Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 44 (2023), No. 4, pp. 745–770
DOI : 10.47974/JIOS-1313

Constrained optimization of engineering design problems: Analyses with Gauss map-based chaotic particle swarm optimization

Hasan Koyuncu
Department of Electrical and Electronics Engineering
Faculty of Engineering and Natural Sciences
Konya Technical University
42250 Konya
Turkey

Abstract

Constrained optimization rises as a challenging issue concerning the evaluation of restrictions, objective and constraints of a model. For this purpose, various optimization algorithms are specifically generated or improved to achieve the best design. Performance of algorithms is strictly concerned with the search capability of the phenomena used. Herein, a state-of-the-art approach can provide worse results on constrained optimization while its performance is remarkable on a different type of optimization problem.

Many engineering design problems are categorized as constrained and nonlinear. Decision variables, constraint functions and objective function always change from one problem to another. This condition reveals the necessity of robust optimization algorithms. With this inspiration, after seeing its remarkable performance on different areas (global optimization, continuous function optimization, hybrid classifier design, etc.), this paper examines a state-of-the-art technique named Gauss map-based chaotic particle swarm optimization (GM-CPSO) on constrained optimization of engineering design problems. GM-CPSO is firstly adapted to operate for constrained optimization. Then, penalty function method is utilized to form the fitness output of optimization algorithm. Six challenging design problems are handled that are gear train design, I-shaped beam design, tension / compression spring design, three-bar truss design, tubular column design, and car side impact design. In experiments, GM-CPSO is compared with the state-of-the-art studies

---

This work is supported by the Coordinatorship of Konya Technical University's Scientific Research Projects

---

E-mail: `hkoyuncu@ktun.edu.tr`
`ORCID: 0000-0003-4541-8833`

handling the design problems. As a result, GM-CPSO achieves the best results recorded in the literature or enhances the optimum result on the specified design problem.

Subject Classification: *46N10, 62P30, 65K10, 65D15.*

Keywords: *Chaotic, Constrained optimization, Engineering design, Gauss map, Metaheuristic, Swarm intelligence.*

1. Introduction

Stochastic algorithms reserve a wide usage area in the literature, *i.e.* global function optimization, continuous function optimization, hybrid classifier design, image processing [1-4]. Constrained optimization arises as an important application area for usage of these approaches, and it is considered to enhance the common design problems in engineering. To improve the design of engineering systems, constrained optimization is frequently handled with robust stochastic algorithms for the achievement of optimum conditions.

Boudjehem and Boudjehem [5] suggested the improved heterogeneous particle swarm optimization (IHPSO) that enhances the velocity update of PSO by using a control parameter and the information difference between global and individual best positions. Herein, these concepts are utilized to enhance the exploration ability and to better keep a trade-off between exploration – exploitation. In experiments, IHPSO was prior to PSO and six efficient PSO-based algorithms on benchmark function optimization. Furthermore, IHPSO attained reliable performance on engineering design of a suspension system including six parameters. Singh et. al. [6] performed the design of an eddy current braking system by comparing PSO, experimental adjustment, and a fuzzy logic controller. According to the results, PSO overcame other approaches by yielding a better convergence on the output of system. Gandomi et. al. [7] generated cuckoo search (CS) algorithm and tested the technique on optimization of some engineering problems, *i.e.* gear train (GT) design, tubular column (TC) design, I-shaped beam (ISB) design, car side impact (CSI) design. In experiments, CS was compared with adaptive response surface method (ARSM), improved ARSM (I-ARSM), *etc.* Dong et. al. [8] utilized the modified oracle penalty method and composite differential evaluation (CoDE) to design a robust algorithm named MoCoDE. On different engineering problems like tension / compression spring (TCS) design, MoCoDE was evaluated with efficient approaches, *i.e.* co-evolutionary PSO (CPSO), co-evolutionary DE (CDE), novel modified DE (NMDE), coupled eagle strategy & DE

(ES-DE). Guedria [9] handled the accelerated PSO (APSO) and proposed the improved APSO (IAPSO) to solve the mechanical engineering problems. In analyses, IAPSO was examined with APSO and mine blast algorithm (MBA) on nonlinear optimization problems involving GT and TCS designs. Singh and Chaudhary [10] designed the modified JAVA (M-JAVA) and performed the optimization of mixed-variable problems, *i.e.* GT design, helical spring design. In trials, M-JAYA was compared with basic and improved algorithms like JAYA, PSO, evolutionary algorithm (EA), evolutionary programming (EP), genetic algorithm (GA), simulated annealing (SA), hybrid particle swarm branch and bound (HPB), modified PSO (MPSO), hybrid swarm intelligence approach (HSIA), *etc.* Talatahari and Azizi [11] suggested chaos game optimization (CGO) and evaluated the approach on constrained mathematical and engineering problems like the designs of ISB, TCS and three-bar truss (TBT) structures. Wu et. al. [12] handled the firefly algorithm (FA) and examined two derivatives: Levy-flight FA (LF-FA) and logarithmic spiral-Lévy flight FA (LS-LF-FA). Moreover, an adaptive logarithmic spiral-Levy FA (AD-IFA) was proposed and compared with FA, LF-FA and LS-LF-FA methods on optimization problems involving TBT design. Azizi et. al. [13] generated atomic orbital search (AOS) algorithm and tried to solve the engineering problems, *i.e.* designs of ISB, TBT and TC structures. On constrained optimization, Gupta et. al. [14] compared nine optimization techniques that are salp swarm algorithm (SSA), multi-verse optimizer (MVO), moth-flame optimizer (MFO), atom search optimization (ASO), ecogeography-based optimization (EBO), queuing search algorithm (QSA), equilibrium optimizer (EO), evolutionary strategy (ES), and hybrid self-adaptive orthogonal genetic algorithm (HSOGA). In experiments, nine approaches were evaluated according to their performance on mechanical design problems, *i.e.* the designs of TCS and TC structures. Zhang et. al. [15] determined JAYA algorithm and offered enhanced JAYA (EJAYA) to perform the optimization of some engineering problems including TCS and CSI designs. In trials, EJAYA was compared with 19 optimization algorithms and obtained the best result on TCS design near of the seven techniques. Braik [16] produced chameleon swarm algorithm (CSA) and evaluated it on optimization of five engineering problems involving TCS design. Fan et. al. [17] determined grey wolf optimizer (GWO) and suggested beetle antenna strategy-based GWO (BGWO) for global optimization. In analyses, BGWO was operated on constrained optimization of four engineering problems implicating TCS design. Cuong-Le et. al. [18] inspired from CS algorithm and proposed the new movement strategy of CS (NMS-CS) for global optimization. In tests with engineering design problems, NMS-CS outperformed 13 optimization approaches on TCS design. Cheng et. al. [19] considered FA algorithm and generated the hybrid FA with grouping attraction (HFA-GA) to better realize the optimization of problems. HFA-GA solved four engineering design problems including TBT and TC structures.

