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Estimation of finite population mean in sample surveys: A new estimator

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Abstract

Utilizing supplementary information in simple random sampling, this research paper discussed a new method for finding the finite population mean of a predictive variable, and the properties of the suggested method have been investigated. The suggested estimator's advantages over traditional estimators are demonstrated using theoretical asymptotic techniques and empirical analysis. The recommended estimator outperforms the customary unbiased, ratio, product, and regression estimators, as well as many other known population mean estimators.

Subject Classification: *62D05.*

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1. Introduction

One of the most broadly used and well-accepted areas of statistics that deals with the estimation of population parameters for the finite population is sampling theory. In sampling theory, the selection of random samples using diverse techniques as well as estimation of parameters are discussed. Estimation of parameters is a necessity in many fields including industry, population studies, economics, agriculture, medical science, etc.

For the estimation of population parameters, there is a need to draw a random sample. A random sample can be drawn using many techniques. Sample survey is a method for inferring the feature of a finite population by studying a part of the population [Pal and Singh (2022)].

In survey sampling, numerical information (data) about the population is frequently used in the design and estimation techniques. The data, also known as supplementary data, is frequently obtained from bureaucrat sources such as national censuses. We can use the information in survey design, estimator construction, or both stages, depending on the forms of auxiliary data. Ratio, product, difference, and regression are all words that come to us once we apply the additional (ancillary) information to construct efficient estimators. In the current work, our main objective is to envisage the method of estimation of unknown characteristic (mean) of the predictive variable of the finite population which is more efficient than traditional methods of estimation on hand. Recently Pal and Singh (2018), Pal et. al. (2020), and Pal and Singh (2022) have given significant contributions to sample random sampling.

Here we are considering a finite population $\{\Theta = (\Theta_1, \Theta_2, ..., \Theta_N)\}$ of 'N' identifiable units and v indicate the main (predictive) variable and u indicate (ancillary variable) supplementary information picking values

(v_i) and (u_i) correspondingly on the i^{th} unit Θ_i of the population Θ and a sample of size $n(<N)$ is taken from Θ by simple random sampling without replacement (SRSWOR).

The customary unbiased estimator is given through

$$\overline{v} = n^{-1}\sum_{i-1}^{n} v_i \tag{1.1}$$

is extensively used to find the unknown $\overline{V} = N^{-1}\sum_{i=1}^{N} v_i$ of v.

The variance (Var) of $\overline{v}$ in SRSWOR is given by

$$Var(\overline{v}) = \eta S_v^2 = \eta \overline{V}^2 C_v^2, \tag{1.2}$$

where where $\eta = (1-f)n^{-1}$, $S_v^2 = (\sum_{i=1}^{N}(v_i - \overline{V})^2)(N-1)^{-1}$ (population mean square of v), and $C_v^2 = (S_v^2 / \overline{V}^2)$ (coefficient of variation of v).

However, data (information) from an ancillary (supplementary) variable that is substantially connected with the main (predictive) variable and has simpler values for reporting all population units are occasionally used. The supplementary (ancillary) data is usually simple to collect, whereas the variable (changeable) of interest can be costly to collect. The use of additional (known) information to improve estimators in sample surveys has been extensively discussed. Indirect methods are estimating methods that rely on information provided by an ancillary variable. Cochran (1940) proposed the ratio method as

$$t_R = \overline{v}\left(\frac{\overline{U}}{\overline{u}}\right), \tag{1.3}$$

is used to estimate $\overline{V}$ of v; here this is positively (high) associated with a supplementary (ancillary) variable u, for example, v maybe COVID-19 positive cases, and u is the sample testing [Pal and Pal 2021, Mukherjee et. al.(2022)], where $\overline{U}$ and $\overline{u}$, the population mean, and sample mean respectively of u are assumed to be known.

The $MSE(t_R)$ is given by

$$MSE(t_R) = \eta \overline{U}^2 [C_v^2 + C_u^2 - 2KC_u^2], \tag{1.4}$$

where the coefficient of variation of u is

$$C_u^2 = (S_u^2 / \overline{U}^2),\ S_u^2 = (N-1)^{-1}\sum_{i=1}^{N}(u_i - \overline{U})^2,$$
$$K = \rho(C_v / C_u),\ \rho = (S_{uv} / S_u S_v)$$

(correlation coefficient between v and u) and

$$S_{uv} = (N-1)^{-1}\sum_{i=1}^{N}(u_i - \bar{U})(v_i - \bar{V})$$

(covariance between u and v).

