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Image quality assessment: Intelligent feature selection using soft computing techniques

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Abstract

Image Quality Assessment (IQA), the quantification of a human's perception of qualityof an image, is a rudimentary problem visual communication system, and beholds unparalleled significance in diverse practical applications like image compression, restoration, and, rendering. The limitations of conventional IQA parameters have motivated the researchers to go for IQA techniques based on the Soft Computing meta-heuristic algorithms that are able to extract features intelligently. In current paper, Whale Optimization Algorithm (WOA) has been applied, which is a meta-heuristic algorithm grounded on swarm behaviour. This technique mimics the demeanor of humpback whales for finding an optimal solution. For the purpose of extensive analysis and testing in Matlab, Colourlab Image Database: Image Quality (CID:IQ) benchmark dataset is applied. Tabular, as well as graphical outcomes, are defended for validating the effectiveness of WOA as compared to traditional methods. WOA understandably surmounts Artificial Neural Networks (ANN) based optimization with regards to Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE), and other popular IQA parameters. This amelioration is capable of motivating the researchers in the time to come for opting the soft computing mechanisms in several image processing arenas that includes but is not limited to the agricultural sciences, medicine, remote sensing, etc., for assessing the images in pre-processing stages to achieve ameliorated results.

Subject Classification: *68T20.*

Keywords: *Image quality assessment, Intelligent feature selection, Meta-heuristic algorithms, Whale optimization algorithm (WOA).*

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1. Introduction

Perceptual Image Quality Assessment (IQA) has a critical function in the imaging systems due to the various erosions in quality brought in at different phases of image acquisition, compression, transmission and exhibition. IQA is required for ensuring and improving the image quality before delivery to the end-users. IQA parameters can be employed as an examining standard for optimization goals of imaging systems [1]. IQA can be executed subjectively as well as objectively, and objective IQA is generally favored given its higher efficiency and ease in deployment. Subjective IQA is, by and large, reckoned as more precise and authentic since the Human Visual System (HVS) is the final recipient of images [2]. Nevertheless, subjective IQA is time-devouring and costly, and, is subject to different factors such as the tempers and working considerations of the perceivers, the display devices employed, the predominant illuminating considerations and the viewing distance of perceivers. Over the past few years, there has been a heightening concern in formulating objective IQA techniques which are able to automatically anticipate human behavior for measuring quality of images [2]. Such IQA techniques have extensive practical applications in valuating, controlling, designing and optimizing imaging systems. Banking on the accessibility of a perfective quality source image, these IQA techniques are assorted into different algorithms: Full-Reference (FR), in which the source image is completely available while assessing the distorted image, Reduced-Reference (RR), in which only partial data regarding the source image is accessible, and, No-Reference (NR), which does not allow any access to the source image [3].

While conventional image accuracy parameters such as Mean Square Error (MSE) or Peak Signal-to-Noise Ratio (PSNR) metrics are frequently disputed since these do not have any considerateness for image features and HVS, these are, at the same time, popularly employed as IQA metrics [4]. Nonetheless, a substantive melioration in the subsisting IQA metrics is yet awaited, as these parameters denounce HVS in one way or the other, proving discrepant execution in the presence of varying degradations for various standard image databases. Contrary to this, there are several metrics, say, SSIM (Structural Similarity Index Measure) and MSSIM (Mean Structural Similarity Index Measure), that render meliorated outcomes, but then, this demand a price: these are likely to be contaminated with various noises, which can be abridged by raising the count of calculated samples and, accordingly, heightening their computing times. Besides, these aggregative image datasets constitute several irrelevant and superfluous characteristics and the conventional IQA metrics are incapable of managing this huge and wide-ranging feature set [4].

