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Analytical solutions and numerical simulation of COVID-19 fractional order mathematical model by Caputo and conformable fractional differential transform method

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Abstract

In this paper, we will discuss an analytical solution and numerical simulation of fractional order mathematical model on COVID-19 under Caputo and conformable sense with the help of fractional differential transform method for different values of q, where $q \in (0,1]$. The underlying mathematical model on COVID-19 consists of four compartments, like, the susceptible class, the healthy class,the infected class and the quarantine class. We show the reliability and simplicity of the methods by comparing the solution of given model obtained by FDTM with the solution obtained by CFDTM graphically and numerically.

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Further, we analyse the stability of model using Lyapunov direct method under Caputo sense. We conclude that the use of fractional epidemic model provides better understanding and biologically deeper insights about the disease dynamics.

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1. Introduction

The idea of models on the Corona virus disease has been discussed by some authors using the various methods : Adams-Moulton type, Laplace transform coupled with Adomain decomposition method, generalized -Bashforth-Moulton method, q-homotopy analysis transform method which can be found in [1, 2, 3, 4, 5]. The differential transform method was applied in different fields by many researchers, as can be seen in [6, 7]. In this paper, we will study a new application of the fractional differential transform method to obtain approximate solutions for the system of fractional order mathematical model on COVID-19 (1.1). For the concept of Caputo and conformable fractional derivatives, the reader can refer to [8, 9, 10, 11].

Vijander Singh et. al. [13], evaluate and forecast the registered, deceased, deaths and death counts per reported case using Corona-19's world health statistics for the world population. They believe that COVID-19's regular mortality is strongly connected with the number of verified cases and that an emergency scenario may arise before a suitable vaccine is developed. They also estimate that the number of COVID-19 cases in Western Europe and Spain will rise in the following weeks and they provide some recommendations for reducing COVID-19 transmission among humans. Vaibhav Bhatnagar et. al. [14], addressed a descriptive study of COVID-19 with a specific focus on India and they analyzed different attributes such as age, gender, travel history, communication method and current status. They also discovered a strong link between gender and the patient's transmission Type. The Conformable Fractional Differential Transform Method (CFDTM) is used to obtain analytical and approximate-analytical solutions of nonlinear fractional logistic equations and the results are analyzed and graphically compared with the Homotopy Perturbation Method (HPM), which can be found in [15].

Recently, some researchers developed a new mathematical model [12] based on susceptible individuals S, healthy or resistant individuals H, infected and quarantine individuals I, Q respectively. All the parameters involved in the model are assumed to be non-negative. The susceptible individuals initially move to the infectious class with a constant flow rate. The suspected or infected individuals move to the quarantine class and confirmed cases are send back to the infected compartment for further treatment. In this paper, we investigate the COVID-19 new model under Caputo fractional derivative and Conformable fractional derivative as follows:

$$\begin{cases} {}^{c}D^{q_1}S(t)=\lambda-\gamma S(t)I(t)-(d+\mu)S(t), \\ {}^{c}D^{q_2}H(t)=\alpha-\beta H(t)I(t)+\theta I(t)-(d+\mu)H(t), \\ {}^{c}D^{q_3}I(t)=\gamma S(t)I(t)+\beta H(t)I(t)+\delta Q(t)-(d+\mu+\eta+\theta)I(t), \\ {}^{c}D^{q_4}Q(t)=\eta I(t)-(d+\mu+\delta)Q(t), \\ S(0)\geq 0, H(0)\geq 0, I(0)\geq 0, Q(0)\geq 0. \end{cases} \tag{1.1}$$

where, ${}^{c}D^{q_i}$ $0<q_i\leq 1$ for $i=1,2,3,4$ is the Caputo's derivative of fractional order and q show fractional time derivative.

In the above model:

$\lambda\rightarrow$ Recruitment rate susceptible,

$\gamma\rightarrow$ Disease transmission rate,

$d\rightarrow$ Natural death rate,

$\alpha\rightarrow$ Recruitment rate of healthy human,

$\beta\rightarrow$ Transmission rate of healthy human,

$\mu\rightarrow$ Disease related death rate infected or suspected individuals,

$\delta\rightarrow$ Rate at which quarantine people get infection,

$\theta\rightarrow$ Cure rate of infected people in the quarantine class.

The study of the mathematical models under fractional derivatives instead of usual ordinary derivatives produces more significant results which are more helpful in understanding.

2. Stability analysis

Now consider the following fractional system as a particular case of model (1.1)

$${}_{t_0}^{C}D_t^{q}l(t)=Al(t)+\phi(t,l(t)), \tag{2.6}$$

where $0<q\leq 1$, $l(t)=(l_1(t),l_2(t))^T$, $\phi(t,l(t))=(sinl_1(t),sinl_2(t))^T$,

$$A = \begin{pmatrix} -3 & 1 \\ 1 & -2 \end{pmatrix}.$$

Here ϕ is Lipschitz continuous with Lipschitz constant $L = 1$. We select $V(t) = l^T l$, where $P = I_2$. A straightforward computation shows that

$$\begin{aligned} {}_{t_0}^{C}D_t^q V(t) &\le 2l^T(t)\,{}_{t_0}^{C}D_t^q l(t) \\ &= [Al + \phi(t,l)]^T l + l^T[Al + \phi(t,l)] \\ &= -6l_1^2 + 4l_1 l_2 - 4l_2^2 + 2l_1 sinl_1 + 2l_2 sinl_2 \\ &\le -4(l_1 - \frac{1}{2}l_2)^2 - l_2^2. \end{aligned}$$

Hence, ${}_{t_0}^{C}D_t^q V(t)$ is negative define, which implies the trivial solution of system (2.6) is Mittag- Leffler stability according to Theorem 3.1 [12]. Furthermore, since $A^T P + PA + \eta I = \begin{pmatrix} -3 & 1 \\ 1 & -2 \end{pmatrix} < 0$, the trivial solution of system (2.6) is also asymptotically stable according to Theorem 3.1 [12] .

Hence we conclude that our model (1.1) is Mittag-Leffler and asymptotically stable according to Theorem 3.1[12]and Theorem 3.3[12].