In the literature, various optimization approaches were generated to better solve different problems. The pursuit behind to produce new techniques, was related to design robust algorithms operating a qualified search ability. At this point, the improvements of PSO were frequently considered in the literature since this technique involves a remarkable search ability to catch a tradeoff between exploration – exploitation. Chen et. al. [20] utilized chaotic Sine map to tune the inertia weight of PSO and suggested Sine map-based chaotic PSO (SM-CPSO). Furthermore, dynamic weights were considered to enhance the position update of PSO, and dynamic weight PSO (DW-PSO) were offered. Chaotic dynamic weight PSO (CDW-PSO) were proposed by combining two improvements that are the Sine map-based inertia update and the dynamic weights-based position update. In tests on global function optimization, CDW-PSO outperformed SM-CPSO, DW-PSO, and nearly 20 optimization algorithms including basic, chaotic and hybrid ones. Herein, Sine map was selected according to the usage suggestion in krill herd (KH) algorithm. Koyuncu [21] examined 10 chaotic maps including Sine map to reveal the coherent one with the PSO dynamics. According to the experiments on global function optimization, Gauss map arose as the unique one that is coherent with PSO for different type of function optimizations. Then, Gauss map-based chaotic PSO (GM-CPSO) was compared with SM-CPSO, DW-PSO, and CDW-PSO on global function optimization. In trials, GM-CPSO surpassed SM-CPSO and DW-PSO algorithms, and its performance was nearly same with the CDW-PSO. This situation ascertained that the coherent chaotic map can change from one technique to another, and its choice should be tested to find the coherent one with the handled optimization algorithm. Besides, GM-CPSO was tested on hybrid classifier design to classify the epileptic seizure. In experiments, the classifier operating GM-CPSO obtained better scores than the classifiers consisting of SM-CPSO, DW-PSO and CDW-PSO algorithms. In another study of Koyuncu [3], GM-CPSO was compared with two state-of-the-art approaches that are CDW-PSO and black widow optimization (BWO) on continuous function optimization. In experiments, GM-CPSO yielded much better fitness values than other algorithms. For COVID-19 classification in X-Ray images, Koyuncu and Barstugan [22] utilized GM-CPSO to form an optimized classifier and designed the framework based on this classifier. Dursun et. al. [23] compared CDW-PSO and GM-CPSO techniques to design a maximum power extraction framework for permanent magnet synchronous generator (PMSG)-based wind energy conversion system (WECS). According to the experiments, the highest efficiency was observed with the framework employing GM-CPSO algorithm.

As seen in the literature studies, GM-CPSO algorithm preceded many state-of-the-art optimization methods on different tasks like continuous function optimization [3], global optimization [21], hybrid classifier design [2,21,22], efficiency-based framework design [23], *etc.* However, its performance is unknown for a problem including constrained optimization. In this study, GM-CPSO algorithm is evaluated on constrained engineering design problems. Besides, the results of the state-of-the-art studies are compared with GM-CPSO for every design problem. As a result, this paper contributes to the literature on the following subjects:

- Application of a state-of-the-art stochastic algorithm on constrained optimization
- A case study to explain the adaptation of a PSO-variant for solving the constrained problems
- A comprehensive study analyzing and explaining the compeller engineering design problems

Section 2 defines the design of GM-CPSO and its adaptation for constrained optimization. Besides, penalty function method is explained to be used in fitness evaluation to solve the constrained problems. Section 3 comprehensively determines and explains the engineering design problems, presents the experimental analyses, and briefly interprets the results. Section 4 summarizes the proposed study and concludes the paper.

2. Methods

In this section, GM-CPSO is comprehensively explained with its adaptation to handle the constrained optimization problems. Then, penalty function method is briefly determined to be used for solution of engineering design problems.

2.1 *Gauss Map-based Chaotic Particle Swarm Optimization*

Gauss map-based chaotic PSO (GM-CPSO) operates the same velocity and position (location) updates in PSO. Velocity and position arrangements can be defined as in (1) and (2) [3,21].

$$V_i(t+1) = \omega V_i(t) + c_1 r_1(t)(X_{pbest(i)}(t) - X_i(t)) + c_2 r_2(t)(X_{gbest}(t) - X_i(t)) \quad (1)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (2)$$

In (1) and (2); $V_i(t)$ and $X_i(t)$ signify the current velocity and position vectors, $V_i(t+1)$ and $X_i(t+1)$ are the new velocity and position vectors, $X_{pbest(i)}(t)$ and $X_{gbest}(t)$ add up to the personal best solution found with i^{th} particle and global best solution found in swarm, respectively. c_1 and c_2 are acceleration constants that guide the particle movements towards the local and global best solutions. Arrangement of these constants are performed according to (3) [3,21]. Besides, velocity and position values should be limited according to the problem or user choice. This restriction is performed via (4) in which velocity and position values are respectively limited in the ranges [V_{min}, V_{max}] and [X_{min}, X_{max}]. Herein, parameter k is defined as the restriction coefficient to prevent the velocity values to be more than position values. $r_1(t)$ and $r_2(t)$ are random numbers produced in the range of [0,1]. With the usage of random numbers, solution variety is upgraded [3,21]. Inertia weight is symbolized as 'ω', and its objective is to restrict the effect of $V_i(t)$ to the new velocity. In GM-CPSO, Gauss / Mouse map is utilized to arrange the inertia weight specified in (5). To apply a good chaotic change, it's advised to assign the first value ω(*0*) as equal to '0.7' [3,21]. ω(*i*) and ω(*i+1*) respectively indicate the current inertia weight and the next one. The change of Gauss map is adjusted according to the maximum iteration number [3,21].

$$c_1 + c_2 \leq 4 \quad (3)$$

$$[V_{\min}, V_{\max}] = k * [X_{\min}, X_{\max}] \quad (4)$$

$$\omega(i+1) = \begin{cases} 1 & \omega(i) = 0 \\ \frac{1}{\mathrm{mod}(\omega(i),1)} & \text{otherwise} \end{cases}, \quad \omega(0) = 0.7 \quad (5)$$

The change of Gauss map is illustrated in Figure 1 for 100 iterations. As seen in Figure 1, the output of map is zero, and this situation eliminates the usage of velocity information for the assignment of new position. For

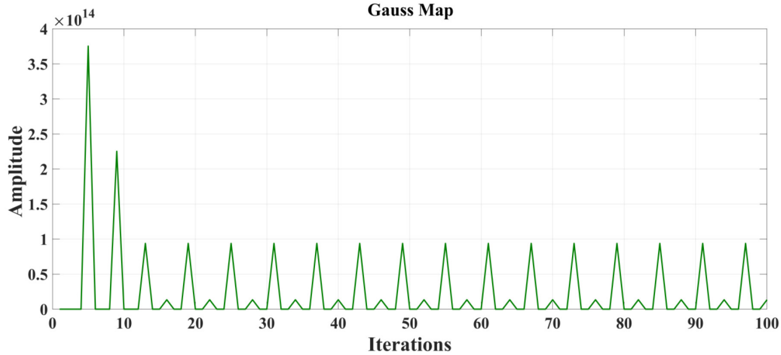

Figure 1

Gauss (Mouse) map

other situations ($\omega \neq 0$), velocity information is transferred to position update. This improvement provides diversity for solutions and upgrades the system performance by handling different viewpoints [3,21].