Therefore, it is a prerogative that rather than concentrating on the recognized perceptual IQA models, novel and intelligent feature-extraction algorithms, employing soft computing approaches, should be employed which are adequate to reduce the data variability, whilst decimating the undesirable and extra characteristics from the data [5][6]. Some of the common examples of metaphor based meta-heuristic algorithms are GA (Genetic Algorithm) [7], PSO (Particle Swarm Optimization) [8], Harmony Search (HS) [9], Simulated Annealing (SA) [10], Teaching–Learning-Based Optimization (TLBO) [11], Grey Wolf Optimization (GWO) Algorithm [12], Artificial Neural Networks (ANN) [13], Whale Optimization Algorithm (WOA) [14] and others. Common examples of non-metaphor established meta-heuristics are Tabu Search (TS) [15] and Variable Neighborhood Search (VNS) [16].

2. Current Work

2.1 *WOA Procedure*

Mirjalili and Lewis put forward WOA for simulating the preying nature of humpback whales, that is achieved by dual cardinal attacking techniques; trailing of the hunt with a random or best search agent as well as simulation of the bubble net hunt technique. These whales have an extremely noteworthy hunting technique known as the bubble net attacking technique. For this, they encircle the fish in a winding-shaped manner, thereby producing clear-cut bubbles in a helical manner. This scrounging is executed depicted in Figure 1 [14].

The WOA begins by allotting population of humpback whales with a stochastic solution and presuming an optimal measure of objective function as a minimal or maxima (dependent on problem), and hence calculation of objective functions is done for all search agents. In all iterations, each search agent update its location, banking on either optimal solution determined when $|\vec{B}|< 1$ or on a arbitrarily selected search agent when $|\vec{B}|> 1$. For accomplishing exploration as well as exploitation stages, the worth of bis reduced from 2 to 0. Furthermore, WOA can select either a circular or a spiral motion with the measure of a random number p in [0, 1] with a fifty percent chance to choose either of the two methods, i.e., if p is larger than 0.5, the search agent will alter its location by Equation (5), else it uses

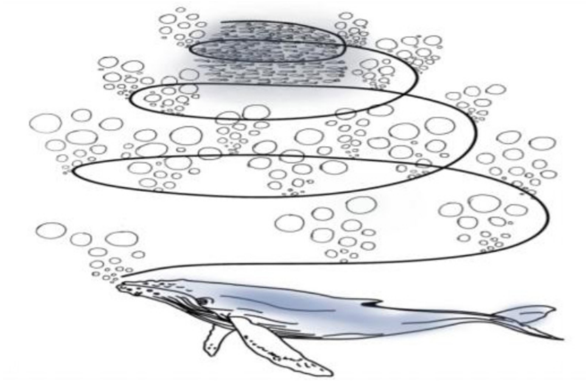

Figure 1

Feeding demeanor of a Whale: Bubble-net Hunt Technique

Equation (1). Eventually, WOA terminates by enforcing endpoint criteria [14]. The pseudocode of the WOA technique is given in Figure 2.

WOA algorithm comprises of two fundamental stages; encircling of the prey and spiral position updating are carried out in the first stage, also known as the exploitation stage. This is followed by random exploring for a hunt in the succeeding stage, also known as the exploration stage [14]. The mathematical framework of both these stages is described as below:

2.2 *Bubble Net Attacking Technique (Exploitation Stage):*

Two strategies are contrived to mathematically model this demeanour of whales. These two strategies are:

(A) Shrinking Encircling of Hunt. The humpback whales first find out the location of the hunt, and afterwards, they circle around them. At the outset, they identify the placement of optimal blueprint in search space, thereby assuming that the currently most adept solution is hunt or nearer to optimal solution. After this, rest of the search agents seek to alter their placements in accordance with leading search agent. This demeanor is presented in the accompanying equations:

$$\vec{W}(i+1) = \overrightarrow{W^*}(i) - \vec{A}.\vec{D} \tag{1}$$

$$\vec{D} = |\vec{C}.\overrightarrow{W^*}(i) - \vec{W}(i)| \tag{2}$$

Here, $\vec{W}(i)$ is indicative of the whale's previous optimal position in iteration i, $\vec{W}(i+1)$ is its present location, $\vec{D}$ is distance among humpback

```
BEGIN
  1. Yield a starting population, W_w , with(w = 1, 2, 3, ......, n);
  2. Compute the fitness value of all solutions;
     W* = The most adept search agent;
  3. while (i< Maximum number of iterations)do:
        For all solutions do:
            Modify values ofb, B, C, L, p;
            If (1)(p < 0.5)do:
                If (2) (|B| < 1)do:
                Modify the current search agent's location with Equation (1);
                Else if (2)(|B| ≥ 1)do:
                   Choose an arbitrary search agent(W_rand);
                  Modify the current search agent's location with Equation (7);
                End if (2)
            Else if (1)(p ≥ 0.5)do:
                Modify the current search agent's location with Equation (5);
            End if (1)
        End for
        Ascertain whatsoever search agent is outside the search space;
        Correct the location;
        Compute the fitness of all search agents;
        ModifyW* in case of a more adept solution i = i + 1;
     end while;
  4. Return W*;
END
```

Figure 2

Pseudo Code of WOA Algorithm

whale and its hunt. The coefficient vectors, $\vec{A}$ and *C*, are computed as given below:

$$\vec{B} = 2.\vec{a}.\vec{r} + \vec{b} \tag{3}$$

$$\vec{C} = 2.\vec{r} \tag{4}$$

For applying shrinking, measure of $\vec{b}$ is brought down in Equation (3); thereby reducing the oscillatory interval of $\vec{B}$ by $\vec{b}$. The measure of $\vec{B}$ can be in the range $(-b, b)$, in which the measure of *b* is reduced from 2 to 0 after a number of iterations. By randomly selecting the measures of $\vec{B}$ in range $(-1, 1)$, the fresh location of search agent is ascertained in interval from original location of search agent to position of currently leading search agent.

(B) Spirally Updating Location. Once distance within whale positioned at (*P*, *Q*) and hunt positioned at (P^*, Q^*) has been ascertained, a spiral equation is yielded for positions of whale as well as hunt for imitating the helix shape motion of whales, given by:

$$\vec{W}(i+1) = e^{fg}.\cos(2\pi g).\overrightarrow{D^*} + \overrightarrow{W^*}(i) \tag{5}$$

$$\vec{D} = |\overrightarrow{W^*}(i) - \vec{W}(i)| \tag{6}$$

Here, f is a constant to identify the contour of the spiral and g is any arbitrary number in the interval of [–1, 1].

$$\vec{W}(i+1) = \begin{cases} \overrightarrow{W^*} - \vec{B}.\vec{D} & \text{if } p < 0.5 \\ e^{fg}.\cos(2\pi g).\overrightarrow{D^*} + \overrightarrow{W^*}(i) & \text{if } p \geq 0.5 \end{cases} \tag{7}$$

Where p is any arbitrary figure in interval (0, 1).

2.3 *Seeking for Hunt (Exploration Stage)*

In the hunt search stage [14], a similar technique can be employed, that depends on the variance of vector $\vec{B}$. The humpback whales in reality utilize random search technique for discovering their hunt banking on each other's location. Nonetheless, for obliging the search agents to displace further from a local humpback whale, the algorithm utilizes the vector $\vec{B}$, having stochastic values less than or greater than 1.

This procedure assists the WOA for performing a global search and overcoming the issue of local optima. The mathematical equations are as follows:

$$\vec{W}(i+1) = \overrightarrow{W_{rand}} - \vec{B}.\vec{D} \tag{8}$$

$$\vec{D} = |\vec{C}.\overrightarrow{W_{rand}} - \vec{W}| \tag{9}$$

In which $\overrightarrow{W_{rand}}$ is the random location vector (i.e., a ergodic humpback whale) selected from the current population.