3. Analytical Solution of Model using FDTM

In this section, we try to find approximate solution of given model by using fractional differential transform method.Now we apply FDTM on model system (1.1), then by using Theorem 1,2,4,5 [7], the first equation of given model is transformed into

$$\frac{\Gamma\left(q_1+1+\frac{k}{\xi_1}\right)}{\Gamma\left(1+\frac{k}{\xi_1}\right)} S(k+q_1\xi_1) = \lambda\delta(k) - \gamma\sum_{l=0}^{k} S(l)I(k-l) - (d+\mu)S(k)$$

this implies

$$S(k+q_1\xi_1) = \frac{\Gamma(1+\frac{k}{\xi_1})\left[\lambda\delta(k) - \gamma\sum_{l=0}^{k} S(l)I(k-l) - (d+\mu)S(k)\right]}{\Gamma(q_1+1+\frac{k}{\xi_1})} \tag{1}$$

Second equation of model (1.1) is transformed into

$$\frac{\Gamma(q_2+1+\frac{k}{\xi_2})}{\Gamma(1+\frac{k}{\xi_2})} H(k+q_2\xi_2) = \alpha\delta(k) - \beta\sum_{l=0}^{k} H(l)I(k-l) + QI(k) - (d+\mu)H(k)$$

this implies

$$H(k+q_2\xi_2)=\frac{\Gamma(1+\frac{k}{\xi_2})\left[\alpha\delta(k)-\beta\sum_{l=0}^{k}H(l)I(k-l)+QI(k)-(d+\mu)H(k)\right]}{\Gamma(q_2+1+\frac{k}{\xi_2})} \tag{2}$$

Third equation of model (1.1) is transformed into

$$\frac{\Gamma(q_3+1+\frac{k}{\xi_3})}{\Gamma(1+\frac{k}{\xi_3})}I(k+q_3\xi_3)=\gamma\sum_{l=0}^{k}S(l)I(k-l)+\beta\sum_{l=0}^{k}H(l)I(k-l)+\delta Q(k)-(d+\mu+\eta+\theta)I(k)$$

this implies

$$I(k+q_3\xi_3)=\frac{\Gamma(1+\frac{k}{\xi_3})\left[\gamma\sum_{l=0}^{k}S(l)I(k-l)+\beta\sum_{l=0}^{k}H(l)I(k-l)+\delta Q(k)-(d+\mu+\eta+\theta)I(k)\right]}{\Gamma(1+\frac{k}{\xi_3})} \tag{3}$$

Fourth equation of model (1.1) is transformed into

$$\frac{\Gamma(q_4+1+\frac{k}{\xi_4})}{\Gamma(1+\frac{k}{\xi_4})}Q(k+q_4\xi_4)=\eta I(k)-(d+\mu+\delta)Q(k)$$

this implies

$$Q(k+q_4\xi_4)=\frac{\Gamma(1+\frac{k}{\xi_4})[\eta I(k)-(d+\mu+\delta)Q(k)]}{\Gamma(q_4+1+\frac{k}{\xi_4})} \tag{4}$$

Subject to initial conditions from [12]and using equation (4)[7] which reduces to

$$S(0)=43994, H(0)=0, I(0)=1, Q(0)=1.$$

Now, we take values for parameters as follows:

$$\alpha=0.535,\ \beta=0.0056,\ \gamma=0.5944,\ \delta=0.27,\ \eta=0.0025,$$

$$d=0.025,\ \lambda=0.0043217,\ \mu=3.5,\ \theta=0.5.$$

Therefore, the series solutions of the transformed expressions, when $k=6$ and $q_1=\frac{1}{2}$, $q_2=\frac{1}{2}$, $q_3=\frac{1}{2}$, $q_4=\frac{1}{2}$ and $\xi_1=\xi_2=\xi_3=\xi_4=2$, for $S(t), H(t), I(t)$ and $Q(t)$ are computed as

$$
\begin{aligned}
s(t) = {} & 43994 + -204494.89185370135t^2 - 682979444.2845785t^4 \\
& -13440867153594.955t^6 - 2.3351275811327456e + 17t^8 \\
& -3.6728833676035553e + 21t^{10} - 5.316566806977249e + 25t^{12} \\
& -7.166073653626551e + 29t^{14} + \ldots.
\end{aligned}
$$

$$
\begin{aligned}
h(t) = {} & 0 + 1.1678724379438556t^2 + 13069.483879000001t^4 \\
& +2093553216.6433747t^6 + 4461859791405.322t^8 \\
& +7.0253126449744776e + 16t^{10} + 1.0170268658299638e + 21t^{12} \\
& +1.3710002839853866e + 25t^{14} + \ldots.
\end{aligned}
$$

$$
\begin{aligned}
i(t) = {} & 1 + 29502.91324836731t^2 + 683512970.9388622t^4 \\
& +13440581950685.072t^6 + 2.335082696829314e + 17t^8 \\
& +3.67281276322261e + 21t^{10} + 5.316464595779258e + 25t^{12} \\
& +7.165935868099082e + 29t^{14} + \ldots.
\end{aligned}
$$

$$
\begin{aligned}
q(t) = {} & 1 - 4.279377991209731t^2 + 79.75822775t^4 \\
& +1285208.6343461156t^6 + 22330643694.280605t^8 \\
& +351263489534993.94t^{10} + 5.085114079353075e + 18t^{12} \\
& +6.854990712769763e + 22t^{14} + \ldots.
\end{aligned}
$$