In the literature, many efficient PSO derivatives use (1) for velocity update, and this arrangement can operate three varieties according to the algorithm: 1-An alternating inertia weight, 2-Random number $r_1(t)$ to be multiplied with '$c_1(X_{pbest(i)}(t)-X_i(t))$', 3-Random number $r_2(t)$ to be multiplied with '$c_2(X_{gbest}(t)-X_i(t))$' [3,21]. The first item changes the effect of $V_i(t)$ into $V_i(t+1)$, and the first variety is applied with Gauss map for GM-CPSO. In (1), the second and third varieties are applied with random numbers that are diversely generated for every particle. Herein, this generation seems as a scalar value multiplied with all values stated in vectors '$c_1(X_{pbest(i)}(t)-X_i(t))$' and '$c_2(X_{gbest}(t)-X_i(t))$' [3,21].

For constrained problems, the decision variables of $X_i(t)$ can be defined and limited in different ranges. Herein, to adjust the velocity and position restrictions according to the defined problem, every variable of a particle should be restricted in X_{min} and X_{max} that are respectively the vectors holding minimum and maximum values of variables. This condition is the first step to adapt the GM-CPSO for constrained optimization problems.

The second adaptation is to determine the $r_1(t)$ and $r_2(t)$ that should not provide the same variety for all variables of the vectors '$c_1(X_{pbest(i)}(t)-X_i(t))$' and '$c_2(X_{gbest}(t)-X_i(t))$'. Herein, the usage of the same random number of all decision variables decreases the variety to be ensured. Besides, concerning the $r_1(t)$ and $r_2(t)$, parameter t signifies that their values are changed during iterations. Herein, the generation of $r_1(t)$ and $r_2(t)$ can be thought as a vector including different random values for all parameters inside $X_i(t)$. By using this idea, (1) can be transformed to (6). Position update can be defined as in (2) or in (7) which performs the same process.

$$\vec{V}_i(t+1) = \omega\vec{V}_i(t) + c_1\vec{r}_1(t)(\vec{X}_{pbest(i)}(t) - \vec{X}_i(t)) + c_2\vec{r}_2(t)(\vec{X}_{gbest}(t) - \vec{X}_i(t)) \tag{6}$$

$$\vec{X}_i(t+1) = \vec{X}_i(t) + \vec{V}_i(t+1) \tag{7}$$

In (6), random numbers are generated as a vector including different values in range of [0,1]. With this process, better convergence is fulfilled to solve the constrained optimization problems. Furthermore, (1) is not demolished and is coherent with the general velocity update. The unique difference is to interpret the second and third varieties in velocity update.

Figure 2 presents the flowchart of GM-CPSO for constrained optimization problems. In algorithm, penalty function method is utilized for fitness evaluation.

2.2 *Penalty Function Method*

Penalty function (PF) method is usually used to define the error of the fitness evaluation on constrained optimization problems. Herein, objective function stays as the main part of fitness evaluation. In addition, all constraint functions are added into the fitness evaluation according to two situations: 1-Satisfaction, 2-Violation [9,24].

As seen in the name of method, if one constraint function is not satisfied, the error of this constraint is multiplied with penalty constant which effects the fitness evaluation. If constrained function is satisfied, the error of this constraint is not considered for fitness evaluation. This process seems similar with the *punish or reward logic*. The description of PF method is as follows [9,24]:

Minimize: $f(\vec{x})$

Subject to: $g_k(\vec{x}) \le 0, \quad k = 1, \ldots, m$

where $f(x)$ and $g_k(x)$ are respectively the objective to minimize and k^{th} inequality constraint. m is the number of inequality constraints to be handled. By considering the objective function $f(x)$ and the evaluation of constraint functions $g_k(x)$, the combined function (penalized objective function) is revealed that transforms the constrained problem into the unconstrained one. Penalized objective function is shown as $f_p(x)$ and defined in (8) [9,24].

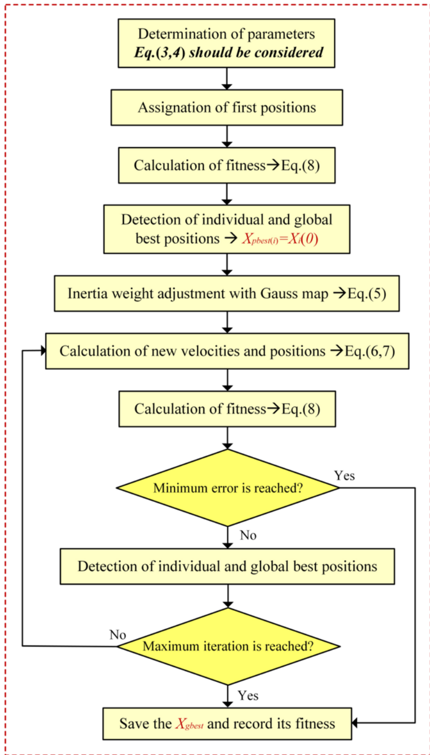

Figure 2

Flowchart of GM-CPSO on constrained optimization

$$f_p(\vec{x}) = f(\vec{x}) + \lambda \sum_{k=1}^{m} \delta_k [g_k(\vec{x})]^2, \quad \lambda > 0 \quad \begin{cases} \delta_k = 1, & \text{if constraint } g_k \text{ is violated} \\ \delta_k = 0, & \text{if constraint } g_k \text{ is satisfied} \end{cases} \tag{8}$$

In (8), λ (penalty constant) is generally chosen as the high degreed fold of 10. In this study, we choose it as equal to 10^{15} according to the advice in the study of Guedria [9]. This arrangement is kept fixed for all engineering problems.

3. Experimental Analyses and Interpretations

In this study, a state-of-the-art algorithm is evaluated for optimization of six challenging engineering problems: gear train design, I-shaped beam design, tension / compression spring design, three-bar truss design, tubular column design, and car side impact design. Table 1 presents the parameter settings of GM-CPSO for every design problem. Independent

Table 1

Parameter settings of GM-CPSO for engineering design problems

Design Problems / Parameters	NFE	c_1/c_2	k
Gear train	2046	2 / 2	0.05
I-shaped beam	8540	2 / 2	0.1
Tension / Compression spring	79400	1.8 / 2.2	0.21
Three-bar truss	223041	2 / 2	0.00005
Tubular column	53500	2 / 2	0.05
Car side impact	76000	0.5/3.5	0.1

(NFE: Number of fitness evaluations = population size × maximum iteration number)

run is chosen as 100 for all trials, and the number of fitness evaluation (NFE) is diversely adjusted for every problem according to the necessity of design complexity.