3. Related Works

The WOA is by and large employed in continuous function optimization problems. Research outcomes establish that WOA is superordinate to other optimization algorithms like gravitational search and differential evolution in terms of algorithm stability and solution accuracy [14][17]. Consequently, researchers continue to improve the WOA. Literature [18–31] analyzed several function optimization problems by using WOA. Trivedi et al. [18] put forward an adaptive technique for a novel globally optimized adaptive WOA. For increasing the diverseness of population, Ling et al. [19] contrived improved LWOA by bringing in Levy flight technique. For balancing the abilities of exploitation and exploration of WOA, Kaur et. al [20] presented the principle of chaos in WOA. For improving the global search ability of WOA, Wu et. al [21] assumed opposite learning strategy for initializing population of the whales. Chen et al. [22] put forward a dual-adaptive weighing algorithm that ameliorated exploration capability in the early stages and exploitation capability in the later stages. Huang et al. [23] purported IWOA grounded on elite guidance techniques and chaotic weights.

Ding et al. [24] aggregated WOA with simulated annealing and adaptive weight. The latter is employed for adjusting the convergence speed and the former is applied for enhancing optimization ability of WOA globally. Bozorgi et al. [25] presented a differential evolution technique for improving local search capability of WOA. Aziz et al. [26] employed WOA to resolve multi-threshold image segmentation issues. The authors in [27] employed WOA to solve the position problem of radial network capacitors. The authors in [28] expended WOA in neural networks to optimize weights. Ya Li et al. [29] utilized the IWOA for solving Knapsack problem. The authors in [30] clubbed WOA with simulated annealing technique for feature extraction problems. Oliva et al. [31] coalesced WOA and chaotic mapping technique for predicting parameters of solar cell.

Because of an unbalance amongst exploitation and exploration, authors in [32] purported a new evaluating technique founded on WOA. IWOA purpurted in [33] used a newer control variable, inertia weight. This variable is used for adjusting the currently leading solution. For evaluating the quality of IWOA, several benchmark functions were employed for testing in [14]. In [34], the mean of ABC as well as FOA were higher as compared to IWOA and WOA. A Modified WOA is developed in [35] that are employed in cryptanalysis of the Merkle–Hellman knapsack cryptosystems. This used a sigmoid function for converting the continuous into a discrete one. Here, dual keys- private and public are employed. Public key encrypted the plaintext, and private key decrypted it [36].

4. Experimental Results and Analysis

In present section, the functioning of WOA on IQA parameters when equated to images employing no optimization and images employing ANN Optimization is presented. For the purport of analysis, Colourlab Image Database: Image Quality (CID: IQ) is being deliberated [37]. The CID: IQ dataset incorporates 690 images that are degraded by different noises (originally 23 good quality images). Figure 3 depicts the five arbitrarily chosen test images of the CID: IQ database with no degradation that have been used in the present work. Apart from original 23 images, rest of the images include 6 aberrations on original images: JPEG compressing, JPEG2000 compressing, Poisson noise, Gaussian blurring, ΔE gamut mapping (DeltaE) degradation and SGCK gamut mapping (SGCK) distortion, over 5 different levels of quality degradation, from a low to a high level of quality debasement.

(1) (2) (3) (4) (5)

Figure 3 (1)-(5)

CID: IQ Database Sample Test Images

For the purpose of analysis, the following IQA metrics have been deliberated: mathematical metrics such as entropy, MSE, PSNR, Root Mean Square Error (RMSE), Percentage Fit Error (PFE), Mean Bias (MB), and, high level metrics like MSSIM.