On the other hand, when u is negatively correlated (high) with v (for example, during the pandemic COVID-19, the lockdown and growth in the economy of a country (Pal and Pal 2021). Robson (1957) proposed and Murthy (1964) rediscovered the product method of an estimate, defined as

$$t_P = \bar{v}\left(\frac{\bar{u}}{\bar{U}}\right). \tag{1.5}$$

The $MSE(t_P)$ is given by

$$MSE(t_P) = \eta\bar{U}^2[C_v^2 + C_u^2 + 2KC_u^2], \tag{1.6}$$

Furthermore, when the predictive variable is linked with an ancillary variable, Hansen et. al. (1953) presented a different technique of population mean finding in the same scenario when the predictive variable v is closely related to an ancillary variable u as

$$t_{\lambda} = [\bar{v} + \lambda(\bar{U} - \bar{u})], \tag{1.7}$$

Here λ is a suitably chosen constant.

The $MSE(t_{\lambda})$ is given by

$$MSE(t_{\lambda}) = \eta[S_v^2 + \lambda^2 S_u^2 - 2\lambda S_{uv}], \tag{1.8}$$

which is least when

$$\lambda = \left(\frac{S_{uv}}{S_u^2}\right). \tag{1.9}$$

Therefore, the lowest amount $MSE(t_{\lambda})$ is written as:

$$MSE_{\min}(t_{\lambda}) = \eta\bar{U}^2(1-\rho^2)C_v^2. \tag{1.10}$$

The customary linear regression method [Watson (1937)] of estimation, defined as

$$t_{\mathrm{Re}g} = [\bar{v} + \beta(\bar{U} - \bar{u})], \tag{1.11}$$

Here β is preassumed to be known.

The MSE (Mean squared error) [or variance] of t_{λ} is shown as:

$$MSE(t_{\mathrm{Re}g}) = Var(t_{\mathrm{Re}g}) = \eta\bar{V}^2(1-\rho^2)C_v^2 = MSE_{\min}(t_{\lambda}). \tag{1.12}$$

The difference estimator performs equally to the typical linear regression estimator at their optimum condition, according to (1.10) and (1.12). Contrarily, when the relationship between the predictive variable, and the ancillary variable is a straight line passing through the origin, the classic ratio and product approaches have efficiency comparable to the traditional linear regression estimator. This pushes authors to develop an estimate that is more effective than the conventional regression estimator (for figuring out the population mean of the predictive variable).

2. The suggested estimator

The new development of the estimator for $\bar{V}$ of v in SRS is given as

$$\ell = \bar{v}\left(\frac{a+b\bar{U}+cS_u^2}{a+b\bar{u}+cs_u^2}\right), \tag{2.1}$$

where

$$s_u^2 = (n-1)^{-1}\sum_{i=1}^{n}(u_i-\bar{u})^2$$

(the usual unbiased estimator of S_u^2), and (a,b,c) are either (parameters) or (real numbers) or related to u such as C_u, $\beta_1(u)$, $\beta_2(u)$ and ρ between v and u.

For finding the *Bais(BS)* and *MSE* of the newly developed estimator ℓ of $\bar{V}$, we have defined

$$\tau_0 = \{(\bar{v}-\bar{V})/\bar{V}\},\ \tau_1 = \{(\bar{u}-\bar{U})/\bar{U}\},\ \tau_2 = \{(s_u^2-S_u^2)/S_u^2\}.$$

such that $[E(\tau_0)=E(\tau_1)=E(\tau_2)=0]$, and to the *FDA*, we have

$$E(\tau_0^2)=\eta C_v^2,\ E(\tau_1^2)=\eta C_u^2, E(\tau_2^2)=\eta\lambda_{04}^*,$$
$$E(\tau_0\tau_1)=\eta K C_u^2,\ E(\tau_0\tau_2)=\eta\lambda_{12}C_v, E(\tau_1\tau_2)=\eta\lambda_{03}C_u,$$

Here $\lambda_{rs}^*=(\lambda_{rs}-1)$, $\lambda_{rs}=(\mu_{rs}/\mu_{20}^{r/2}\mu_{02}^{s/2})$, $\mu_{rs}=N^{-1}\sum_{i=1}^{N}(v_i-\bar{V})^r(u_i-\bar{U})^s$.