Therefore, the series solution of the transformed expressions, when $k = 6$ and $q_1 = 0.3,\ q_2 = 0.4,\ q_3 = 0.6,\ q_4 = 0.7,\ \xi_1 = \xi_2 = \xi_3 = \xi_4 = 10$ and using equation (4)[7], initial conditions of given model are reduces to

$$S(0) = 43994, S(1) = S(2) = 0,$$

$$H(0) = H(1) = H(2) = H(3) = 0,$$

$$I(0) = 1, I(1) = I(2) = I(3) = I(4) = I(5) = 0,$$

$$Q(0) = 1, Q(1) = Q(2) = Q(3) = Q(4) = Q(5) = Q(6) = 0,$$

for $S(t), H(t), I(t)$ and $Q(t)$ are obtained as:

$$
\begin{aligned}
s(t) = {} & 43994 - 201932.9210682691t^{10} - 819588036.05357t^{40/3} \\
& -21692091405660.13t^{50/3} - 5.695492785548731e + 17t^{20} \\
& -1.3505476753304897e + 22t^{70/3} - 3.359001065324167e + 26t^{80/3} \\
& -8.156550160887602e + 30t^{30} + \ldots
\end{aligned}
$$

$$h(t) = 0 + 1.1665076154155385t^{10} + 15701.867117328291t^{50/4}$$
$$+3442357990.894716t^{15} + 10700536335456.986t^{70/4}$$
$$+2.582996787274982e+17t^{20} + 6.424394739441411e+21t^{90/4}$$
$$+1.5600702312748967e+26t^{25} + \ldots$$

$$i(t) = 1 + 29262.25735332224t^{10} + 800929837.5555459t^{70/6}$$
$$+21691708290148.32t^{80/6} + 5.291217720706947e+17t^{90/6}$$
$$+1.350535627973157e+22t^{100/6} + 3.3589364954930755e$$
$$+26t^{1100/6} + 8.15639337383579e+30t^{120/6} + \ldots$$

$$q(t) = 1 - 4.173826035448501t^{10} + 90.90319410083866t^{80/7}$$
$$+2074135.928152912t^{90/7} + 48662117055.841675t^{100/7}$$
$$+1291506307912265.5t^{110/7} + 3.212175606898621e+19t^{120/7}$$
$$+7.800302139534849e+23t^{130/7} + \ldots$$

Therefore, the series solution of the transformed expressions, when $k = 6$ and $q_1 = q_2 = q_3 = q_4 = 0.8$, $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 10$ and using equation (4)[7], initial conditions of given model are reduces to

$$S(0) = 43994, S(1) = S(2) = S(3) = S(4) = S(5) = S(6) = S(7) = 0,$$

$$H(0) = H(1) = H(2) = H(3) = H(4) = H(5) = H(6) = H(7) = 0,$$

$$I(0) = 1, I(1) = I(2) = I(3) = I(4) = I(5) = I(6) = I(7) = 0,$$

$$Q(0) = 1, Q(1) = Q(2) = Q(3) = Q(4) = Q(5) = Q(6) = Q(7) = 0,$$

for $S(t), H(t), I(t)$ and $Q(t)$ are computed as:

$$s(t) = 43994 - 194580.2417059126t^{10} - 725354477.9582386t^{90/8}$$
$$-17423745277896.578t^{100/8} - 3.905994053546499e+17t^{110/8}$$
$$-8.221758137795173e+21t^{120/8} - 3.683763337851668e+26t^{130/8}$$
$$-6.926853679862806e+30t^{140/8} + \ldots.$$

$$h(t) = 0 + 1.1112497686219136t^{10} + 13880.37185535794t^{90/8}$$
$$+2708987275.50775t^{100/8} + 7462974268676.504t^{110/8}$$
$$+1.5724699137811232e+17t^{120/8} + 7.045833486564776e+21t^{130/8}$$
$$+1.3246922101927448e+26t^{140/8} + \ldots.$$

$$i(t) = 1 + 28072.505571448954t^{10} + 725921106.9410487t^{90/8}$$
$$+17423380432702.893t^{100/8} + 3.9059189793838445e + 17t^{110/8}$$
$$+8.221600104594644e + 21t^{120/8} + 3.683692527227321e + 26t^{130/8}$$
$$+6.926720548300939e + 30t^{140/8} + ...$$
$$q(t) = 1 - 4.0718983067619385t^{10} + 84.70677724873059t^{90/8}$$
$$+1666000.0180418086t^{100/8} + 37350490031.25452t^{110/8}$$
$$+786229281554522.4t^{120/8} + 3.5228948003016438e + 19t^{130/8}$$
$$+6.623408474131018e + 23t^{140/8} + ...$$

Therefore, the series solutions of the transformed expressions, when $k = 6$ and $q_1 = q_2 = q_3 = q_4 = 1$ and $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 1$ for $S(t), H(t), I(t)$ and $Q(t)$ are obtained as:

$$s(t) = 43994 - 181228.8792783t^1 - 341489722.14228904t^2$$
$$-2977573434823.655t^3 - 1.945188026546866e + 16t^4$$
$$-1.0162820450677952e + 20t^5 - 4.4225071453039736e + 23t^6$$
$$-1.6481614693744282e + 27 + ...$$
$$h(t) = 0 + 1.0350000000000001t^1 + 6534.741939500001t^2$$
$$+467054951.48506397t^3 + 371775398546.0024t^4$$
$$+1944868479597809.0t^5 + 8.467514733534964e + 18t^6$$
$$+3.158301744434283e + 22 + ...$$
$$i(t) = 1 + 26146.276100000003t^1 + 341756485.46943116t^2$$
$$+2977507032055.2954t^3 + 1.9451506276111144e + 16t^4$$
$$+1.0162624991420808e + 20t^5 + 4.4224220467754366e + 23t^6$$
$$+1.6481297284362924e + 27 + ...$$
$$q(t) = 1 - 3.7925t^1 + 39.87911387500001t^2 + 854239.8724364223t^3$$
$$+1860131434.9555857t^4 + 9724341298296.441t^5$$
$$+4.233812015171553e + 16t^6 + 1.5792069121826908e + 20 + ...$$

4. Analytical Solutions of Model using CFDTM

In this section, we try to find approximate solution of given model by using conformable fractional differential transform method. Now we apply CFDTM on model system (1.1), then by using Theorem 3.1, 3.2, 3.3, 3.4, 3.7 [9], the first equation is transformed into

$$q(k+1)S_q(k+1) = \lambda\delta(k) - \gamma\sum_{l=0}^{k} S_q(l)I_q(k-l) - (d+\mu)S_q(k)$$

after some simplification this implies that

$$S_q(k+1)=\frac{\lambda\delta(k)-\gamma\sum_{l=0}^{k}S_q(l)I_q(k-l)-(d+\mu)S_q(k)}{q(k+1)} \tag{5}$$