3.1 *Gear train design problem*

Gear train (GT) design aims to minimize the gear ratio by changing the teeth number (T_i) of gearwheels i. x_1 (T_d), x_2 (T_b), x_3 (T_a) and x_4 (T_f) respectively signify the teeth number of gear D, B, A and F. Gear ratio is calculated as $(x_1 \times x_2)/(x_3 \times x_4)$ and forms the main part of objective to minimize [7,9,10]. The decision variables / parameters consist of integer values, and among iterations of optimization algorithm, these parameters should be rounded before the fitness evaluation. GT design is illustrated in Figure 3. The definition of decision variables, the objective to minimize and restrictions can be defined as follows for GT design [7]:

$$\textit{Consider: } \vec{x} = [x_1, x_2, x_3, x_4] = [T_d, T_b, T_a, T_f]$$

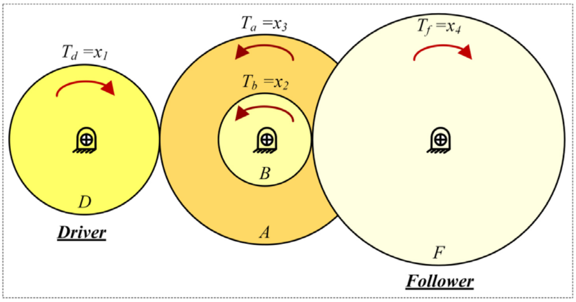

Figure 3

Gear train design problem

Table 2

Comparison of the best solutions of literature studies for GT design

Algorithm	***Study***	x_1	x_2	x_3	x_4	**Optimum Result**
CS	Gandomi et. al. [7]	16	19	43	49	*2.700857e-12*
MBA	Guedria [9]	16	19	43	49	*2.700857e-12*
APSO		16	19	43	49	*2.700857e-12*
IAPSO		16	19	43	49	*2.700857e-12*
SA	Singh and Chaudhary [10]	30	15	52	60	2.36e-6
EP		30	15	52	60	2.36e-6
HSIA		16	19	43	49	*2.700857e-12*
MPSO		16	19	43	49	*2.700857e-12*
JAYA		16	19	43	49	*2.700857e-12*
M-JAYA		16	19	43	49	*2.700857e-12*
GM-CPSO	This study	16	19	43	49	*2.700857e-12*

Objective to minimize: $f(\vec{x}) = \left(\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}\right)$

Where $12 \le x_i \le 60$

Table 2 presents the comparison of optimum results of various algorithms for GT design. As seen in Table 2, GM-CPSO achieves the best result observed in the literature along with the eight efficient approaches. Herein, it's seen that decision parameters (x_1,x_2,x_3,x_4) stay same for all techniques obtaining the best score.

3.2 I-shaped beam design problem

I-shaped beam (ISB) design aims the minimization of vertical deflection of an I-shaped beam. ISB problem evaluates both the cross-sectional area and the constraints under a specific load. ISB design is considered using four decision variables which are x_1 (b), x_2 (h), x_3 (t_w) and x_4 (t_f) that respectively signify the width of flanges, height of web, thickness of web and thickness of flanges. Moreover, two inequality design constraints ($g_1(x)$, $g_2(x)$) are evaluated beside the objective to minimize. ISB design is presented in Figure 4. The definition of decision parameters,

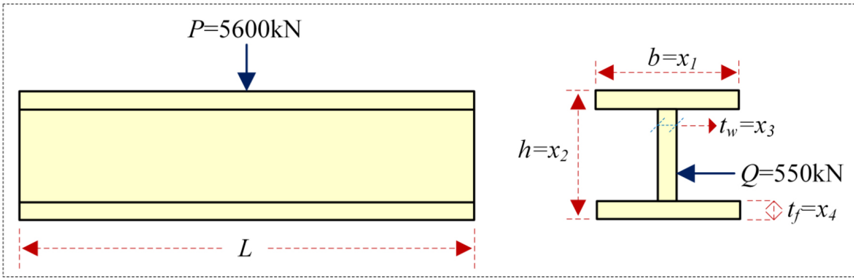

Figure 4

I-shaped beam design problem

the objective to minimize, inequality constraints and restrictions can be defined as follows for ISB design [7,11,13]:

$$\textit{Consider: } \vec{x} = [x_1, x_2, x_3, x_4] = [b, h, t_w, t_f]$$

$$\textit{Objective to minimize: } f(\vec{x}) = \frac{5000}{\frac{x_3(x_2-2x_4)^3}{12} + \frac{x_1 x_4^3}{6} + 2x_1 x_4 \left(\frac{x_2-x_4}{2}\right)^2}$$

$$\textit{Subject to: } g_1(\vec{x}) = 2x_1x_3 + x_3(x_2 - 2x_4) - 300 \le 0$$

$$g_2(\vec{x}) = \frac{180000x_2}{x_3(x_2-2x_4)^3 + 2x_1x_3(4x_4^2 + 3x_2(x_2-2x_4))} + \frac{15000x_1}{(x_2-2x_4)x_3^3 + 2x_3x_1^3} - 6 \le 0$$

$$\textit{Where } 10 \le x_1 \le 50,\ 10 \le x_2 \le 80,\ 0.9 \le x_3 \le 5,\ 0.9 \le x_4 \le 5$$

Table 3 shows the comparison of optimum results of different algorithms for ISB design.

Table 3

Comparison of the best solutions of literature studies for ISB design

Algorithm	***Study***	x_1	x_2	x_3	x_4	$g_1(x)$	$g_2(x)$	**Optimum Result**
ARSM	Gandomi et. al. [7]	37.05	80	1.71	2.31	*NA*	*NA*	0.0157
I-ARSM		48.42	79.9	0.9	2.4	*NA*	*NA*	0.131
CS		50	80	0.9	2.3217	*NA*	*NA*	0.0130747
CGO	Talatahari and Azizi [11]	50	80	0.9	2.3218	*NA*	*NA*	0.01307412

Contd...

AOS	Azizi et. al. [13]	50	80	0.9	2.3218	*NA*	*NA*	0.01307412
GM-CPSO	This study	50	80	1.7647	5	-6.0609e-06	-1.8163	*0.00662596*

As seen in Table 3, GM-CPSO reveals the best score observed among the state-of-the-art literature studies. In variable evaluation of GM-CPSO, it's seen that the change of two decision parameters (x_3,x_4) yields better objective value for the obtainment of the best score. In other words, the change of thicknesses about web and flanges, provide a sharp decrease on fitness evaluation.