To find the *Bias(BS)* and *MSE* of the newly developed estimator ℓ, we explore (2.1) in terms of τ's, we have

$$\ell = \bar{V}(1+\tau_0)\left[1+\frac{b\bar{U}\tau_1+cS_u^2\tau_2}{a+b\bar{U}+cS_u^2}\right]^{-1}$$
$$=\bar{V}(1+\tau_0)(1-\Omega_1\tau_1-\Omega_2\tau_2+\Omega_1^2\tau_1^2+\Omega_2^2\tau_2^2+2\Omega_1\Omega_2\tau_1\tau_2-...)$$
$$\approx\bar{V}(1+\tau_0-(\Omega_1\tau_1+\Omega_2\tau_2)-(\Omega_1\tau_0\tau_1+\Omega_2\tau_0\tau_2)+\Omega_1^2\tau_1^2+\Omega_2^2\tau_2^2+2\Omega_1\Omega_2\tau_1\tau_2)$$

or

$$(\ell - \bar{V}) = \bar{V}(\tau_0 - (\Omega_1\tau_1 + \Omega_2\tau_2) - (\Omega_1\tau_0\tau_1 + \Omega_2\tau_0\tau_2) + \Omega_1^2\tau_1^2 + \Omega_2^2\tau_2^2 + 2\Omega_1\Omega_2\tau_1\tau_2), \tag{2.2}$$

where

$$\Omega_1 = b\bar{U}(a + b\bar{U} + cS_u^2)^{-1}, \ \Omega_2 = cS_u^2(a + b\bar{U} + cS_u^2)^{-1}.$$

The *Bias* (*BS*) of developed estimator ℓ up to *FDA*, as

$$\begin{aligned} Bias(\ell) &= \eta\bar{V}(\Omega_1^2C_u^2 + \Omega_2^2\lambda_{04}^* + 2\Omega_1\Omega_2\lambda_{03}C_u - (\Omega_1KC_u^2 + \Omega_2\lambda_{12}C_v)) \\ &= \eta\bar{V}(\Omega_1C_u(\Omega_1C_u + \Omega_2\lambda_{03} - KC_u) + \Omega_2\{\Omega_2\lambda_{04}^* + \Omega_1\lambda_{03}C_u - \lambda_{12}C_v\}). \end{aligned} \tag{2.3}$$

Expectation and square of (2.2), we require

$$\begin{aligned} (\ell - \bar{V})^2 &= \bar{V}^2(\tau_0 - \Omega_1\tau_1 - \Omega_2\tau_2)^2 \\ &= \bar{V}^2(\tau_0^2 + \Omega_1^2\tau_1^2 + \Omega_2^2\tau_2^2 + 2\Omega_1\Omega_2\tau_1\tau_2 - 2\Omega_1\tau_0\tau_1 - 2\Omega_2\tau_0\tau_2). \end{aligned} \tag{2.4}$$

The MSE of newly developed estimator ℓ up to *FDA*, as

$$MSE(\ell) = \eta\bar{V}^2(C_v^2 + \Omega_1^2C_u^2 + \Omega_2^2\lambda_{04}^* + 2\Omega_1\Omega_2\lambda_{03}C_u - 2\theta_1KC_u^2 - 2\theta_2\lambda_{12}C_v), \tag{2.5}$$

which is minimized for

$$\left.\begin{aligned} \Omega_1 &= \frac{(\lambda_{04}^*KC_u - \lambda_{03}\lambda_{12}C_v)}{C_u(\lambda_{04}^* - \lambda_{03}^2)} = \Omega_{1(opt)} \\ \Omega_2 &= \frac{(\lambda_{12}C_v - \lambda_{03}KC_u)}{(\lambda_{04}^* - \lambda_{03}^2)} = \Omega_{2(opt)} \end{aligned}\right\}, \text{(say)}. \tag{2.6}$$

