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On eccentricity-based topological indices of line and para-line graphs of some convex polytopes

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Abstract

Graph theory has been studied different areas such as mathematics, information and chemistry sciences. It is about descriptors in quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) studies in the chemical network. Let $G = (V(G), E(G))$ be a graph without directed and multiple edges and without loops. A lot of topological indices have been defined for QSPR/QSAR studies. There are several types of these indices such as degree-based indices, eccentricity-based indices, and so on. The eccentricity-based topological indices are very important QSPR/QSAR studies, also recently these indices have been studied in many papers. In this paper, some eccentricity-based topological indices namely the connective eccentricity index $\xi^{ce}(G)$, the eccentric connectivity index $\xi^{c}(G)$, the modified eccentric connectivity index $\xi_{c}(G)$, the total eccentricity index $\xi(G)$, the Zagreb eccentricity indices $M_1^*(G)$, $M_1^{**}(G)$, $M_2^*(G)$, the average eccentricity index $avec$(G), the eccentricity based geometric-arithmetic index $GA_4(G)$ and new version of ABC index such as $ABC_5(G)$ have been computed for line and para-line graphs of some convex polytopes D_n, Q_n and R_n which are geometric graphs.

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1. Introduction

Graph theory's diverse applications in natural science (Mathematics, Chemistry, Biology, and so on), especially it has become an important component of the mathematical chemistry sciences. A chemical graph is a graph in which the atoms of the molecule represented by the vertex and the covalent bonds between these atoms are represented by edges of the corresponding vertices [5, 6, 34]. In chemical graph theory, a lot of graphical invariants have been used for obtaining correlations of chemical structures with various chemical reactivity, physical and biological [2]. These are called topological indices of graphs in the mathematical chemistry. The topological indices have been the numerical indices based on the topology of the atoms and their bonds [1, 2]. A topological index is a mathematical value associated with chemical regulation impersonate for the relationship of a chemical structure with some chemical reactivity, physical and biological activities [14]. The chemical structures can be also the communication networks. So, the topological indices can be used to compute vulnerability values of chemical graphs. The graph theory provide different perspective to chemists through valuable tools such as topologic index. Researchers in these two fields have been working intensively in recent years. In theoretical chemistry, they proposed many topological indices that have found some applications especially in QSPR/QSAR research [5, 18, 35]. Furthermore, the topological indices also can compute the vulnerability of a molecular graph same as the network vulnerability parameters.

Let $G = (V(G), E(G))$ be a simple undirected graph of order n and size m. Now, we give some definitions are related to graphs. Let $v \in V(G)$, *the open neighborhood of* v is $N_G(v) = \{u \in V(G) \mid uv \in E(G)\}$ and *closed neighborhood of* v is $N_G[v] = N_G(v) \bigcup \{v\}$. The *degree of vertex* v is the size of its open neighborhood in G, it is denoted by $\deg G(v)$ [7]. The sum of the degrees of the vertices which is the vertex v neighbor's is denoted by $\delta_G(v)$, that is $\delta_G(v) = \sum_{u \in N_G(v)} \deg_G(u)$ [1]. The *distance* $d_G(u, v)$ between two vertices u and v in G is the length of a shortest path between them [13]. The *eccentricity value* of the vertex $u \in V(G)$ denoted by $\varepsilon_G(u)$, that is the largest between *vertex* u and any other *vertex* v of G, $\varepsilon_G(u) = \max_{v \in V(G)} d(u,v)$ [13].

The first topological index is the Wiener index in chemical graph theory [15]. The Wiener index was defined as follows:

$$W(G) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_G(v_i, v_j). \tag{1.1}$$

Then, many topological indices were proposed after the Wiener index. Now, the eccentricity-based topological indices terminology will be given.

In 2000, Gupta et al. [37] introduced a topological index namely *connective eccentricity* index denoted by $\xi^{ce}(G)$ for the graph G. It is defined as follows:

$$\xi^{ce}(G) = \sum_{u \in V(G)} (\deg_G(u) / \varepsilon_G(u)). \tag{1.2}$$

The *eccentric connectivity index* $\xi^{c}(G)$ of given graph G was defined by Sharma et al [42]. It is defined as follows:

$$\xi^{c}(G) = \sum_{u \in V(G)} (\deg_G(u).\varepsilon_G(u)). \tag{1.3}$$

The modified eccentric connectivity index $\xi_c(G)$ was defined in [1] as follows:

$$\xi_c(G) = \sum_{u \in V(G)} (\varepsilon_G(u).\delta_G(u)), \tag{1.4}$$

where $\delta_G(u) = \sum_{v \in N_G(u)} \deg_G(v),$ that is the sum of degrees of vertices which is the vertex u neighbors.

The total eccentricity index is presented by Farooq et al. [32], which is defined as follows:

$$\xi(G) = \sum_{u \in V(G)} \varepsilon_G(u). \tag{1.5}$$

The first Zagreb index and second Zagreb index of graphs were defined in [16] . Then the first, second and third *Zagreb eccentricity* indices $M_1^{*}(G)$, $M_1^{**}(G)$, $M_2^{*}(G)$ were defined in [8] and [25]. The definition of first Zagreb eccentricity index is as follows:

$$M_1^{*}(G) = \sum_{uv \in E(G)} (\varepsilon_G(u) + \varepsilon_G(v)). \tag{1.6}$$

The definition of second Zagreb eccentricity index is as follows:

$$M_1^{**}(G) = \sum_{u \in V(G)} (\varepsilon_G(u))^2. \tag{1.7}$$

The definition of third Zagreb eccentricity index is as follows:

$$M_2^*(G) = \sum_{uv \in E(G)} (\varepsilon_G(u).\varepsilon_G(v)). \tag{1.8}$$

In [3], the *average eccentricity* index for any graph G was defined as follows:

$$avec(G) = \frac{1}{|V(G)|} \sum_{u \in V(G)} \varepsilon_G(u). \tag{1.9}$$

The eccentricity based geometric-arithmetic index for the any graph G was defined in [26] as follows:

$$GA_4(G) = \sum_{uv \in E(G)} \left(\frac{2\sqrt{\varepsilon_G(u).\varepsilon_G(v)}}{\varepsilon_G(u) + \varepsilon_G(v)} \right). \tag{1.10}$$