3.3 *Tension / Compression spring design problem*

Tension / Compression spring (TCS) design aims the minimization of tension weight under some constraints which are shear stress, surge frequency and minimum deflection. There exist three variables to arrange in TCS design. x_1 (d), x_2 (D) and x_3 (P) respectively indicate the wire width / diameter, the mean winding diameter and the number of active windings. Furthermore, four inequality design constraints ($g_1(x)$, $g_2(x)$, $g_3(x)$, $g_4(x)$) are considered alongside the minimization of objective function ($f(x)$). TCS design is shown in Figure 5. The definition of decision variables, objective function, inequality constraints and restrictions are as follows for TCS design [8,9,11]:

$$\textit{Consider: } \vec{x} = [x_1, x_2, x_3] = [d, D, P]$$

$$\textit{Objective to minimize: } f(\vec{x}) = (x_3 + 2)x_2x_1^2$$

$$\textit{Subject to: } g_1(\vec{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \le 0$$

$$g_3(\vec{x}) = 1 - \frac{140.45x_1}{x_2^2 x_3} \le 0$$

$$g_4(\vec{x}) = \frac{x_2 + x_1}{1.5} - 1 \le 0$$

$$\textit{Where } 0.05 \le x_1 \le 2,\ 0.25 \le x_2 \le 1.3,\ 2 \le x_3 \le 15$$

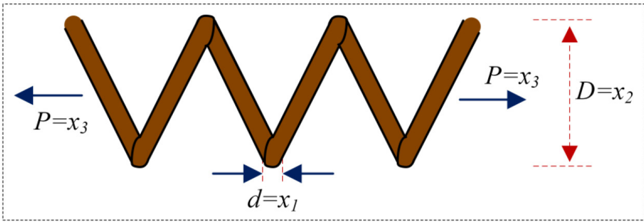

Figure 5

Tension / Compression spring design problem

Table 4 shows the optimum result comparison of literature algorithms for TCS design. As seen in Table 4, GM-CPSO succeeds the best result among the significant approaches. Herein, it's revealed that the change at decision variables, especially the change at number of active windings (x_3), is crucial for achievement of the best objective value.

3.4 *Three-bar truss design problem*

Three-bar truss (TBT) design is a nonlinear fractional programming problem and can be summarized as the design of a 3-bar planar truss. In TBT problem, every truss member is subject to stress constraints, and the minimization is considered for the volume of a loaded 3-bar truss. Two decision variables are declared that are respectively the cross sectional areas for oblique and straight bars: x_1 (A_1) and x_2 (A_2). Besides, three inequality design constraints ($g_1(x)$, $g_2(x)$, $g_3(x)$) are determined with the objective function to be minimized. TBT design is illustrated in Figure 6. The definition of decision parameters, objective to minimize, inequality constraints and restrictions are as follows for TBT design [7,11,13,19]:

Consider: $\vec{x} = [x_1, x_2] = [A_1, A_2]$

Objective to minimize: $f(\vec{x}) = 100(2\sqrt{2}x_1 + x_2)$

Subject to: $g_1(\vec{x}) = 2\dfrac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} - 2 \le 0$

$$g_2(\vec{x}) = 2\frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} - 2 \le 0$$

$$g_3(\vec{x}) = 2\frac{1}{\sqrt{2}x_2 + x_1} - 2 \le 0$$

Where $0 \le x_1, x_2 \le 1$

Table 4
Comparison of the best solutions of literature studies for TCS design

Algorithm	***Study***	x_1	x_2	x_3	$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$	Optimum Result
MoCoDE	Dong et. al. [8]	0.0517	0.3574	11.2480	-6.0916e-8	-4.4751e-7	–4.0552	–0.7272	0.012665
IAPSO		0.0517	0.3566	11.2942	-1.9e-10	-4.64e-10	–4.0536	–1.0917	0.012665
CGO	Talatahari and Azizi [11]	0.0517	0.3561	11.3266	-4.92e-9	-2.98e-8	–4.0525	–0.7282	0.012665
SSA	Gupta et. al. [14]	0.0513	0.3472	11.8708	*NA*	*NA*	*NA*	*NA*	0.012668
MVO		0.0512	0.3448	12.0476	*NA*	*NA*	*NA*	*NA*	0.012693
MFO		0.0509	0.3371	12.5418	*NA*	*NA*	*NA*	*NA*	0.012678
ASO		0.0527	0.3823	9.9346	*NA*	*NA*	*NA*	*NA*	0.012684
EBO		0.0534	0.3977	9.2519	*NA*	*NA*	*NA*	*NA*	0.012737
QSA		0.0517	0.3567	11.2900	*NA*	*NA*	*NA*	*NA*	0.012665
EO		0.0513	0.3480	11.8220	*NA*	*NA*	*NA*	*NA*	0.012667
ES		0.0520	0.3632	10.9199	*NA*	*NA*	*NA*	*NA*	0.012666
HSOGA		0.0518	0.3600	11.0969	*NA*	*NA*	*NA*	*NA*	0.012665
EJAYA	Zhang et. al. [15]	0.0517	0.3580	11.2130	*NA*	*NA*	*NA*	*NA*	0.012665
CSA	Braik [16]	0.0518	0.3589	11.1650	*NA*	*NA*	*NA*	*NA*	0.012665
BGWO	Fan et. al. [17]	0.0516	0.3551	11.3846	*NA*	*NA*	*NA*	*NA*	0.012665
NMS-CS	Cuong-Le et. al. [18]	0.0516	0.3567	11.2890	*NA*	*NA*	*NA*	*NA*	0.012665
GM-CPSO	This study	0.05	0.3744	8.5512	-3.0847e-06	-1.6942e-04	-4.8597	-0.7171	*0.009875*

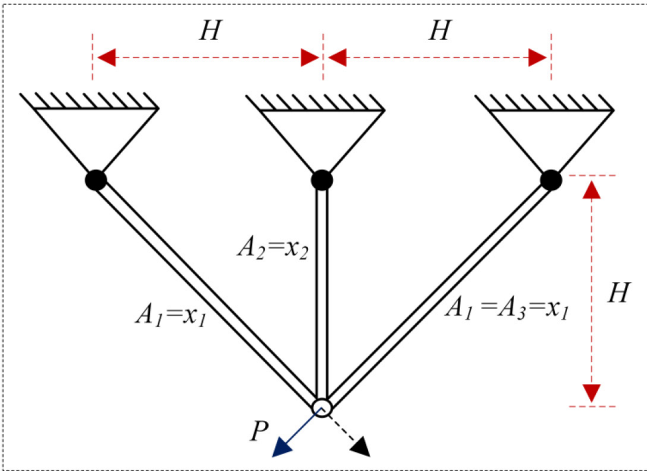

Figure 6

Three-bar truss design problem

Table 5 reveals the optimum results of the literature studies for TBT design.