Second equation of given model (1.1) is transformed into

$$q(k+1)H_q(k+1)=\alpha\delta(k)-\beta\sum_{l=0}^{k}H_q(l)I_q(k-l)+QI_q(k)-(d+\mu)H_q(k)$$

after some simplification we get

$$H_q(k+1)=\frac{\alpha\delta(k)-\beta\sum_{l=0}^{k}H_q(l)I_q(k-l)+QI_q(k)-(d+\mu)H_q(k)}{q(k+1)} \tag{6}$$

Third equation of model (1.1) is transformed into

$$q(k+1)I_q(k+1)=\gamma\sum_{l=0}^{k}S_q(l)I_q(k-l)+\beta\sum_{l=0}^{k}H_q(l)I_q(k-l)+\delta Q_q(k)-(d+\mu+\eta+\theta)I_q(k)$$

this implies

$$I_q(k+1)=\frac{\gamma\sum_{l=0}^{k}S_q(l)I_q(k-l)+\beta\sum_{l=0}^{k}H_q(l)I_q(k-l)+\delta Q_q(k)-(d+\mu+\eta+\theta)I_q(k)}{q(k+1)} \tag{7}$$

Fourth equation of model (1.1) is transformed into

$$q(k+1)Q_q(k+1)=\eta I_q(k)-(d+\mu+\delta)Q_q(k)$$

after some calculation and simplification, we have

$$Q_q(k+1)=\frac{\eta I_q(k)-(d+\mu+\delta)Q_q(k)}{q(k+1)} \tag{8}$$

By using definition (3.2) [9], the initial conditions of given model are reduces to

$$S(0)=43994, H(0)=0, I(0)=1, Q(0)=1.$$

Now, we take values for parameters are as follows:

$$\alpha=0.535,\ \beta=0.0056,\ \gamma=0.5944,\ \delta=0.27,\ \eta=0.0025,$$

$$d=0.025,\ \lambda=0.0043217,\ \mu=3.5,\ \theta=0.5.$$

Thus, the series solution of the transformed expressions, when $k=6$ and $q_1=\frac{1}{2}$, $q_2=\frac{1}{2}$, $q_3=\frac{1}{2}$, $q_4=\frac{1}{2}$, for $S(t), H(t), I(t)$ and $Q(t)$ are obtained as:

$$\begin{aligned}s(t)=&\ 43994-362457.7585566t^{\frac{1}{2}}-1365958888.5691562t\\&-23820587478589.24t^{\frac{3}{2}}-3.1123008424749856e+17t^{2}\\&-3.2521025442105115e+21t^{\frac{5}{2}}-2.830404572988943e+25t^{3}\\&-2.10964668079509e+29t^{\frac{7}{2}}+\ldots\end{aligned}$$

$$h(t) = 0 + 2.0700000000000003t^{\frac{1}{2}} + 26138.967758000003t$$
$$+3736439611.8805118t^{\frac{3}{2}} + 5948406376736.038t^{2}$$
$$+6.223579134700689e + 16t^{\frac{5}{2}} + 5.419209429451661e + 20t^{3}$$
$$+4.0426262328678816e + 24t^{\frac{7}{2}} + \ldots.$$

$$i(t) = 1 + 52292.552200000006t^{\frac{1}{2}} + 1367025941.8777246t$$
$$+23820056256442.363t^{\frac{3}{2}} + 3.1122410041716326e + 17t^{2}$$
$$+3.2520399972482275e + 21t^{\frac{5}{2}} + 2.8303501099306783e + 25t^{3}$$
$$+2.1096060523942762e + 29t^{\frac{7}{2}} + \ldots$$

$$q(t) = 1 - 7.585t^{\frac{1}{2}} + 159.51645550000003t + 2277972.993163793t^{\frac{3}{2}}$$
$$+29770747866.798428t^{2} + 311178908421901.44t^{\frac{5}{2}}$$
$$+2.7096396897210363e + 18t^{3} + 2.021384847589842e + 22t^{\frac{7}{2}} + \ldots.$$

For $q_1 = 0.3, q_2 = 0.4, q_3 = 0.6, q_4 = 0.7$ and hence approximate values of $S(t), H(t), I(t)$ and $Q(t)$ are obtained as:

$$s(t) = 43994 - 604096.2642610001t^{\frac{3}{10}} - 1895091361.2202196t^{\frac{3}{5}}$$
$$-27552751961158.49t^{\frac{9}{10}} - 2.998238770238271e + 17t^{\frac{6}{5}}$$
$$-2.608460402856581e + 21t^{\frac{3}{2}} - 1.889233356555197e + 25t^{\frac{9}{5}}$$
$$-1.1708036160359847e + 29t^{\frac{21}{10}} + \ldots$$

$$h(t) = 0 + 2.5875000000000004t^{\frac{4}{10}} + 27224.284986458337t^{\frac{4}{5}}$$
$$+6085168866.184664t^{\frac{12}{10}} + 4292147542678.396t^{\frac{8}{5}}$$
$$+3.7473829817806136e + 16t^{2} + 2.7167526543688303e + 20t^{\frac{12}{5}}$$
$$+1.6864744339479867e + 24t^{\frac{14}{5}} + \ldots.$$

$$
\begin{aligned}
i(t) = 1 &+ 43577.12683333334t^{\frac{6}{10}} + 949173956.4474993t^{\frac{6}{5}} \\
&+ 13777881525218.57t^{\frac{18}{10}} + 1.499292856317466e+17t^{\frac{12}{5}} \\
&+ 1.3043812150367266e+21t^{\frac{15}{10}} + 9.447261634298385e+24t^{\frac{18}{5}} \\
&+ 5.854697774811895e+28t^{\frac{21}{5}} + ...
\end{aligned}
$$

$$
\begin{aligned}
q(t) = 1 &- 5.417857142857144t^{\frac{7}{10}} + 92.50256067176872t^{\frac{7}{5}} \\
&+ 1129801.8304290469t^{\frac{21}{10}} + 12300148648.24998t^{\frac{16}{5}} \\
&+ 107079010004356.11t^{\frac{35}{10}} + 7.763206363687738e+17t^{\frac{21}{5}} \\
&+ 4.819430193659376e+21t^{\frac{49}{10}} + ...
\end{aligned}
$$