A new version of the ABC index namely $ABC_5(G)$ for the any graph G was defined in [24] as follows:

$$ABC_5(G) = \sum_{uv \in E(G)} \left(\sqrt{\frac{\varepsilon_G(u) + \varepsilon_G(v) - 2}{\varepsilon_G(u).\varepsilon_G(v)}} \right). \tag{1.11}$$

These topological indices play an important role in chemical graph theory and particularly in theoretical chemistry. Some topological indices and some eccentricity based topological indices have been studied in some papers, readers can be seen them in [4, 9-11, 27, 28, 31, 35, 38, 39, 41, 43, 46, 47]. The first topological index on the basis of line graph was introduced by Bertz in 1981 (see [36]). The subdivision $S(G)$ of a graph G can be obtained by replacing each edge of G by a path of length 2, or we can say by inserting an additional vertex between each pair of vertices of G. For more details on the topological indices of $L(S(G))$ we refer to the articles [12, 20-23, 28, 31, 29, 40, 48].

This paper is organized as follows: In Section 2, the definitions of some convex polytopes D_n, Q_n, R_n and their subdivisions $S(D_n)$, $S(Q_n)$ and $S(R_n)$ are given. Then, the exact solutions of some eccentricity based topological indices are given for the line and para-line graphs of some convex polytopes D_n, Q_n and R_n in Section 3. Finally, in Section 4 we present our conclusions.

2. On Combinatorial Properties of Some Convex Polytopes and Their Subdivisions

In this section, some polytopes are determined according to combinatorial aspects. Firstly, we give definitions of some convex polytopes denoted by D_n, Q_n and R_n. Then the definitions of subdivisions $S(D_n)$, $S(Q_n)$ and $S(R_n)$ of the polytopes D_n, Q_n and R_n are given. Furthermore, the vertex and edge partitions of these polytopes are given. The degre-based topological indices of some convex polytopes have been studied in some papers, readers can be seen them in [12, 28, 29, 33, 44, 48].

2.1. *The convex polytope D_n, the subdivision $S(D_n)$, the line graph $L(D_n)$ and the para-line graph $L(S(D_n))$*

The graph of convex polytope D_n consists of 5-sided faces and n-sided face as defined in [18]. Clearly, $|V(D_n)| = 4n$ and $|E(D_n)| = 6n$. The subdivision $S(D_n)$ by inserting an additional vertex between each pair of adjacent vertices of D_n. Clearly, $|V(S(D_n))| = 10n$ and $|E(S(D_n))| = 12n$. Furthermore, we have $|V(L(D_n))| = 6n$, $|E(L(D_n))| = 12n$, $|V(L(S(D_n)))| = 12n$ and $|E(L(S(D_n)))| = 18n$. The polytope D_8, the graphs $S(D_8)$, $L(D_4)$ and $L(S(D_4))$ are shown in Figure 1.

2.2. *The convex polytope Q_n, the subdivision $S(Q_n)$, the line graph $L(Q_n)$ and the para-line graph $L(S(Q_n))$*

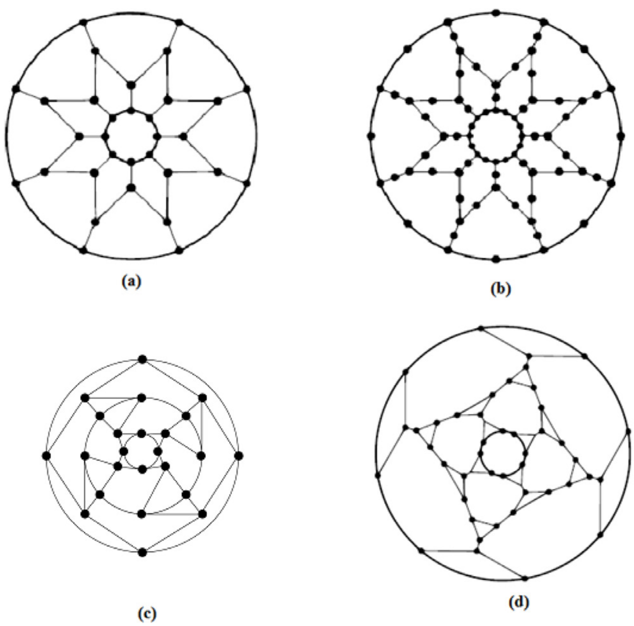

Figure 1

(a) The convex polytope D_8 (b) The graph $S(D_8)$

(c) The graph $L(D_4)$ (d) The graph $S(L(D_4))$

The graph of convex polytope Q_n consists of 3-sided faces, 4-sided faces, 5-sided faces and n-sided face as defined in [3]. Clearly, $|V(Q_n)| = 4n$ and $|E(Q_n)| = 7n$. The subdivision $S(Q_n)$ by inserting an additional vertex between each pair of adjacent vertices of Q_n. Clearly, $|V(S(Q_n))| = 11n$ and $|E(S(Q_n))| = 14n$. Furthermore, we have $|V(L(Q_n))| = 7n$, $|E(L(Q_n))| = 19n$, $|V(L(S(Q_n)))| = 14n$ and $|E(L(S(Q_n)))| = 26n$. The polytope Q_8, the graphs $S(Q_8)$, $L(Q_4)$ and $L(S(Q_4))$ are shown in Figure 2.