Talking about the RMSE, Figure 4 compares the RMSE values of WOA with ANN Optimization and no optimization. Clearly, WOA results in least

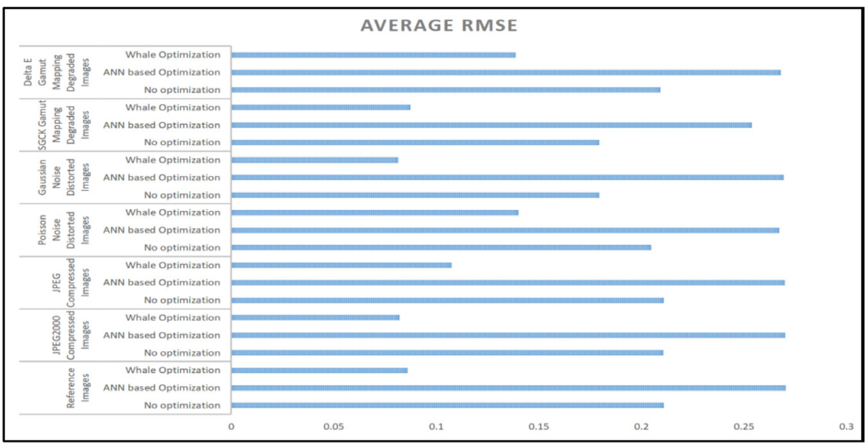

Figure 4

RMSE Curves for different degradations in CID: IQ Dataset

RMSE, with an improvement of around 48.6% over no optimization and 61.2% in case of ANN Optimization. Figure 4 shows the RMSE graphs, as an average for all the dataset images under different types of degradations.

Moving on further to the SNR/PSNR, that has been shown in Figure 5; we can observe that WOA has the highest SNR as compared to the other two techniques. The SNR/PSNR is improved by as much as 29.8% as compared to no optimization and by 40.1% when compared with ANN Optimization Figure 5

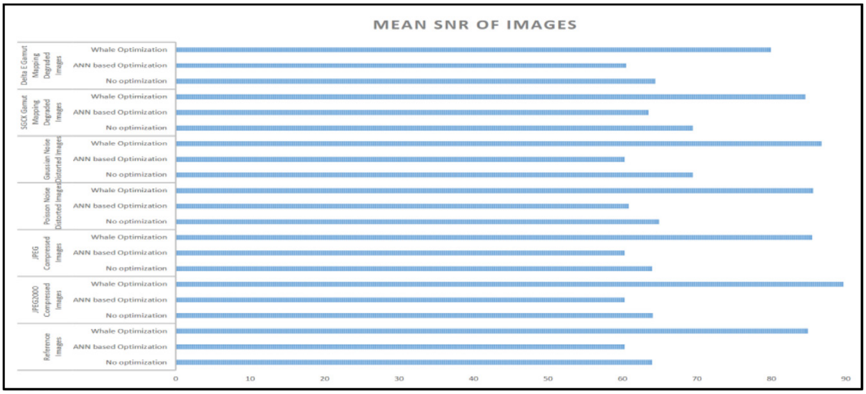

Figure 5

SNR/PSNR Curves for different degradations in CID: IQ Dataset

Next, we move onto the Mean Bias comparison. Figure 6 depicts the MB value for all five selected database images in different types of degradations. Here it can be seen that the MB for WOA is almost similar to that of ANN Optimizationand is the closest to zero. The improvement of WOA over ANN Optimization is marginal (average 3.6%). However, WOA is outperforming ANN Optimization in many different parameters significantly. WOA performs better as compared to no optimization in terms of MB by an average of 61.4%, which is very significant.

During the experimental study, it has been observed that without incorporating optimization, the metrics perform poorly. Also, because of the obvious reasons that no additional procedures are being performed in the case of no optimization, hence, it bounds to give the best results. So, we are altogether removing the no optimization scenario while talking of the total execution time. Figure 8 shows the execution times of algorithm run of the five selected images in 6 different types of degradations in a graphical manner. It can be clearly seen that WOA takes lesser time to