For $q_1 = 0.8, q_2 = 0.8, q_3 = 0.8, q_4 = 0.8$ and hence approximate values of $S(t), H(t), I(t)$ and $Q(t)$ are obtained as:

$$
\begin{aligned}
s(t) = 43994 &- 226536.099097875t^{\frac{4}{5}} - 533577690.8473266t^{\frac{8}{5}} \\
&- 5815573114889.951t^{\frac{12}{5}} - 4.748994205436685e+16t^{\frac{16}{5}} \\
&- 3.101446670732986e+20t^{4} - 1.6870525914364716e+24t^{\frac{24}{5}} \\
&- 7.859046313148332e+27t^{\frac{28}{5}} + ...
\end{aligned}
$$

$$
\begin{aligned}
h(t) = 0 &+ 1.2937500000000002t^{\frac{4}{5}} + 10210.53428046875t^{\frac{8}{5}} \\
&+ 912216702.1192657t^{\frac{12}{5}} + 907654781606.4512t^{\frac{16}{5}} \\
&+ 5935267576885878.0t^{4} + 3.2301005300591358e+19t^{\frac{24}{5}} \\
&+ 1.5059956285610361e+23t^{\frac{28}{5}} + ...
\end{aligned}
$$

$$
\begin{aligned}
i(t) = 1 &+ 32682.845125000003t^{\frac{4}{5}} + 533994508.5459862t^{\frac{8}{5}} \\
&+ 5815443421982.999t^{\frac{12}{5}} + 4.748902899431812e+16t^{\frac{16}{5}} \\
&+ 3.101387021301487e+20t^{4} + 1.6870201289240594e+24t^{\frac{24}{5}} \\
&+ 7.858894960560939e+27t^{\frac{28}{5}} + ...
\end{aligned}
$$

$$q(t) = 1 - 4.740625t^{\frac{4}{5}} + 62.31111542968751t^{\frac{8}{5}} + 556145.750284129t^{\frac{12}{5}}$$
$$+4542655619.32349t^{\frac{16}{5}} + 29676333276929.992t^{4}$$
$$+1.6150711117512205e + 17t^{\frac{24}{5}} + 7.530245362184356e + 20t^{\frac{28}{5}} + \ldots\ldots$$

For $q_1 = q_2 = q_3 = q_4 = 1$ and hence approximate values of $S(t), H(t), I(t)$ and $Q(t)$ are obtained as:

$$s(t) = 43994 - 181228.8792783t - 1365958888.5691562t^{2}$$
$$-107217992520952.2t^{3} - 1.1212350713573536e + 19t^{4}$$
$$-1.4656859966044585e + 24t^{5} - 2.2991644515586486e + 29t^{6}$$
$$-4.207738078551882e + 34t^{7} + \ldots..$$

$$h(t) = 0 + 1.0350000000000001t + 26138.967758000003t^{2}$$
$$+16575159853.59114t^{3} + 214197553639755.1t^{4}$$
$$+2.8026540629606425e + 19t^{5} + 4.3963783531973526e + 24t^{6}$$
$$+8.045832599916284e + 29t^{7} + \ldots..$$

$$q(t) = 1 + 26146.276100000003t + 1367025941.8777246t^{2}$$
$$+107215840839691.12t^{3} + 1.1212135240501512e + 19t^{4}$$
$$+1.4656578299360666e + 24t^{5} + 2.299120267958349e + 29t^{6}$$
$$+4.20765721793823e + 34t^{7} + \ldots..$$

$$i(t) = 1 - 3.7925t + 159.51645550000003t^{2} + 10250878.469237069t^{3}$$
$$+1072002800061.7483t^{4} + 1.4013134925313771e + 17t^{5}$$
$$+2.1981676658218506e + 22t^{6}$$
$$+4.022876525686686e + 27t^{7} + \ldots..$$

5. Numerical Simulation and Discussion

In this section, we provide numerical simulation by using tables and graphs. The mathematical analysis of epidemic COVID-19 model with non linear system of fractional differential equation has been presented. We observe that fractional order COVID -19 model has more degree of freedom as compared to ordinary derivatives. The compression for some different values of q has been obtained and numerical simulation are shown by using FDTM and CFDTM. Through a novel methods FDTM and CFDTM, we have derived approximate solution for the corresponding COVID-19 model under investigation. Further, some numerical solution have been presented graphically for different fractional orders by using Python software. We plot the solutions of given model (1.1) for different fractional order by using Python software, which is shown in Figures.

Table 1

Table of susceptible class (population) at different values of q using FDTM

S.N.	q = 0.5	q = 0.3	q = 0.8	q = 1
1	4.39940000e+04	4.39940000e+04	4.39940000e+04	4.39940000e+04
2	-1.17408973e+48	-8.75802906e+69	-4.06033142e+53	-2.10967499e+36
3	-1.92362834e+52	-9.40386209e+78	-7.52637913e+58	-2.70036587e+38
4	-5.61563383e+54	-1.80320052e+84	-9.08203213e+61	-4.61381792e+39
5	-3.15167256e+56	-1.00973200e+88	-1.39511781e+64	-3.45645672e+40
6	-7.16607371e+57	-8.15655016e+90	-6.92685484e+65	-1.64816589e+41
7	-9.20065433e+58	-1.93617181e+93	-1.68347968e+67	-5.90567374e+41

Table 2

Table of resistive class (population) at different values of q using FDTM

S.N.	q = 0.5	q = 0.4	q = 0.8	q = 1
1	0.00000000e+00	0.00000000e+00	0.00000000e+00	0.00000000e+00
2	2.24624728e+43	5.23472717e+58	7.76498199e+48	4.04268043e+31
3	3.68025103e+47	1.75648294e+66	1.43934552e+54	5.17459626e+33
4	1.07437293e+50	4.43532461e+70	1.73684876e+57	8.84126308e+34
5	6.02972309e+51	5.89377870e+73	2.66802473e+59	6.62346102e+35
6	1.37100029e+53	1.56007023e+76	1.32469243e+61	3.15831021e+36
7	1.76025259e+54	1.48824798e+78	3.21948827e+62	1.13167915e+37