2.3. *The convex polytope R_n, the subdivision $S(R_n)$, the line graph $L(R_n)$ and the para-line graph $L(S(R_n))$*

The graph of convex polytope R_n is obtained as a combination of the graph of a prism and the graph of antiprism as defined in [19]. Clearly, $|V(R_n)| = 3n$ and $|E(R_n)| = 6n$. The subdivision $S(R_n)$ by inserting an additional vertex between each pair of adjacent vertices of R_n. Clearly, $|V(S(R_n))| = 9n$ and $|E(S(R_n))| = 12n$. Furthermore, we have $|V(L(R_n))| = 6n$, $|E(L(R_n))| = 19n$, $|V(L(S(R_n)))| = 12n$ and $|E(L(S(R_n)))| = 25n$. The polytope R_8 , the graphs $S(R_8)$, $L(R_4)$ and $L(S(R_4))$ are shown in Figure 3.

2.4 *The vertex partitions of line and para-line graphs of some convex polytopes*

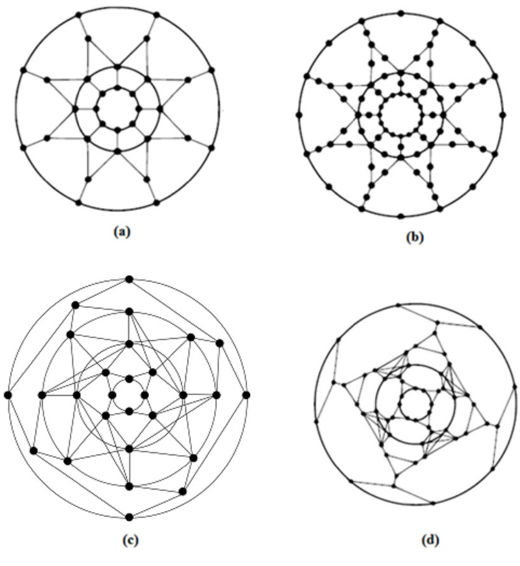

Figure 2

(a) The convex polytope Q_8 (b) The graph $S(Q_8)$

(c) The graph $L(Q_4)$ (d) The graph $S(L(Q_4))$

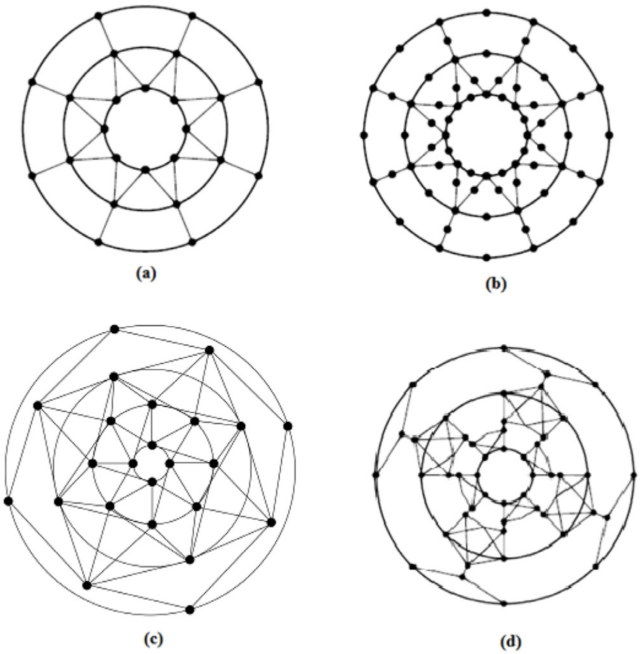

Figure 3

(a) The convex polytope R_8 (b) The graph $S(R_8)$

(c) The graph $L(R_4)$ (d) The graph $S(L(R_4))$

Let $n \geq 4$. We partition $V(G)$ into subsets based on the degrees of vertices, sum of the degrees of neighbors of vertices, the eccentricity values of vertices and frequencies of vertices in G. By using the C programming language we have developed to find the vertex partitions for these classes of convex polytopes. The vertex partition of $L(D_n)$, $L(Q_n)$ and $L(R_n)$ with respect to degrees and eccentricity values are shown in Tables 1, 2, 3, 4, 5 and 6. Furthermore, the vertex partition of $L(S(D_n))$, $L(S(Q_n))$ and $L(S(R_n))$ are shown in Tables 7, 8, 9, 10 and 11.

Table 1

The partitions of vertices in $G \cong L(D_n)$ for n is odd

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	4	$6n$	16	$\frac{n+5}{2}$

Table 2

The partitions of vertices in $G \cong L(D_n)$ for n is even

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	**4**	**4**n	**16**	$\frac{n+4}{2}$
2	**4**	**2**n	**16**	$\frac{n+6}{2}$

Table 3

The partitions of vertices in $G \cong L(Q_n)$ for n is odd

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	4	n	16	$\frac{n+5}{2}$
2	4	n	20	$\frac{n+3}{2}$
3	4	n	20	$\frac{n+5}{2}$
4	6	n	36	$\frac{n+3}{2}$
5	6	$2n$	38	$\frac{n+3}{2}$
6	8	n	52	$\frac{n+3}{2}$

Table 4

The partitions of vertices in $G \cong L(Q_n)$ for n is even

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	4	n	16	$\frac{n+4}{2}$
2	4	$2n$	20	$\frac{n+4}{2}$
3	6	n	36	$\frac{n+4}{2}$
4	6	$2n$	38	$\frac{n+2}{2}$
5	8	n	52	$\frac{n+2}{2}$

Table 5

The partitions of vertices in $G \cong L(R_n)$ for n is odd

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	4	n	20	$\frac{n+3}{2}$
2	6	n	38	$\frac{n+1}{2}$
3	6	n	40	$\frac{n+3}{2}$
4	7	$2n$	48	$\frac{n+3}{2}$
5	8	n	56	$\frac{n+1}{2}$