Table 5

Comparison of the best solutions of literature studies for TBT design

Algorithm	*Study*	x_1	x_2	$g_1(x)$	$g_2(x)$	$g_3(x)$	Optimum Result
CS	Gandomi et. al. [7]	0.7887	0.4090	-0.00029	-0.26853	-0.73176	263.9716
CGO	Talatahari and Azizi [11]	0.7887	0.4083	0	–1.4641	–0.5359	*263.8958*
FA	Wu et. al. [12]	*NA*	*NA*	*NA*	*NA*	*NA*	282.84
LF-FA		*NA*	*NA*	*NA*	*NA*	*NA*	282.84
LS-LF-FA		*NA*	*NA*	*NA*	*NA*	*NA*	282.84
AD-IFA		*NA*	*NA*	*NA*	*NA*	*NA*	282.84
AOS	Azizi et. al. [13]	0.7887	0.4083	0	–1.4641	–0.5359	*263.8958*
HFA-GA	Cheng et. al. [19]	0.7887	0.4083	*NA*	*NA*	*NA*	*263.8958*
GM-CPSO	This study	0.7887	0.4082	-1.0841e-09	-1.4642	-0.5358	*263.8958*

As seen in Table 5, GM-CPSO succeeds the best result beside three efficient algorithms. If a detailed analysis is performed about the decision parameters of CS *vs* others, it's seen that the small change of x_2 can dramatically change the float part of the best result. This effect is not only observed on the output of objective function, but also revealed on constraints $g_1(x)$ and $g_2(x)$. This situation also proves the significance of technique selection for better convergence.

3.5 *Tubular column design problem*

Tubular column (TC) design aims to minimize the costs related to material and construction of a tubular column loaded with P=2500 kgf. The column properties should perform a yield stress (σ_y) of 500 kgf/cm^2, elasticity modulus (E) of 850000 kgf/cm^2, density (ρ) of 0.0025 kgf/cm^3, and the length (L) of 250 cm. The stress included in the column should be kept lower than the buckling and yield stresses (constraints $g_1(x)$ and $g_2(x)$). The average column diameter is limited within the range [2 cm, 14 cm] (constraints $g_3(x)$ and $g_4(x)$), and the thickness of columns should be in the range of [0.2 cm, 0.8 cm] (constraints $g_5(x)$ and $g_6(x)$). Two decision parameters are specified that are the average column diameter 'x_1 (d)' and average column thickness 'x_2 (t)'. TC design is presented in Figure 7. The definition of decision variables, objective function, inequality constraints and restrictions are as follows for TC design [7,13,14,19]:

Consider: $\vec{x} = [x_1, x_2] = [d, t]$

Objective to minimize: $f(\vec{x}) = 9.8x_1x_2 + 2x_1$

Subject to: $g_1(\vec{x}) = \frac{2500}{500\pi x_1 x_2} - 1 \le 0$

$$g_2(\vec{x}) = \frac{125\times10^7}{850000\pi^3 x_1 x_2 (x_1^2 + x_2^2)} - 1 \le 0$$

$$g_3(\vec{x}) = \frac{2}{x_1} - 1 \le 0$$

$$g_4(\vec{x}) = \frac{x_1}{14} - 1 \le 0$$

$$g_5(\vec{x}) = \frac{0.2}{x_2} - 1 \le 0$$

$$g_6(\vec{x}) = \frac{x_2}{0.8} - 1 \le 0$$

Where $2 \le x_1 \le 14,\ 0.2 \le x_2 \le 0.8$

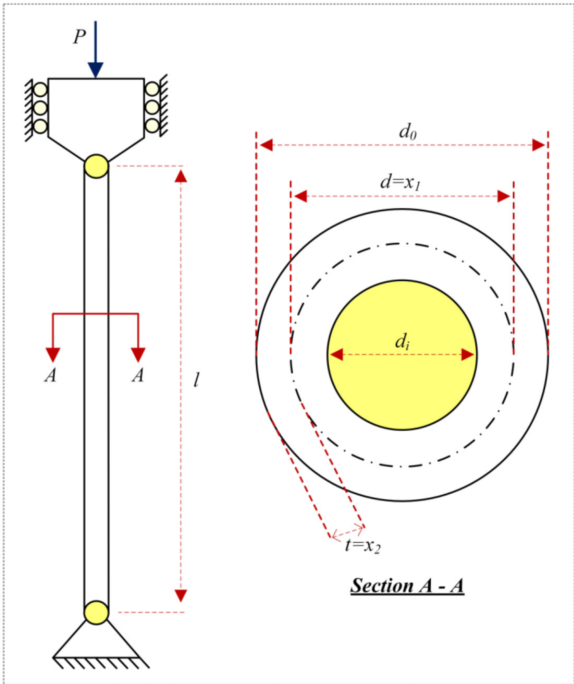

Figure 7

Tubular column design problem

Table 6 reveals the optimum results of the literature studies for TC design.

As seen in Table 6, GM-CPSO achieves the best result among the state-of-the-art techniques for TC design. If the decision parameters of GM-CPSO *vs* others are examined in detail, it's revealed that a small variety on variables can effect positively the cost of system. During the minimization of objective function, the most important part of solution is to prevent a violation among inequality constraints which is directly related to the techniques' search ability.

3.6 *Car side impact design problem*

Car side impact (CSI) design aims to minimize the weight of a car by considering the vehicle crash safety design of side impact. For CSI design, eleven variables are adjusted that are the width of B-Pillar inner (x_1), width of B-Pillar reinforcement (x_2), width of floor side inner (x_3), width of cross members (x_4), width of door beam (x_5), width of door beltline

Table 6

Comparison of the best solutions from literature for TC design

Algorithm	***Study***	x_1	x_2	$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$	$g_5(x)$	$g_6(x)$	Optimum Result
CS	Gandomi et. al. [7]	5.4514	0.2920	-0.0241	-0.1095	-0.6331	-0.6106	-0.3150	-0.6351	26.5325
AOS	Azizi et. al. [13]	5.4512	0.2920	-3.64e-06	-2.47e-06	-0.6331	-0.6106	-0.3150	-0.6350	26.5314
SSA	Gupta et. al. [14]	5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
MVO		5.4513	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5323
MFO		5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
ASO		5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
EBO		5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
QSA		5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
EO		5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
ES		5.4855	0.2901	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
HSOGA		5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
HFA-GA	Cheng et. al. [19]	5.4512	0.2920	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	26.5313
GM-CPSO	This study	5.4538	0.2918	-2.25e-05	-9.72e-04	-0.6333	-0.6105	-0.3147	-0.9635	*26.5051*