Table 3

Table of infected class (population) at different values of q using FDTM

S.N.	q = 0.5	q = 0.6	q = 0.8	q = 1
1	8.70421891e+08	1.00000000e+00	1.00000000e+00	1.00000000e+00
2	1.17406715e+48	8.55260073e+56	4.06025338e+53	2.10963436e+36
3	1.92359135e+52	8.96805014e+62	7.52623448e+58	2.70031386e+38
4	5.61552586e+54	2.98210678e+66	9.08185757e+61	4.61372907e+39
5	3.15161196e+56	9.40368158e+68	1.39509100e+64	3.45639015e+40
6	7.16593592e+57	8.15639353e+70	6.92672171e+65	1.64813415e+41
7	9.20047742e+58	3.12696550e+72	1.68344732e+67	5.90556000e+41

Table 4

Table of quarantined class (population) at different values of q using FDTM

S.N.	q = 0.5	q = 0.7	q = 0.8	q = 1
1	1.00000000e+00	1.00000000e+00	1.00000000e+00	1.00000000e+00
2	1.12312189e+41	1.13265411e+48	3.88246018e+46	2.02141194e+29
3	1.84012264e+45	4.41219007e+53	7.19667047e+51	2.58738995e+31
4	5.37185628e+47	8.22091982e+56	8.68417486e+54	4.42078842e+32
5	3.01485683e+49	1.71874409e+59	1.33400178e+57	3.31184803e+33
6	6.85499076e+50	1.08384853e+61	6.62340959e+58	1.57921115e+34
7	8.80124920e+51	3.20241375e+62	1.60973135e+60	5.65859653e+34

Table 5

Table of susceptible class (population) at different values of q using CFDTM

S.N.	q = 0.5	q = 0.3	q = 0.8	q = 1
1	4.39940000e+04	0.00000000e+00	2.29675648e+02	1.00000000e+00
2	-7.54792786e+33	1.44637729e+29	7.54778250e+33	7.23214306e+26
3	-8.53943052e+34	1.63637473e+30	8.53926606e+34	8.18216401e+27
4	-3.52977177e+35	6.76395146e+30	3.52970379e+35	3.38209574e+28
5	-9.66120273e+35	1.85133518e+31	9.66101668e+35	9.25700438e+28
6	-2.10967499e+36	4.04268043e+31	2.10963436e+36	2.02141194e+29
7	-3.99346063e+36	7.65249872e+31	3.99338372e+36	3.82638514e+29

Table 6

Table of resistive class (population) at different values of q using CFDTM

S.N.	$q = 0.5$	$q = 0.4$	$q = 0.8$	$q = 1$
1	4.39940000e+04	0.00000000e+00	1.00000000e+00	1.00000000e+00
2	-6.31939606e+31	7.41113728e+27	1.70546212e+34	1.14301325e+28
3	-2.70915098e+32	5.16135504e+28	3.13446992e+35	3.41267508e+29
4	-6.34778486e+32	1.60626377e+29	1.72086079e+36	2.48851810e+30
5	-1.16142682e+33	3.59454437e+29	5.76086213e+36	1.01891953e+31
6	-1.85567390e+33	6.71414707e+29	1.47064856e+37	3.04087438e+31
7	-2.72133081e+33	1.11865858e+30	3.16278506e+37	7.42995616e+31

Table 7

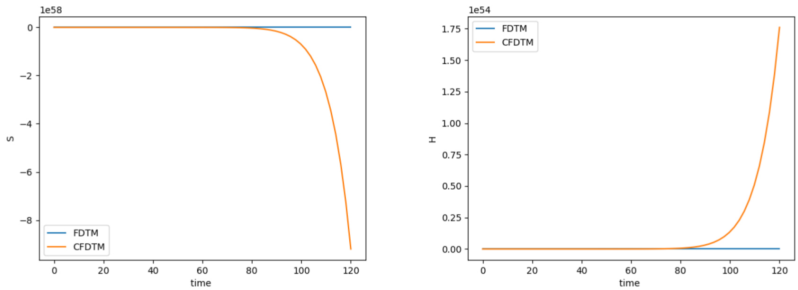
Table of infected class (population) at different values of q using CFDTM

S.N.	$\mu = 0.5$	$\mu = 0.6$	$\mu = 0.8$	$\mu = 1$
1	4.39940000e+04	0.00000000e+00	1.00000000e+00	1.00000000e+00
2	-1.51756154e+35	2.90803860e+30	1.51753231e+35	1.45407090e+28
3	-7.36055689e+36	1.41047221e+32	7.36041514e+36	7.05261130e+29
4	-7.12886473e+37	1.36607403e+33	7.12872744e+37	6.83061253e+30
5	-3.57006871e+38	6.84117083e+33	3.56999996e+38	3.42070680e+31
6	-1.24558162e+39	2.38685508e+34	1.24555763e+39	1.19346989e+32
7	-3.45769634e+39	6.62583644e+34	3.45762975e+39	3.31303579e+32

Table 8

Table of quarantined class (population) at different values of q using CFDTM

S.N.	$\mu = 0.5$	$\mu = 0.3$	$\mu = 0.8$	$\mu = 1$
1	4.39940000e+04	0.00000000e+00	1.00000000e+00	1.00000000e+00
2	-5.38590621e+43	1.02986685e+39	5.38580271e+43	5.14928336e+36
3	-6.89395901e+45	1.31822939e+41	6.89382653e+45	6.59108180e+38
4	-1.17789747e+47	2.25231840e+42	1.17787484e+47	1.12614807e+40
5	-8.82426693e+47	1.68733351e+43	8.82409735e+47	8.43658413e+40
6	-4.20773831e+48	8.04583304e+43	4.20765745e+48	4.02287675e+41
7	-1.50770870e+49	2.88296742e+44	1.50767972e+49	1.44146946e+42