Table 6

The partitions of vertices in $G \cong L(R_n)$ for n is even

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	4	n	20	$\frac{n+2}{2}$
2	6	n	38	$\frac{n+2}{2}$
3	6	n	40	$\frac{n+2}{2}$
4	7	$2n$	48	$\frac{n+2}{2}$
5	8	n	56	$\frac{n+2}{2}$

Table 7

The partitions of vertices in $G \cong L(S(D_n))$ for n is odd

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	3	$12n$	9	$n+5$

Table 8

The partitions of vertices in $G \cong L(S(D_n))$ for n is even

Type of vertices	$\deg_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	3	$4n$	9	$n+4$
2	3	$8n$	9	$n+5$

Table 9

The partitions of vertices in $G \cong L(S(Q_n))$

Type of vertices	$\mathbf{deg}_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	3	n	9	$n+3$
2	3	n	9	$n+4$
3	3	$4n$	9	$n+5$
4	3	$2n$	11	$n+3$
5	3	n	11	$n+4$
6	5	$3n$	23	$n+3$
7	5	$2n$	25	$n+3$

Table 10

The partitions of vertices in $G \cong L(S(R_n))$ for n is odd

Type of vertices	$\mathbf{deg}_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	3	$2n$	9	$n+3$
2	3	n	11	$n+2$
3	4	$2n$	16	$n+3$
4	4	$2n$	17	$n+3$
5	5	n	23	$n+1$
6	5	$2n$	24	$n+2$
7	5	$2n$	25	$n+2$

Table 11

The partitions of vertices in $G \cong L(S(R_n))$ for n is even

Type of vertices	$\mathbf{deg}_G(u)$	*Frequency*	$\delta_G(u)$	$\varepsilon_G(u)$
1	3	$2n$	9	$n+3$
2	3	n	11	$n+2$
3	4	$2n$	16	$n+3$
4	4	$2n$	17	$n+3$
5	5	n	23	$n+2$
6	5	$2n$	24	$n+2$
7	5	$2n$	25	$n+2$

2.5 *The edge partitions of line and para-line graphs of some convex polytopes*

Let $uv \in E(G)$ and $n \geq 4$. We partition $E(G)$ into subsets based on the degrees of end vertices, the eccentricity values of end vertices and frequencies of vertices in G. By using the C programming language, we have developed to find the edges partitions for these classes of convex polytopes. These edge partitions of the graphs $L(D_n)$, $L(Q_n)$, $L(R_n)$, $L(S(D_n))$, $L(S(Q_n))$ and $L(S(R_n))$ are shown in Tables from 12 to 22.

Table 12

The partitions of edges in $G \cong L(D_n)$ for n is odd

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(4, 4)	$12n$	$\left(\frac{n+5}{2}, \frac{n+5}{2}\right)$

Table 13

The partitions of edges in $G \cong L(D_n)$ for n is even

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(4, 4)	$6n$	$\left(\frac{n+4}{2}, \frac{n+4}{2}\right)$
2	(4, 4)	$4n$	$\left(\frac{n+4}{2}, \frac{n+6}{2}\right)$
3	(4, 4)	$2n$	$\left(\frac{n+6}{2}, \frac{n+6}{2}\right)$

Table 14

The partitions of edges in $G \cong L(Q_n)$ for n is odd

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(4, 4)	$2n$	$\left(\frac{n+3}{2}, \frac{n+5}{2}\right)$
2	(4, 6)	$2n$	$\left(\frac{n+3}{2}, \frac{n+3}{2}\right)$
3	(4, 6)	$2n$	$\left(\frac{n+3}{2}, \frac{n+5}{2}\right)$
4	(4, 6)	$2n$	$\left(\frac{n+5}{2}, \frac{n+5}{2}\right)$

Contd...

5	(6, 6)	$4n$	$\left(\frac{n+3}{2},\frac{n+3}{2}\right)$
6	(6, 8)	$6n$	$\left(\frac{n+3}{2},\frac{n+3}{2}\right)$
7	(8, 8)	n	$\left(\frac{n+3}{2},\frac{n+3}{2}\right)$

Table 15

The partitions of edges in $G \cong L(Q_n)$ for n is even

Type of edges	**$(\deg_G(u), \deg_G(v))$**	***Frequency***	**$(\varepsilon_G(u),\varepsilon_G(v))$**
1	(4, 4)	$4n$	$\left(\frac{n+4}{2},\frac{n+4}{2}\right)$
2	(4, 6)	$2n$	$\left(\frac{n+4}{2},\frac{n+2}{2}\right)$
3	(4, 6)	$2n$	$\left(\frac{n+4}{2},\frac{n+4}{2}\right)$
4	(6, 6)	$2n$	$\left(\frac{n+2}{2},\frac{n+2}{2}\right)$
5	(6, 6)	$2n$	$\left(\frac{n+2}{2},\frac{n+4}{2}\right)$
6	(6, 8)	$4n$	$\left(\frac{n+2}{2},\frac{n+2}{2}\right)$
7	(6, 8)	$2n$	$\left(\frac{n+4}{2},\frac{n+2}{2}\right)$
8	(8, 8)	n	$\left(\frac{n+2}{2},\frac{n+2}{2}\right)$

Table 16

The partitions of edges in $G \cong L(R_n)$ for n is odd

Type of edges	**$(\deg_G(u), \deg_G(v))$**	***Frequency***	**$(\varepsilon_G(u),\varepsilon_G(v))$**
1	(4, 4)	n	$\left(\frac{n+3}{2},\frac{n+3}{2}\right)$
2	(4, 6)	$2n$	$\left(\frac{n+3}{2},\frac{n+1}{2}\right)$
3	(6, 6)	n	$\left(\frac{n+3}{2},\frac{n+3}{2}\right)$

Contd...