reinforcement (x_6), width of roof rail (x_7), B-Pillar inner materials (x_8), floor side inner materials (x_9), height of the barrier (x_{10}) and barrier hitting position (x_{11}). x_7 and x_8 signify the parameters of materials and can own two values which are 0.192 and 0.345 respectively for mild steel and high strength steel. In summary, thickness based parameters ($x_1,\ldots,x_7$), material based parameters (x_8 and x_9), and noise based parameters (x_{10} and x_{11}) are evaluated to achieve the best weight. There exist ten constraints that are abdomen load '$g_1(x)$', model upper chest '$g_2(x)$', model middle chest '$g_3(x)$', model lower chest '$g_4(x)$', rib deflection at upper level '$g_5(x)$', rib deflection at middle level '$g_6(x)$', rib deflection at lower level '$g_7(x)$', pubic symphysis force '$g_8(x)$', velocity of B-Pillar at middle point '$g_9(x)$' and velocity of front door at B-Pillar '$g_{10}(x)$'. CSI design is shown in Figure 8 [25]. The definition of decision parameters, objective function to be minimized, inequality constraints and restrictions are defined as follows for CSI design [7,15]:

$$\textit{Consider: } \vec{x} = [x_1, x_2, ..., x_{11}]$$

$$\begin{aligned}\textit{Objective to minimize}: \ f(\vec{x}) = {} & 1.98 + 4.9x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 \\ & + 1.78x_5 + 2.73x_7\end{aligned}$$

$$\begin{aligned}\textit{Subject to}: \ g_1(\vec{x}) = {} & 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 \\ & + 0.01343x_6x_{10} - 1kN \le 0\end{aligned}$$

$$\begin{aligned}g_2(\vec{x}) = {} & 0.261 - 0.0159x_1x_2 - 0.0188x_1x_8 - 0.0191x_2x_7 + 0.0144x_3x_5 \\ & + 0.0008757x_5x_{10} + 0.08045x_6x_9 + 0.00139x_8x_{11} \\ & + 0.00001575x_{10}x_{11} - 0.32m/s \le 0\end{aligned}$$

$$\begin{aligned}g_3(\vec{x}) = {} & 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 \\ & - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 \\ & + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} + 0.00121x_8x_{11} \\ & - 0.32m/s \le 0\end{aligned}$$

$$\begin{aligned}g_4(\vec{x}) = {} & 0.74 - 0.61x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 \\ & + 0.227x_2^2 - 0.32m/s \le 0\end{aligned}$$

$$\begin{aligned}g_5(\vec{x}) = {} & 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 \\ & + 0.32x_9x_{10} - 32mm \le 0\end{aligned}$$

$$g_6(\vec{x}) = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22x_8x_9 - 32mm \le 0$$

$$g_7(\vec{x}) = 46.36 - 9.9x_2 - 12.9x_1x_8 - 5.057x_1x_2 + 0.1107x_3x_{10} - 32mm \le 0$$

$$g_8(\vec{x}) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 - 4kN \le 0$$

$$g_9(\vec{x}) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10} - 9.9mm/ms \le 0$$

$$g_{10}(\vec{x}) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11}^2 - 15.7mm/ms \le 0$$

Where $0.5 \le x_1, x_2, ..., x_7 \le 1.5,\ x_8, x_9 \in \{0.192, 0.345\},\ -30 \le x_{10}, x_{11} \le 30$

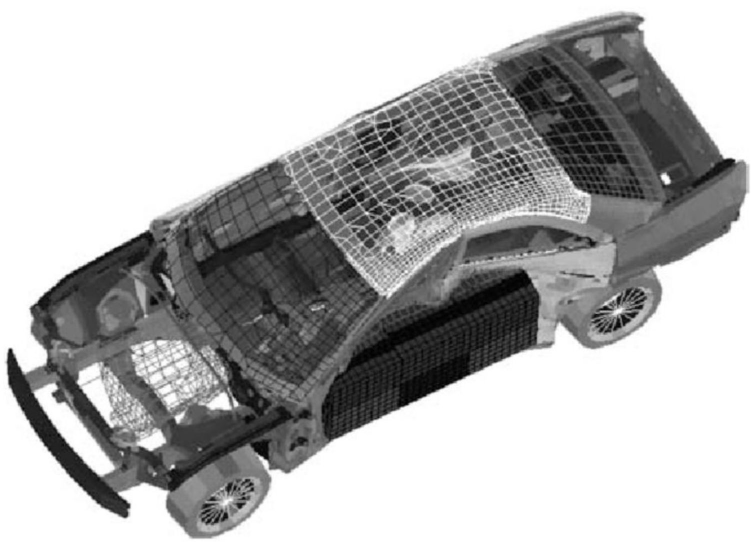

Figure 8

Car side impact design problem [25]

Table 7 reveals the optimum results of the literature studies for CSI design.

As seen in Table 7, the least weight value is observed with the usage of GM-CPSO among the state-of-the-art techniques of the literature. If

Table 7

Comparison of the best solutions from literature for CSI design

Parameters / Outputs	*Study*		
	Gandomi et. al. [7]	**Zhang et. al. [15]**	**This study**
	CS	**EJAYA**	**GM-CPSO**
x_1	0.50000	0.50000000	0.63202922
x_2	1.11643	1.11631315	0.91094701
x_3	0.50000	0.50000000	0.50000000
x_4	1.30208	1.30228464	1.30000000
x_5	0.50000	0.50000022	0.50000000
x_6	1.50000	1.49999999	1.50000000
x_7	0.50000	0.50000006	0.50000000
x_8	0.34500	0.34499999	0.34500000
x_9	0.19200	0.32679979	0.34500000
x_{10}	-19.54935	–19.570927	5.99639496
x_{11}	-0.00431	0.00837595	10.27321227
$g_1(x)$	*NA*	*NA*	-0.29372625
$g_2(x)$	*NA*	*NA*	-0.02719808
$g_3(x)$	*NA*	*NA*	-0.08816698
$g_4(x)$	*NA*	*NA*	-3.66245e+04
$g_5(x)$	*NA*	*NA*	-0.70228984
$g_6(x)$	*NA*	*NA*	-1.12649427
$g_7(x)$	*NA*	*NA*	-0.05086406
$g_8(x)$	*NA*	*NA*	-0.00761029
$g_9(x)$	*NA*	*NA*	-0.16180746
$g_{10}(x)$	*NA*	*NA*	-0.19514406
Optimum Result	22.84294	22.8429707	*22.110960*

an in-depth analysis is performed among parameters values achieved by algorithms, it's seen that the change among two thickness-based parameters (x_1,x_2), one material-based parameter (x_9) and two noise-based parameters (x_{10},x_{11}) provides the minimization of weight in comparison with the values recorded by GM-CPSO *vs* other techniques. Besides, CSI design involves more variables than other design problems. Concerning this, GM-CPSO proves its efficiency again for design problems operating both high or low numbered variables. In brief, GM-CPSO enhances the result of CSI design problem according to the literature results.

4. Conclusions

This paper presents a comprehensive study about the usage of a state-of-the-art algorithm on constrained optimization, presents the comprehensive determination of engineering design problems that constitutes constrained optimization, and contributes to the literature about the optimum results for the specified design problems.