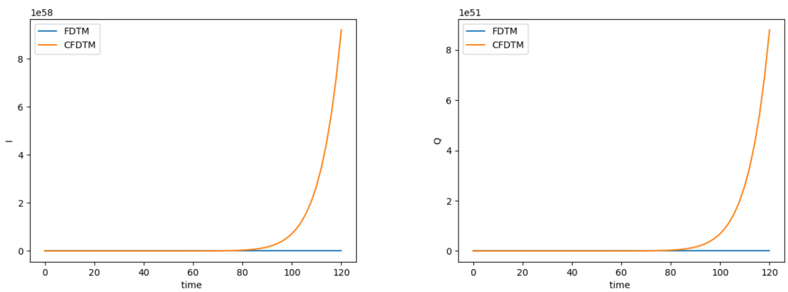


(a) The comparison plots of dynamical behaviour of the susceptible class

(b) The comparison plots of dynamical behaviour of resistive class

Figure 1

The comparison plots of dynamical behaviour of the susceptible class [b]0.48 AG2 The comparison plots of dynamical behaviour of resistive class. The comparison plots of dynamical behaviour of the susceptible and resistive class (population)using FDTM and CFDTM for q=0.5

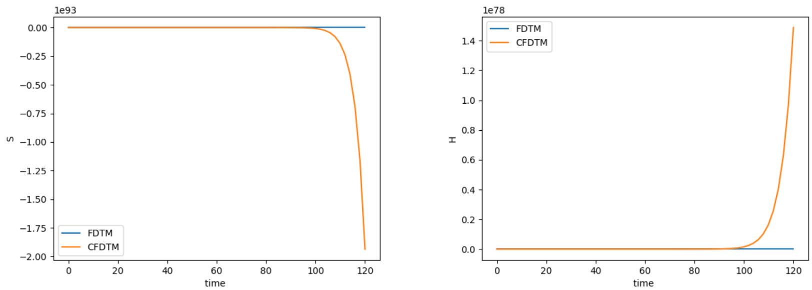


(a) The comparison plots of dynamical behaviour of infected class

(b) The comparison plots of dynamical behaviour of quarantined class

Figure 2

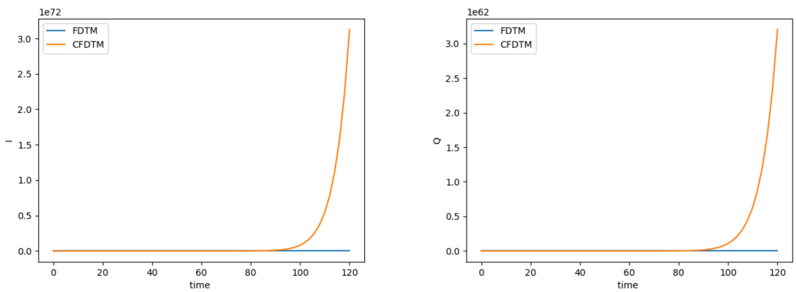
The comparison plots of dynamical behaviour of infected class [b]0.48 AG4 The comparison plots of dynamical behaviour of quarantined class. The comparison plots of dynamical behaviour of the infected and quarantined class (population)using FDTM and CFDTM for q=0.5



(a) The comparison plots of dynamical behaviour of susceptible class (b) The comparison plots of dynamical behaviour of resistive class

Figure 3

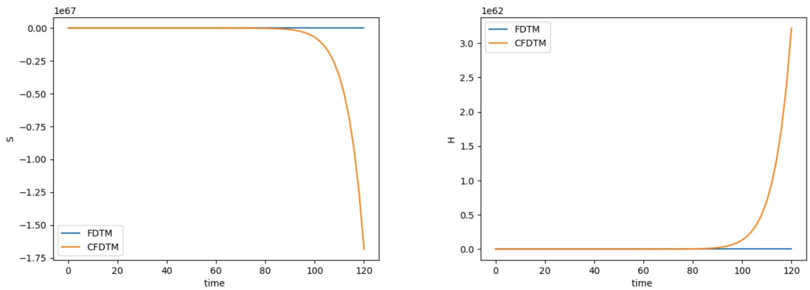
The comparison plots of dynamical behaviour of susceptible class [b]0.48 AG6 The comparison plots of dynamical behaviour of resistive class. The comparison plots of dynamical behaviour of the susceptible and resistive class (population)using FDTM and CFDTM for q=0.3, 0.4



(a) The comparison plots of dynamical behaviour of infected class (b) The comparison plots of dynamical behaviour of quarantined class

Figure 4

The comparison plots of dynamical behaviour of infected class [b]0.48 AG8 The comparison plots of dynamical behaviour of quarantined class. The comparison plots of dynamical behaviour of the infected and quarantined class (population)using FDTM and CFDTM for q=0.6, 0.7

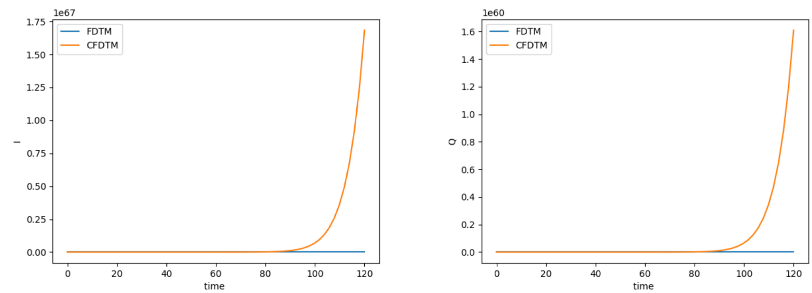


(a) The comparison plots of dynamical behaviour of susceptible class

(b) The comparison plots of dynamical behaviour of resistive class

Figure 5

The comparison plots of dynamical behaviour of susceptible class [b]0.48 AG10 The comparison plots of dynamical behaviour of resistive class. The comparison plots of dynamical behaviour of the susceptible and resistive class (population)using FDTM and CFDTM for q=0.9