4	(6, 7)	$2n$	$\left(\frac{n+1}{2}, \frac{n+3}{2}\right)$
5	(6, 7)	$4n$	$\left(\frac{n+3}{2}, \frac{n+3}{2}\right)$
6	(6, 8)	$2n$	$\left(\frac{n+1}{2}, \frac{n+1}{2}\right)$
7	(7, 7)	$2n$	$\left(\frac{n+3}{2}, \frac{n+3}{2}\right)$
8	(7, 8)	$4n$	$\left(\frac{n+3}{2}, \frac{n+1}{2}\right)$
9	(8, 8)	n	$\left(\frac{n+1}{2}, \frac{n+1}{2}\right)$

Table 17

The partitions of edges in $G \cong L(R_n)$ for n is even

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(4, 4)	n	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
2	(4, 6)	$2n$	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
3	(6, 6)	n	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
4	(6, 7)	$6n$	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
5	(6, 8)	$2n$	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
6	(7, 7)	$2n$	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
7	(7, 8)	$4n$	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$
8	(8, 8)	n	$\left(\frac{n+2}{2}, \frac{n+2}{2}\right)$

Table 18

The partitions of edges in $G \cong L(S(D_n))$ for n is odd

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(3, 3)	$18n$	$(n+5, n+5)$

Table 19

The partitions of edges in $G \cong L(S(D_n))$ for n is even

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(3, 3)	$2n$	$(n+4, n+4)$
2	(3, 3)	$8n$	$(n+4, n+5)$
3	(3, 3)	$8n$	$(n+5, n+5)$

Table 20

The partitions of edges in $G \cong L(S(Q_n))$

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(3, 3)	$3n$	$(n+3, n+3)$
2	(3, 3)	n	$(n+3, n+4)$
3	(3, 3)	$4n$	$(n+4, n+5)$
4	(3, 3)	$4n$	$(n+5, n+5)$
5	(3, 5)	$2n$	$(n+3, n+3)$
6	(3, 5)	n	$(n+4, n+3)$
7	(5, 5)	$11n$	$(n+3, n+3)$

Table 21

The partitions of edges in $G \cong L(S(R_n))$ for n is odd

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(3, 3)	$2n$	$(n+2, n+3)$
2	(3, 3)	$2n$	$(n+3, n+3)$
3	(3, 5)	n	$(n+2, n+1)$
4	(4, 4)	$7n$	$(n+3, n+3)$
5	(4, 5)	$2n$	$(n+3, n+2)$
6	(5, 5)	$4n$	$(n+2, n+1)$
7	(5, 5)	$7n$	$(n+2, n+2)$

Table 22

The partitions of edges in $G \cong L(S(R_n))$ for n is even

Type of edges	$(\deg_G(u), \deg_G(v))$	*Frequency*	$(\varepsilon_G(u), \varepsilon_G(v))$
1	(3, 3)	$2n$	$(n+2, n+3)$
2	(3, 3)	$2n$	$(n+3, n+3)$
3	(3, 5)	n	$(n+2, n+2)$
4	(4, 4)	$7n$	$(n+3, n+3)$
5	(4, 5)	$2n$	$(n+3, n+2)$
6	(5, 5)	$11n$	$(n+2, n+2)$

3. The vertex eccentricity-based topological indices of the line and para-line graphs of some convex polytopes

In this section we compute the eccentricity-based topological indices of the line and para-line graphs of some convex polytopes discussed in Section 2. By using the MAPLE program results of following theorems have been obtained.

Theorem 3.1: *Let $L(D_n)$ be a line graph of a convex polytope of D_n, where $n \geq 4$. Then,*

(a) $$\xi^{ce}(L(D_n)) = \begin{cases} \frac{48n}{n+5}, & \text{if } n \text{ is odd;} \\ \frac{48n^2+256n}{n^2+10n+24}, & \text{if } n \text{ is even.} \end{cases}$$

(b) $\xi^{c}(L(D_n)) = 12n^2 + 58n + (2n)(-1)^{n+1}$.

(c) $\xi_c(L(D_n)) = 48n^2 + 232n + (8n)(-1)^{n+1}$.

(d) $\xi(L(D_n)) = 3n^2 + \frac{29}{2}n + \left(\frac{1}{2}n\right)(-1)^{n+1}$.

(e) $M_1^*(L(D_n)) = 12n^2 + 58n + (2n)(-1)^{n+1}$.

(f) $$M_1^{**}(L(D_n)) = \begin{cases} \frac{3}{2}n^3 + 15n^2 + \frac{75}{2}n, & \text{if } n \text{ is odd;} \\ \frac{3}{2}n^3 + 14n^2 + 34n, & \text{if } n \text{ is even.} \end{cases}$$

(g) $$M_2^*(L(D_n)) = \begin{cases} 3n^3 + 30n^2 + 75n, & \text{if } n \text{ is odd;} \\ 3n^3 + 28n^2 + 66n, & \text{if } n \text{ is even.} \end{cases}$$

(h) $avec(L(D_n)) = \begin{cases} \frac{n+5}{2}, & \text{if } n \text{ is odd;} \\ \frac{3n+14}{6}, & \text{if } n \text{ is even.} \end{cases}$

(i) $GA_4(L(D_n)) = \begin{cases} 12n, & \text{if } n \text{ is odd;} \\ 8n+\frac{4n}{n+5}\sqrt{n^2+10n+24}, & \text{if } n \text{ is even.} \end{cases}$

(j) $ABC_5(L(D_n)) = \begin{cases} \frac{24n}{n+5}\sqrt{n+3}, & \text{if } n \text{ is odd;} \\ \frac{12n}{n+4}\sqrt{n+2} + 8n\sqrt{\frac{n+3}{n^2+10n+24}} + \frac{4n}{n+6}\sqrt{n+4}, & \text{if } n \text{ is even.} \end{cases}$

Proof: The values of $\xi^{ce}(L(D_n))$, $\xi^{c}(L(D_n))$, $\xi_c(L(D_n))$, $\xi(L(D_n))$, $M_1^{**}(L(D_n))$ and $avec(L(D_n))$ can be obtained by the Formulas (1.2), (1.3), (1.4), (1.5), (1.7), (1.9) and using the vertex partitions of $L(D_n)$ as shown the Tables 1 and 2. Similarly, the values of $M_1^*(L(D_n))$, $M_2^*(L(D_n))$, $GA_4(L(D_n))$ and $ABC_5(L(D_n))$ can be obtained by the Formulas (1.6), (1.8), (1.10), (1.11) and using the edge partitions of $L(D_n)$ as shown the Tables 12 and 13.

