IR(G)$ *holds for every graph G.*

Proof: Corollary 3.6 extends the chain in equation (1) which is the required extended chain in proposition

4. Realization and characterisation

Theorem 4.1: $\gamma_t(G) \leq 2\gamma_{it2}(G) - 1$ *holds for graph with* $\gamma_{it2}(G) \leq \gamma_t(G)$

Proof: Let S be a $\gamma_{it2}(G)$ set. By definition of $\gamma_{it2}(G)$ set, each pair of vertices in S has a common neighbour not in S. Choose a pair x and y with common neighbour z. For other vertices of S choose one *neighbour not in S*. The result forms a minimal total dominating set.

Theorem 4.2: *For every pair* $\gamma_{it2}(G), \gamma_t(G)$ *of integers there exists a connected non-complete graph G with* $\gamma_t(G)$ *= Total domination number and* $\gamma_{it2}(G)$ *= Independent semitotal domination number.*

Proof: For a graph G which is connected, non-complete and for the prescribed values of γ_t and γ_{it2}, The following cases exist.

Case (i) $\gamma_t(G) = \gamma_{it2}(G)$ $C_{\gamma_t} o K_1$ is the required graph when O(G) = $2\gamma_t$ and $C_{\gamma_{t+1}}$ with γ_t pendant vertices is the required graph when O(G) = $2\gamma_t$ + 1 When $\gamma_t = \gamma_{it2} = 2$. C_4 is the required graph. $\therefore 3 \leq \gamma_t = \gamma_{it2} \leq n-1$ and $C_{\gamma_t} o K_1$ has $\gamma_t = \gamma_{it2}$.

Case (ii) $\gamma_t(G) < \gamma_{it2}(G)$

when $\gamma_{it2} = 3K_{1,3}$ has $\gamma_t = 2$

For $\gamma_t = 2$ and any $\gamma_{it2} > 2$

K_1, γ_{it2} is the required graph

when $\gamma_{it2} = n-1 K_{1,n-1}$ has $\gamma_t = 2$

$\therefore$ for $3 \leq \gamma_t < \gamma_{it2} \leq n-2$

Consider F = $\overline{K_{\gamma_{it2}}} \cup P_{\gamma_t}$

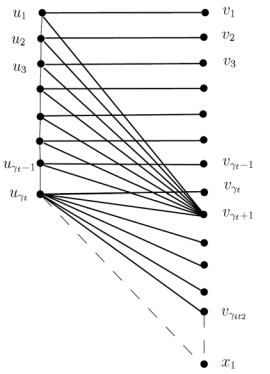
$$O(F) = \gamma_{it2} + \gamma_t$$

Let $V(\overline{K_{\gamma_{it2}}}) = \{v_1, v_2, ...v_{\gamma_{it2}}\}$

$$V(P_{\gamma_t}) = \{u_1, u_2 ..... u_{\gamma_t}\}$$

Construct F_1 from F by joining v_1 to u_1, v_2 to $u_2 ..... v_{\gamma_t}$ to u_{γ_t} respectively. Since $\gamma_t < \gamma_{it2}$, join the vertex v_{γ_t+1} to all the vertices of P_{γ_t}.

Now join the rest of the vertices in $\overline{K_{\gamma_{it2}}}$ to u_{γ_t} ((ie)join the vertices starts from v_{γ_t+2} to u_{γ_t}) Resulting graph F_1 has an independent semitotal domination number γ_{it2} and total domination number γ_t. Construct F_2 from F_1 by introducing a vertex x_1 adjacent to $v_{\gamma_{it2}}$ and u_{γ_t}. Construct F_3 from F_2 by introducing a vertex x_2 adjacent to $v_{\gamma_{it2}}$ and u_{γ_t}. Proceeding in the same manner,we can construct a graph of n vertices with prescribed value γ_{it2} and γ_t.



Construction for $\gamma_t < \gamma_{it2}$

Case (iii) $\gamma_{it2}(G) < \gamma_t(G)$

For prescribed values of γ_{it2} and $\gamma_t \leq 2\gamma_{it2} - 1$ there exists a non-complete connected graph with *independent semitotal domination number* γ_{it2} and *total domination number* γ_t.