In this paper, design problems are comprehensively defined concerning the model parameters and their transform to mathematical phenomena. Furthermore, the usage of GM-CPSO is adapted to solve constraint optimization problems, and this process is explicitly presented. For four engineering designs (ISB, TCS, TC, CSI), GM-CPSO changes the best (optimum) results of the literature, and it obtains the same-best results recorded in the literature for two design problems (GT, TBT). As seen in experiments, GM-CPSO attains remarkable scores on engineering design problems and proves its efficiency for the issue of constrained optimization in addition to the previous successes on different issues, *i.e.* continuous function optimization, global optimization, hybrid classifier design [2,3,21,22].

For future work, some specific problems will be designed concerning the area of electrical and electronics engineering. Regarding this, GM-CPSO will be utilized to solve the aforementioned problems utilizing the adaption presented in this paper.

Acknowledgement

This work is supported by the Coordinatorship of Konya Technical University's Scientific Research Projects.

References

[1] H. Koyuncu and R. Ceylan, "Multithresholding of benchmark images by a novel optimization approach", in: *Procs of the 2018 IEEE 13th*

International Scientific and Technical Conference on Computer Sciences and Information Technologies (*CSIT*), pp. 322-325 (2018).

[2] H. Koyuncu, M. Barstuğan, and M. Ü. Öziç, "A comprehensive study of brain tumour discrimination using phase combinations, feature rankings, and hybridised classifiers", *Medical & Biological Engineering & Computing*, vol. 58, no. 12, pp. 2971-2987 (2020).

[3] H. Koyuncu, "A detailed study about CDW-PSO, BWO and GM-CPSO methods on continuous function optimization", *Journal of Information and Optimization Sciences*, vol. 42, no. 4, pp. 753-772 (2021).

[4] A. Khajeh, M. R. Ghasemi, and H. G. Arab, "Modified particle swarm optimization with novel population initialization", *Journal of Information and Optimization Sciences*, vol. 40, no. 6, pp. 1167-1179 (2019).

[5] D. Boudjehem and B. Boudjehem, "Improved heterogeneous particle swarm optimization", *Journal of Information and Optimization Sciences*, vol. 38, no. (3-4), pp. 481-499 (2017).

[6] A. K. Singh, I. Nasiruddin, A. K. Sharma, and A. Saxena, "Implicit control of eddy current braking system using fuzzy logic controller (FLC) and particle swarm optimisation (PSO)", *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 22, no. 2, pp. 253-275 (2019).

[7] A. H. Gandomi, X. S. Yang, and A. H. Alavi, "Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems", *Engineering with Computers*, vol. 29, no. 1, pp. 17-35 (2013).

[8] M. Dong, N. Wang, X. Cheng, and C. Jiang, "Composite differential evolution with modified oracle penalty method for constrained optimization problems", *Mathematical Problems in Engineering*, vol. 2014, pp. 1-15 (2014).

[9] N. B. Guedria, "Improved accelerated PSO algorithm for mechanical engineering optimization problems", *Applied Soft Computing*, vol. 40, pp. 455-467 (2016).

[10] P. Singh and H. Chaudhary, "A modified Jaya algorithm for mixed-variable optimization problems", *Journal of Intelligent Systems*, vol. 29, no. 1, pp. 1007-1027 (2020).

[11] S. Talatahari and M. Azizi, "Optimization of constrained mathematical and engineering design problems using chaos game optimization", *Computers & Industrial Engineering*, vol. 145, pp. 106560 (2020).

[12] J. Wu, Y. G. Wang, K. Burrage, Y. C. Tian, B. Lawson, and Z. Ding, "An improved firefly algorithm for global continuous optimization problems", *Expert Systems with Applications*, vol. 149, pp. 113340 (2020).

[13] M, Azizi, S. Talatahari, and A. Giaralis, "Optimization of engineering design problems using atomic orbital search algorithm", *IEEE Access*, vol. 9, pp. 102497-102519 (2021).

[14] S. Gupta, H. Abderazek, B. S. Yıldız, A. R. Yildiz, S. Mirjalili, and S. M. Sait, "Comparison of metaheuristic optimization algorithms for solving constrained mechanical design optimization problems", *Expert Systems with Applications*, vol. 183, pp. 115351 (2021).

[15] Y. Zhang, A. Chi, and S. Mirjalili, "Enhanced Jaya algorithm: A simple but efficient optimization method for constrained engineering design problems", *Knowledge-Based Systems*, vol. 233, pp. 107555 (2021).

[16] M. S. Braik, "Chameleon Swarm Algorithm: A bio-inspired optimizer for solving engineering design problems", *Expert Systems with Applications*, vol. 174, pp. 114685 (2021).

[17] Q. Fan, H. Huang, Y. Li, Z. Han, Y. Hu, and D. Huang, "Beetle antenna strategy based grey wolf optimization", *Expert Systems with Applications*, vol. 165, pp. 113882 (2021).

[18] T. Cuong-Le, H. L. Minh, S. Khatir, M. A. Wahab, M. T. Tran, and S. Mirjalili, "A novel version of Cuckoo search algorithm for solving optimization problems", *Expert Systems with Applications*, vol. 186, pp. 115669 (2021).

[19] Z. Cheng, H. Song, J. Wang, H. Zhang, T. Chang, and M. Zhang, "Hybrid firefly algorithm with grouping attraction for constrained optimization problem", *Knowledge-Based Systems*, vol. 220, pp. 106937 (2021).

[20] K. Chen, F. Zhou, and A. Liu, "Chaotic dynamic weight particle swarm optimization for numerical function optimization", *Knowledge-Based Systems*, vol. 139, pp. 23-40 (2018).

[21] H. Koyuncu, "GM-CPSO: A new viewpoint to chaotic particle swarm optimization via Gauss map", *Neural Processing Letters*, vol. 52, no. 1, pp. 241-266 (2020).

[22] H. Koyuncu and M Barstuğan, "COVID-19 discrimination framework for X-ray images by considering radiomics, selective information, feature ranking, and a novel hybrid classifier", *Signal Processing: Image Communication*, vol. 97, pp. 116359 (2021).

[23] E. H. Dursun, H. Koyuncu, and A. A. Kulaksiz, "A novel unified maximum power extraction framework for PMSG based WECS using chaotic particle swarm optimization derivatives", *Engineering Science and Technology, an International Journal*, vol. 24, no. 1, pp. 158-170 (2021).

[24] Z. Michalewicz, "A survey of constraint handling techniques in evolutionary computation methods", *Evolutionary Programming*, vol. 4, pp. 135-155 (1995).

[25] B. D. Youn, K. K. Choi, R. J. Yang, and L. Gu, "Reliability-based design optimization for crashworthiness of vehicle side impact", *Structural and Multidisciplinary Optimization*, vol. 26, no. 3, pp. 272-283 (2004).

Received June, 2022

Revised September, 2022