Theorem 3.2: *Let $L(Q_n)$ be a line graph of a convex polytope of Q_n, where $n \geq 4$. Then,*

(a) $\xi^{ce}(L(Q_n)) = \begin{cases} \frac{76n^2+348n}{n^2+8n+15}, & \text{if } n \text{ is odd;} \\ \frac{76n^2+232n}{n^2+6n+8}, & \text{if } n \text{ is even.} \end{cases}$

(b) $\xi^{c}(L(Q_n)) = 19n^2 + \frac{121}{2}n + \left(\frac{9}{2}n\right)(-1)^{n+1}.$

(c) $\xi_c(L(Q_n)) = 110n^2 + 339n + (27n)(-1)^{n+1}.$

(d) $\xi(L(Q_n)) = \frac{7}{2}n^2 + \frac{47}{4}n + \left(\frac{3}{4}n\right)(-1)^{n+1}.$

(e) $M_1^*(L(Q_n)) = 19n^2 + \frac{121}{2}n + \left(\frac{9}{2}n\right)(-1)^{n+1}.$

(f) $M_1^{**}(L(Q_n)) = \begin{cases} \frac{7}{4}n^3+\frac{25}{2}n^2+\frac{95}{4}n, & \text{if } n \text{ is odd;} \\ \frac{7}{4}n^3+11n^2+19n, & \text{if } n \text{ is even.} \end{cases}$

(g) $M_2^*(L(Q_n)) = \begin{cases} \frac{19}{4}n^3+\frac{65}{2}n^2+\frac{227}{4}n, & \text{if } n \text{ is odd;} \\ \frac{19}{4}n^3+28n^2+43n, & \text{if } n \text{ is even.} \end{cases}$

(h) $avec(L(Q_n)) = \begin{cases} \frac{7n+25}{14}, & \textit{if } n \textit{ is odd;} \\ \frac{7n+22}{14}, & \textit{if } n \textit{ is even.} \end{cases}$

(i) $GA_4(L(Q_n)) = \begin{cases} 15n + \frac{4n}{n+4}\sqrt{n^2+8n+15}, & \textit{if } n \textit{ is odd;} \\ 11n + \sqrt{2}\,n + \frac{6n}{n+3}\sqrt{n^2+6n+8}, & \textit{if } n \textit{ is even.} \end{cases}$

(j) $ABC_5(L(Q_n)) = \begin{cases} \frac{26n}{n+3}\sqrt{n+1} + \frac{4n}{n+5}\sqrt{n+3} + 8n\sqrt{\frac{n+2}{n^2+8n+15}}, & \textit{if } n \textit{ is odd;} \\ \frac{14n}{n+2}\sqrt{n} + \frac{12n}{n+4}\sqrt{n+2} + 12n\sqrt{\frac{n+1}{n^2+6n+8}}, & \textit{if } n \textit{ is even.} \end{cases}$

Proof: The values of $\xi^{ce}(L(Q_n))$, $\xi^{c}(L(Q_n))$, $\xi_c(L(Q_n))$, $\xi(L(Q_n))$, $M_1^{**}(L(Q_n))$ and $avec(L(Q_n))$ can be obtained by the Formulas (1.2), (1.3), (1.4), (1.5), (1.7), (1.9) and using the vertex partitions of $L(Q_n)$ as shown the Tables 3 and 4. Similarly, the values of $M_1^{*}(L(Q_n))$, $M_2^{*}(L(Q_n))$, $GA_4(L(Q_n))$ and $ABC_5(L(Q_n))$ can be obtained by the Formulas (1.6), (1.8), (1.10), (1.11) and using the edge partitions of $L(Q_n)$ as shown the Tables 14 and 15.

Theorem 3.3: *Let $L(R_n)$ be a line graph of a convex polytope of R_n, where $n \geq 4$. Then,*

(a) $\xi^{ce}(L(R_n)) = \begin{cases} \frac{76n^2+132n}{n^2+4n+3}, & \textit{if } n \textit{ is odd;} \\ \frac{76n}{n+2}, & \textit{if } n \textit{ is even.} \end{cases}$

(b) $\xi^{c}(L(R_n)) = 19n^2 + \frac{81}{2}n + \left(\frac{5}{2}n\right)(-1)^{n+1}.$

(c) $\xi_c(L(R_n)) = 125n^2 + \frac{531}{2}n + \left(\frac{31}{2}n\right)(-1)^{n+1}.$

(d) $\xi(L(R_n)) = 3n^2 + \frac{13}{2}n + \left(\frac{1}{2}n\right)(-1)^{n+1}.$

(e) $M_1^{*}(L(R_n)) = 19n^2 + \frac{81}{2}n + \left(\frac{5}{2}n\right)(-1)^{n+1}.$

(f) $M_1^{**}(L(R_n)) = \begin{cases} \frac{3}{2}n^3 + 7n^2 + \frac{19}{2}n, & \textit{if } n \textit{ is odd;} \\ \frac{3}{2}n^3 + 6n^2 + 6n, & \textit{if } n \textit{ is even.} \end{cases}$