Consider F = $\overline{K_{\gamma_{it2}}} \cup K_{\gamma_t}$

$$O(F) = \gamma_{it2} + \gamma_t$$

Let $V(\overline{K_{\gamma_{it2}}}) = \{v_1, v_2, ...v_{\gamma_{it2}}\}$

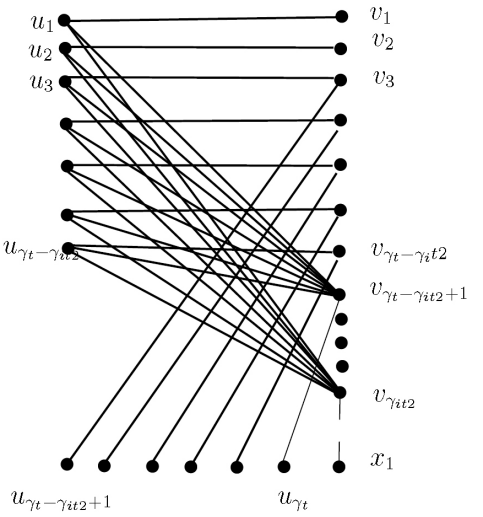
$$V(\overline{K_{\gamma_t}}) = \{u_1, u_2 \ldots\ldots u_{\gamma_t}\}$$

Since $\gamma_t > \gamma_{it2}$ construct F_1 from F by joining v_1 to u_1, v_2 to u_2 ..... $v_{\gamma_t - \gamma_{it2}}$ to $u_{\gamma_t - \gamma_{it2}}$. Also join all the vertices $\{u_1, u_2, ..u_{\gamma_t - \gamma it2}\}$ to each of the vertices $\{v_{\gamma_t - \gamma_{it2}+1}, ...v_{\gamma_{it2}}\}$. Then join distinct vertices of $\{u_{\gamma_t - \gamma_{it2}+1} .....u_{\gamma_t}\}$ to distinct vertices of $\{v_1, v_2, ...v_{\gamma_{it2}}\}$.

Construct F_2 from F_1 by introducing a vertex x_1 adjacent to $v_{\gamma_{it2}}$.

Construct F_3 from F_2 by introducing a vertex x_2 adjacent to $v_{\gamma_{it2}}$.

Proceeding in the same manner, we can construct a graph of n vertices with prescribed value γ_{it2} and γ_t.



Construction for $\gamma_{it2} < \gamma_t$

Theorem 4.3: *Suppose G is a graph of n vertices,* $\gamma_{it2}(G) = n-1$ *iff* $G = K_{1,n-1}$

Proof: Let G be a graph and $o(G) = n$. Let S be minimal independent semitotal dominating set *with n-1 vertices* (ie) $|S| = n-1$.

Claim: $G = K_{1,n-1}$

(ie) G has only one vertex of degree n-1.

Suppose G has at least 2 vertices *u* and *v* of degree n-1. Clearly *uv* is an edge and both *u* and *v* cannot be in S. So at most n-2 vertices of G can be in S. but $|S| = n-1$. *This is a contradiction*.

Let *u* be the vertex of *degree* n-1 in G.

Claim: Rest of n-1 vertices having degree 1

Suppose *there are at least two vertices x and y* of degree more than 1.

Case (i) *xy is an edge*

Then x,y,u all cannot belong to S.

$\therefore |S| < n-1$. This is a contradiction.

Case (ii) *x and y are adjacent* to some other vertices. In that case also $|S| < n-1$.

$\therefore$ Remaining n-1 vertices are of degree 1.

$\because$ G is connected $G = K_{1,n-1}$.

Clearly $G = K_{1,n-1}$. Then $\gamma_{it2}(G) = n-1$.

Theorem 4.4: *Suppose G is a graph of n vertices,* $\gamma_{it2}(G) = n/2$ *iff* $\gamma_{it2}(G) = \alpha_0 = \alpha_1$.

Proof: Let S be a γ_{it2} - set of G. $o(G) = n$.

Suppose $|S| = n/2$ then n is even.

Since S is independent, *to cover the vertices* of S at least $n/2$ edges are needed.

As $|V-S| = n/2$ and S is a *dominating set*, each *v* not in S is *adjacent to at least one vertex* of S.

$\therefore$ edge covering number $\alpha_1(G) = n/2$.

Since S is independent and V-S can have edges, $|V-S|$ vertices are needed to cover all the edges.

vertex covering number $\alpha_0(G) = n/2$.

Conversely, let $\gamma_{it2}(G) = \alpha_0 = \alpha_1$

Claim: $\gamma_{it2}(G) = n/2$.

$$\gamma_{it2}(G) = \alpha_0 = \alpha_1 = \text{k(say)}\ k \geq 2$$

Let $o(G) = n$ and k edges cover n vertices. Suppose these k edges are independent edges, then n=2k $k = n/2$

Since $\gamma_{it2}(G) = k$ G has k independent vertices.