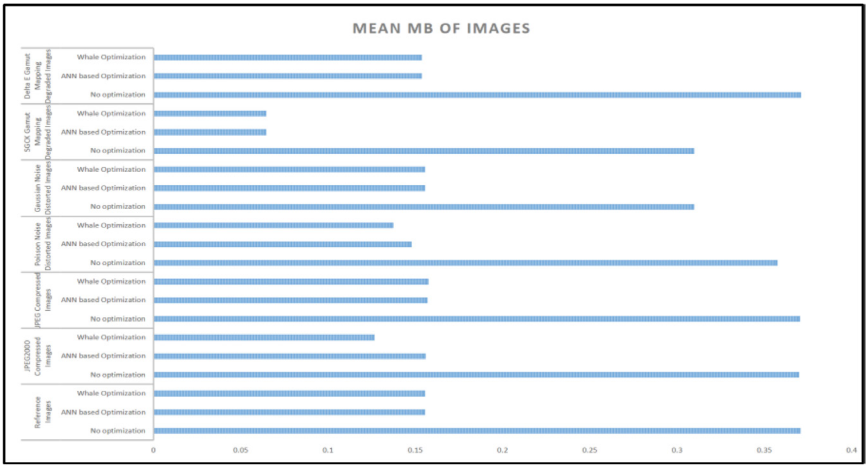

Figure 6

MB Curves for different degradations in CID: IQ Dataset

execute as compared to ANN Optimization. There is, on an average, an improvement of 25.4% in case of WOA over ANN Optimization in terms of execution time.

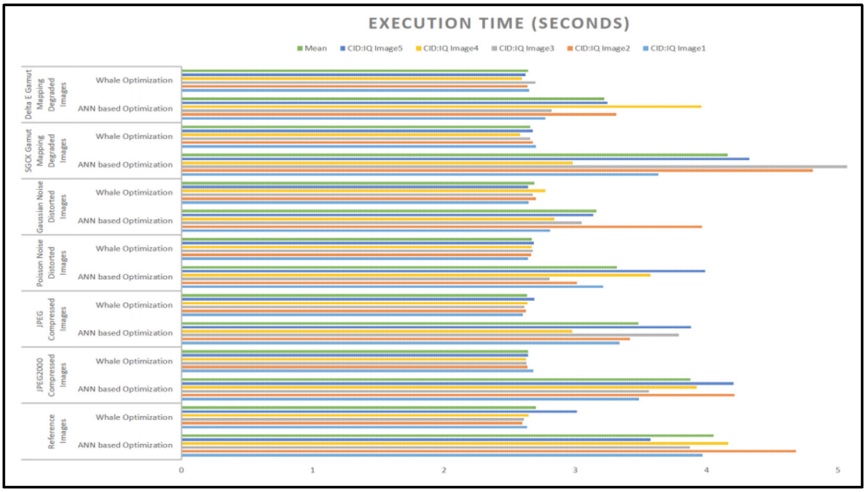

Figure 7

Execution Time Curves for different degradations in CID: IQ Dataset

5. Conclusion and Future Scope

Substantial efforts have been realized the in recent years for developing objective IQA parameters which correlate with the perceived HVS. Regrettably, the researchers have been able to achieve only limited success in this domain. In the present work, the authors have provided insights on the state-of-the-art in IQA and have also highlighted the weaknesses of conventional IQA metrics. Furthermore, the authors have included soft computing meta-heuristic methods for optimizing IQA parameters by intelligently extracting image features with the use of WOA. The experimentation was implemented on 6 different types of debasements from on CID: IQ IQA dataset and WOA results show promising performance when compared to no optimization or some ANN Optimization incorporated. The WOA performance improvement in terms of the different IQA metrics varied from as low as 13.15% to as high as 70.9%.

Despite the remarkable progressions realized in IQA, various challenges continue to prevail and the present work must be elongated further to provide for such issues. The authors did not consider compound degradations, in which images are eroded by multiple degradations simultaneously. Improvements must be encouraged in this domain. Further, the authors have deliberated the lowliest level of degradations in images of CID: IQ dataset. This present work must be expanded to validate the efficacy of WOA for hig-level consequences of debasements on image quality. The present work can also be extended in the area of Video Quality Assessment.

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