(g) $M_2^{*}(L(R_n)) = \begin{cases} \frac{19}{4}n^3 + \frac{43}{2}n^2 + \frac{99}{4}n, & \textit{if } n \textit{ is odd;} \\ \frac{19}{4}n^3 + 19n^2 + 19n, & \textit{if } n \textit{ is even.} \end{cases}$

(h) $avec(L(R_n)) = \begin{cases} \frac{3n+7}{6}, & \textit{if } n \textit{ is odd;} \\ \frac{n+2}{2}, & \textit{if } n \textit{ is even.} \end{cases}$

(i) $GA_4(L(R_n)) = \begin{cases} 11n+\frac{8n}{n+2}\sqrt{n^2+4n+3}, & \textit{if } n \textit{ is odd;} \\ 19n, & \textit{if } n \textit{ is even.} \end{cases}$

(j) $ABC_5(L(R_n)) = \begin{cases} \frac{16n}{n+3}\sqrt{n+1}+\frac{6n}{n+1}\sqrt{n-1}+16n\sqrt{\frac{n}{n^2+4n+3}}, & \textit{if } n \textit{ is odd;} \\ \frac{38n}{n+2}\sqrt{n}, & \textit{if } n \textit{ is even.} \end{cases}$

***Proof*:** The values of $\xi^{ce}(L(R_n))$, $\xi^{c}(L(R_n))$, $\xi_c(L(R_n))$, $\xi(L(R_n))$, $M_1^{**}(L(R_n))$ and $avec(L(R_n))$ can be obtained by the Formulas (1.2), (1.3), (1.4), (1.5), (1.7), (1.9) and using the vertex partitions of R_n as shown the Tables 5 and 6. Similarly, the values of $M_1^*(L(R_n))$, $M_2^*(L(R_n))$, $GA_4(L(R_n))$ and $ABC_5(L(R_n))$ can be obtained by the Formulas (1.6), (1.8), (1.10), (1.11) and using the edge partitions of R_n as shown the Tables 16 and 17.

Theorem 3.4: *Let $L(S(D_n))$ be a para-line graph of a convex polytope D_n, where $n \geq 4$. Then,*

(a) $\xi^{ce}(L(S(D_n))) = \begin{cases} \frac{36n}{n+5}, & \textit{if } n \textit{ is odd;} \\ \frac{36n^2+156n}{n^2+9n+20}, & \textit{if } n \textit{ is even.} \end{cases}$

(b) $\xi^{c}(L(S(D_n))) = 36n^2+174n+(6n)(-1)^{n+1}$.

(c) $\xi_c(L(S(D_n))) = 108n^2+522n+(18n)(-1)^{n+1}$.

(d) $\xi(L(S(D_n))) = 12n^2+58n+(2n)(-1)^{n+1}$.

(e) $M_1^*(L(S(D_n))) = 36n^2+174n+(6n)(-1)^{n+1}$.

(f) $M_1^{**}(L(S(D_n))) = \begin{cases} 12n^3+120n^2+300n, & \textit{if } n \textit{ is odd;} \\ 12n^3+112n^2+264n, & \textit{if } n \textit{ is even.} \end{cases}$

(g) $M_2^*(L(S(D_n))) = \begin{cases} 18n^3+180n^2+450n, & \textit{if } n \textit{ is odd;} \\ 18n^3+168n^2+392n, & \textit{if } n \textit{ is even.} \end{cases}$

(h) $avec(L(S(D_n))) = \begin{cases} n+5, & \textit{if } n \textit{ is odd;} \\ \frac{3n+14}{3}, & \textit{if } n \textit{ is even.} \end{cases}$

(i) $GA_4(L(S(D_n))) = \begin{cases} 18n, & \textit{if } n \textit{ is odd;} \\ 10n+\frac{16n}{2n+9}\sqrt{n^2+9n+20}, & \textit{if } n \textit{ is even.} \end{cases}$

(j) $ABC_5(L(S(D_n))) = \begin{cases} \frac{18n}{n+5}\sqrt{2n+8}, & \textit{if } n \textit{ is odd;} \\ \frac{2n}{n+4}\sqrt{2n+6}+\frac{8n}{n+5}\sqrt{2n+8}+8n\sqrt{\frac{2n+7}{n^2+9n+20}}, & \textit{if } n \textit{ is even.} \end{cases}$

***Proof*:** The values of $\xi^{ce}(L(S(D_n)))$, $\xi^{c}(L(S(D_n)))$, $\xi_c(L(S(D_n)))$, $\xi(L(S(D_n)))$, $M_1^{**}(L(S(D_n)))$ and $avec(L(S(D_n)))$ can be obtained by the Formulas (1.2), (1.3), (1.4), (1.5), (1.7), (1.9) and using the vertex partitions of $(L(S(D_n)))$ as shown the Tables 7 and 8. Similarly, the values of $M_1^{*}(L(S(D_n)))$, $M_2^{*}(L(S(D_n)))$, $GA_4(L(S(D_n)))$ and $ABC_5(L(S(D_n)))$ can be obtained by the Formulas (1.6), (1.8), (1.10), (1.11) and using the edge partitions of $(L(S(D_n)))$ as shown the Tables 18 and 19.

Theorem 3.5: *Let $L(S(Q_n))$ be a para-line graph of a convex polytope (Q_n), where $n \geq 4$. Then,*

(a) $\xi^{ce}(L(S(Q_n))) = \frac{52n^3+438n^2+914n}{n^3+12n^2+47n+60}$.

(b) $\xi^{c}(L(S(Q_n))) = 52n^2+186n.$

(c) $\xi_c(L(S(Q_n))) = 206n^2+710n.$

(d) $\xi(L(S(Q_n))) = 14n^2+52n.$

(e) $M_1^{*}(L(S(Q_n))) = 52n^2+186n.$

(f) $M_1^{**}(L(S(Q_n))) = 14n^3+104n^2+204n.$

(g) $M_2^{*}(L(S(Q_n))) = 26n^3+186n^2+348n.$

(h) $avec(L(S(Q_n))) = n+\frac{26}{7}$.