To cover these k vertices, at least k different edges are needed and since $\alpha_1 = k$, these k edges serve as α_1 set. This implies *V-S has at most k vertices*.

Since $\alpha_0 = k$ implies V-S has exactly k vertices.

$$\therefore o(G) = 2k$$

Thus $\gamma_{it2}(G) = n/2$.

5. Bounds

Theorem 5.1: *For a graph G,* $\gamma_{it2}(G) \le i(G) + (\Delta - 1)k$ *where k = 0 or 1. The bound is also attained.*

Proof: Let S be an $i(G)$-set. If $i(G)$-set is also an independent semitotal dominating set then we are through. Suppose S is not a γ_{it2}-set then there are vertices in S for which *there exists* no vertex in S at distance exactly 2. Clearly any two such vertices are at distance 3 in S.

Case (i) k disjoint pairs are at distance 3 in S

For each pair replace the one with minimum independent dominating set of private neighbours. Now $D = S - S_k \cup T$ where $S_k \subset S$ with k vertices that are removed and T is an *independent dominating set* for the private neighbors of those k vertices in S which are replaced. Then D is an independent semitotal dominating set.

$$\therefore \gamma_{it2}(G) \le |D|$$

$$|D| = |S| - |S_k| + |T|$$

$$\text{(ie) } \gamma_{it2}(G) \le i(G) - k + |T|$$

$$\le i(G) - k + k_1 + k_2 + \cdots + k_k$$

where k_i is the degree of the vertex v_i in S that is removed.

$$\gamma_{it2}(G) \le i(G) - k + \underbrace{\Delta + \Delta + \cdots + \Delta}_{k \text{ - times}}$$

$$\gamma_{it2}(G) = i(G) - k + k\Delta$$

$$\gamma_{it2}(G) = i(G) + (\Delta - 1)k$$

Thus $\gamma_{it2}(G) \le i(G) + (\Delta - 1)k$

Case (ii) k distinct pairs are at distance 3 in S

Here the common vertex is replaced by its independent private neighbours. Similar argument gives $\gamma_{it2}(G) \le i(G) + (\Delta - 1)k$ It is observed that k takes values either 0 or 1 for any connected graph.

The bound is attained. $k = 0$, $\gamma_{it2}(G) = i(G)$ for Petersen graph.

$$k = 1, \ \gamma_{it2}(G) = i(G) + (\Delta - 1)k \text{ for } C_8.$$

Theorem 5.2: *For a cubic graph with* $n \geq 10$, $\gamma_{it2}(G) \leq \lceil n/3 \rceil$. *and this bound is also attained.*

Proof: Let V(G) = $\{q_1, q_2, ...q_n\}$. Choose a vertex q_1 in V(G), q_1 dominates q_2, q_3, q_4 (say).

Now G-q_1 is also connected and degrees of q_2, q_3, q_4 are all equal to 2.

Choose a vertex q_5 (say) of degree 3 that is adjacent to either q_2 or q_3 or q_4.

Let q_5 be adjacent to q_2 (say). Clearly q_5 dominates two other vertices different from q_2 say q_6, q_7. Now q_1, q_5 dominates $\{q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$*set*. Choose a vertex q_8 in V(G)-$\{q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$ such that q_8 dominates 3 vertices not already dominated by $\{q_1, q_5\}$.

Clearly n is even. Let n = 2k and $n \geq 10$ implies $k \geq 5$.

For $k = 5$, choose q_8 so that it dominates q_9 and q_{10} and is at distance two from either q_1 or q_5 in G. Thus q_8 is adjacent to exactly one of q_2 or q_3 or q_4 or q_6 or q_7. Hence $\{q_1, q_5, q_8\}$ forms an independent semitotal dominating set. $\therefore \gamma_{it2}(G) \leq \lceil n/3 \rceil$.

For $k > 5$, choose q_8 so that it dominates next 3 vertices. Repeat the process for the next set of 7 vertices. Thus $\gamma_{it2}(G) \leq \lceil n/3 \rceil$.

For Tietze's graph $\gamma_{it2}(Tietze) = \lceil n/3 \rceil = \lceil 12/3 \rceil = 4$.

6. Application

In an educational institution the teachers are considered as vertices and two teachers are connected by an edge if they share a class. This forms a non-complete connected setup. In order to pass information to students, minimum number of teachers who don't share classes are chosen. This forms an independent dominating set. Also when two teachers chosen are on-duty, it is necessary to have a common back-up so that information can be passed to students through the back-up teacher. Thus in this setup, independent semitotal domination is applied.

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