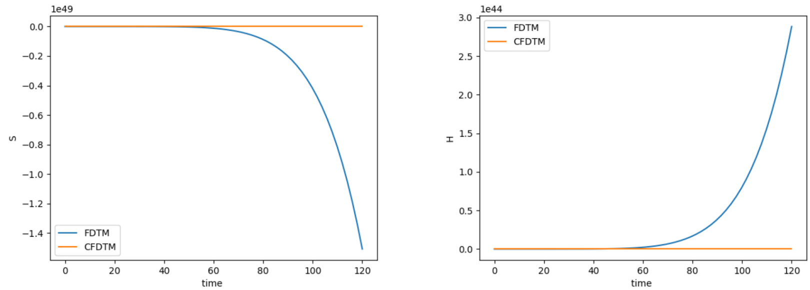


(a) The comparison plots of dynamical behaviour of infected class

(b) The comparison plots of dynamical behaviour of quarantined class

Figure 6

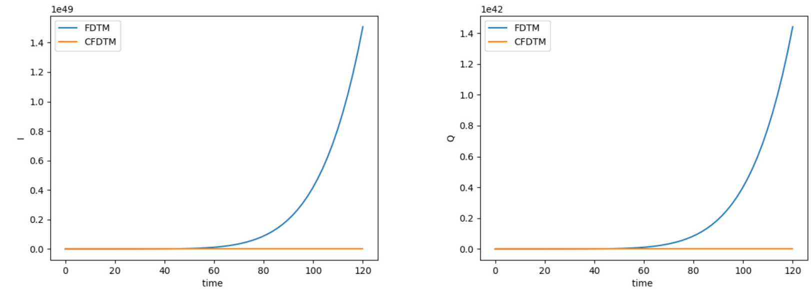
The comparison plots of dynamical behaviour of infected class [b]0.48 AG12 The comparison plots of dynamical behaviour of quarantined class. The comparison plots of dynamical behaviour of the infected and quarantined class (population)using FDTM and CFDTM for q=0.9



(a) The comparison plots of dynamical behaviour of susceptible class (b) The comparison plots of dynamical behaviour of resistive class

Figure 7

The comparison plots of dynamical behaviour of susceptible class [b]0.48 AG14. The comparison plots of dynamical behaviour of resistive class. The comparison plots of dynamical behaviour of the susceptible and resistive class (population)using FDTM and CFDTM for q=1



(a) The comparison plots of dynamical behaviour of infected class (b) The comparison plots of dynamical behaviour of quarantined class

Figure 8

The comparison plots of dynamical behaviour of infected class [b]0.48 AG16. The comparison plots of dynamical behaviour of quarantined class. The comparison plots of dynamical behaviour of the infected and quarantined class (population)using FDTM and CFDTM for q=1

6. Conclusion

In this work, a fractional order corona virus model under Caputo and conformable sense has been investigated and solved by using fractional differential transform method and conformable fractional differential transform method. The analytical solution has been obtained in terms of convergent series with easily computable components in a direct way without using linearisation or perturbation or restrictive assumptions. We obtained the approximate solution of fractional order corona virus model by using FDTM and CFDTM which are in good agreement with real data. The compression for some different values of q has been obtained and numerical simulation are shown. Further, we have analyzed the stability of non-linear model by using Lyapunov direct method under Caputo sense. This analysis plays an important roll to describe the physical behaviour of the systems when the model is constructed in fractional differential equations.

References

[1] Musibau A. Omoloye, Sunday O. Adewale, Aliyu M. Umar, Asimiyu O. Oladapo; analysis of Coronavirus disease model by Differential Transformation Method (DTM), (2021).

[2] Amal Shah, Thabet Abdeljawad, Ibrahim Mahariq and Fahd Jarad; Qualitative Analysis of a Mathematical Model in the Time of COVID-19, (2020).

[3] V. Padmavathi, A. Prakash, K. Alagesan, N. Magesh; Analysis and numerical simulation of novel coronavirus (COVID-19) model with Mittag-Leffler Kernel, (2020).

[4] Mohammed A. Aba Oud, Aatif Ali, Hussam Alrabaiah, Saif Ullah, Muhammad Altaf Khan and Saeed Islam; A fractional order mathematical model for COVID-19 dynamics with quarantine, isolation and environmental viral load, (2021).

[5] Nguyen Huy Tuan, Hakimeh Mohammadi, Shahram Rezapour; A mathematical model for COVID-19 transmission by using the Caputo fractional derivative, (2020).

[6] J.K. Zhou, Differential Transformation and its Applications for Electrical Circuits, (1986).

[7] Aytac Arikoglu, Ibrahim Ozkol; Solution of fractional differential equations by using differential transform method, (2007).

[8] Mostafa Eslami, a and Hadi Rezazadeh; Multi-step conformable fractional differential transform method for solving and stability of the conformable fractional differential systems, (2020).

[9] Emrah Ünal, Ahmet Gökdoğan; Solution of Conformable Fractional Ordinary Differential Equations via Differential Transform Method, (2017).

[10] Shantanu Das, Functional Fractional Calculus, Springer, (2011).

[11] I. Podlubny, Fractional Differential equations, Academic Press USA, (1999).

[12] Abdullah, Saeed Ahmad, Saud Owyed, Abdel-Haleem Abdel-Aty, Emad E. Mahmoud, Kamal Shah, Hussam Alrabaiah; Mathematical analysis of COVID-19 via new mathematical model, (2021).

[13] Vijander Singh, Ramesh Chandra Poonia , Sandeep Kumar, Pranav Dass,Pankaj Agarwal, Vaibhav Bhatnagar and Linesh Raja; Prediction of COVID-19 corona virus pandemic based on time series data using support vector machine, *Journal of Discrete Mathematical Sciences and Cryptography*, (2020).

[14] Vaibhav Bhatnagar, Ramesh Chandra Poonia, Pankaj Nagar, Sandeep Kumar, Vijander Singh, Linesh Raja and Pranav Dass; Descriptive analysis of COVID-19 patients in the context of India, *Journal of Interdisciplinary Mathematics*, (2021).

[15] Hatıra Günerhan, Muhammed Yiğider, Jalil Manafian and Onur Alp Ilhan; Numerical solution of fractional order logistic equations via conformable fractional differential transform method, *Journal of Interdisciplinary Mathematics*, (2021).

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