(i) $GA_4(L(S(Q_n))) = 20n+\frac{4n}{2n+7}\sqrt{n^2+7n+12}+\frac{8n}{2n+9}\sqrt{n^2+9n+20}.$

(j) $ABC_5(L(S(Q_n))) = \frac{16n}{n+3}\sqrt{2n+4}+\frac{4n}{n+5}\sqrt{2n+8}+2n\sqrt{\frac{2n+5}{n^2+7n+12}}+4n\sqrt{\frac{2n+7}{n^2+9n+20}}.$

***Proof*:** The values of $\xi^{ce}(L(S(Q_n)))$, $\xi^{c}(L(S(Q_n)))$, $\xi_c(L(S(Q_n)))$, $\xi(L(S(Q_n)))$, $M_1^{**}(L(S(Q_n)))$ and $avec(L(S(Q_n)))$ can be obtained by the Formulas (1.2), (1.3), (1.4), (1.5), (1.7), (1.9) and using the vertex partitions of $L(S(Q_n))$ as shown the Table 9. Similarly, The values of $M_1^*(L(S(Q_n)))$, $M_2^*(L(S(Q_n)))$, $GA_4(L(S(Q_n)))$ and $ABC_5(L(S(Q_n)))$ can be obtained by the Formulas (1.6), (1.8), (1.10), (1.11) and using the edge partitions of $L(S(Q_n))$ as shown the Table 20.

Theorem 3.6: *Let $L(S(R_n))$ be a para-line graph of a convex polytope R_n, where $n \geq 4$. Then,*

(a) $$\xi^{ce}(L(S(R_n))) = \begin{cases} \frac{50n^3+183n^2+143n}{n^3+6n^2+11n+6}, & \text{if } n \text{ is odd;} \\ \frac{50n^2+128n}{n^2+5n+6}, & \text{if } n \text{ is even.} \end{cases}$$

(b) $\xi^c(L(S(R_n))) = 50n^2 + \frac{239}{2}n + \left(\frac{5}{2}n\right)(-1)^n.$

(c) $\xi_c(L(S(R_n))) = 216n^2 + \frac{1009}{2}n + \left(\frac{23}{2}n\right)(-1)^n.$

(d) $\xi(L(S(R_n))) = 12n^2 + \frac{59}{2}n + \left(\frac{1}{2}n\right)(-1)^n.$

(e) $M_1^*(L(S(R_n))) = 50n^2 + \frac{239}{2}n + \left(\frac{5}{2}n\right)(-1)^n.$

(f) $$M_1^{**}(L(S(R_n))) = \begin{cases} 12n^3 + 58n^2 + 75n, & \text{if } n \text{ is odd;} \\ 12n^3 + 60n^2 + 78n, & \text{if } n \text{ is even.} \end{cases}$$

(g) $$M_2^*(L(S(R_n))) = \begin{cases} 25n^3 + 117n^2 + 143n, & \text{if } n \text{ is odd;} \\ 25n^3 + 122n^2 + 153n, & \text{if } n \text{ is even.} \end{cases}$$

(h) $$avec(L(S(R_n))) = \begin{cases} \frac{12n+29}{12}, & \text{if } n \text{ is odd;} \\ \frac{2n+5}{2}, & \text{if } n \text{ is even.} \end{cases}$$

(i) $$GA_4(L(S(R_n))) = \begin{cases} 16n + \frac{8n}{2n+5}\sqrt{n^2+5n+6} + \frac{10n}{2n+3}\sqrt{n^2+3n+2}, & \text{if } n \text{ is odd;} \\ 21n + \frac{8n}{2n+5}\sqrt{n^2+5n+6}, & \text{if } n \text{ is even.} \end{cases}$$

(j) $$ABC_5(L(S(R_n))) = \begin{cases} \frac{9n}{n+3}\sqrt{2n+4} + \frac{7n}{n+2}\sqrt{2n+2} + 4n\sqrt{\frac{2n+3}{n^2+5n+6}} + 5n\sqrt{\frac{2n+1}{n^2+3n+2}}, & \text{if } n \text{ is odd;} \\ \frac{9n}{n+3}\sqrt{2n+4} + \frac{12n}{n+2}\sqrt{2n+2} + 4n\sqrt{\frac{2n+3}{n^2+5n+6}}, & \text{if } n \text{ is even.} \end{cases}$$

***Proof*:** The values of $\xi^{ce}(L(S(R_n)))$, $\xi^{c}(L(S(R_n)))$, $\xi_c(L(S(R_n)))$, $\xi(L(S(R_n)))$, $M_1^{**}(L(S(R_n)))$ and $avec(L(S(R_n)))$ can be obtained by the Formulas (1.2), (1.3), (1.4), (1.5), (1.7), (1.9) and using the vertex partitions of $L(S(R_n))$ as shown the Tables 10 and 11. Similarly, The values of $M_1^{*}(L(S(R_n)))$, $M_2^{*}(L(S(R_n)))$, $GA_4(L(S(R_n)))$ and $ABC(L(S(R_n)))$ can be obtained by the Formulas (1.6), (1.8), (1.10), (1.11) and using the edge partitions of $L(S(R_n))$ as shown the Tables 21 and 22.

4. Conclusion

In this paper, we have studied and computed some eccentricity-based topological indices for line graphs of some convex polytopes D_n, Q_n, R_n and their subdivisions $S(D_n)$, $S(Q_n)$ and $S(R_n)$. The research work can be continued to derive new architectures from the line graphs of some convex polytopes D_n, Q_n, R_n and their subdivisions $S(D_n)$, $S(Q_n)$ and $S(R_n)$, and also these eccentricity values can be obtained for line and para-line graphs of different types convex polytopes for future works.

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