Here the lowest amount of *MSE* of the newly developed estimator ℓ up to the *FDA* is given by

$$MSE_{\min}(\ell) = \eta\bar{V}^2\left[C_v^2 - K^2C_u^2 - \frac{(\lambda_{03}KC_u - \lambda_{12}C_v)^2}{(\lambda_{04}^* - \lambda_{03}^2)}\right]. \tag{2.7}$$

Remark 2.1: Note that *MSE* of newly developed estimator ℓ at (2.5) and optimum values of constants Ω_1 and Ω_2 at (3.6) involve unknown population parameters such as C_u, C_v, ρ, λ_{04} etc. If we put distinct values (a, b, c) in (2.1), the newly developed estimator ℓ transfer to some recognized estimators of $\bar{V}$ which are specified in Table 1.

Table 1

Several known members of the newly developed estimator ℓ

a	b	c	Estimator
1	0	0	$\ell_0 = \bar{v}$, [The customary unbiased estimator]
0	1	0	$\ell_1 = t_R = \bar{v}\left(\frac{\bar{U}}{\bar{u}}\right)$, [Cochran (1940)]
C_u	1	0	$\ell_2 = \bar{v}\left(\frac{C_u+\bar{U}}{C_u+\bar{u}}\right)$, [Sisodia and Dwivedi (1981)]
C_u	$\beta_2(u)$	0	$\ell_3 = \bar{v}\left(\frac{C_u+\beta_2(u)\bar{U}}{C_u+\beta_2(u)\bar{u}}\right)$, [Upadhyaya and Singh (1999)]
$\beta_2(u)$	C_u	0	$\ell_4 = \bar{v}\left(\frac{\beta_2(u)+C_u\bar{U}}{\beta_2(u)+C_u\bar{u}}\right)$, [Upadhyaya and Singh (1999)]
ρ	1	0	$\ell_5 = \bar{v}\left(\frac{\rho+\bar{U}}{\rho+\bar{u}}\right)$, [Singh and Tailor (2003)]
$\beta_2(u)$	1	0	$\ell_6 = \bar{v}\left(\frac{\beta_2(u)+\bar{U}}{\beta_2(u)+\bar{u}}\right)$, [Singh et. al. (2004)]
ρ	C_u	0	$\ell_7 = \bar{v}\left(\frac{\rho+C_u\bar{U}}{\rho+C_u\bar{u}}\right)$, [Kadilar and Cingi (2006)]
C_u	ρ	0	$\ell_8 = \bar{v}\left(\frac{C_u+\rho\bar{U}}{C_u+\rho\bar{u}}\right)$, [Kadilar and Cingi (2006)]
$\beta_1(u)$	1	0	$\ell_9 = \bar{v}\left(\frac{\beta_1(u)+\bar{U}}{\beta_1(u)+\bar{u}}\right)$, [Yan and Tian (2010)]
$\beta_2(u)$	$\beta_1(u)$	0	$\ell_{10} = \bar{v}\left(\frac{\beta_2(u)+\beta_1(u)\bar{U}}{\beta_2(u)+\beta_1(u)\bar{u}}\right)$, [Yan and Tian (2010)]

Some new members of the newly developed estimator ℓ are also generated which are specified in Table 2.

Table 2

Different new members of the newly developed estimator ℓ

a	b	c	Estimator
C_u	S_u^2	$\beta_2(u)^2$	$\ell_{11} = \bar{v}\left(\frac{C_u+S_u^2\bar{U}+\beta_2(u)^2 S_u^2}{C_u+S_u^2\bar{u}+\beta_2(u)^2 s_u^2}\right)$
ρ	$-\bar{U}$	$-f$	$\ell_{12} = \bar{v}\left(\frac{\rho-\bar{U}^2-fS_u^2}{\rho-\bar{U}\bar{u}-fs_u^2}\right)$
$\beta_2(u)$	$\bar{U}$	$K-1$	$\ell_{13} = \bar{v}\left(\frac{\beta_2(u)+\bar{U}^2+(K-1)S_u^2}{\beta_2(u)+\bar{U}\bar{u}+(K-1)s_u^2}\right)$

Remark 2.2: We would like to note that by entering the appropriate values for the scalars (*a, b, c*) as given in Tables 1 and 2, it is simple to acquire the properties of the estimators that are components of the newly produced estimator from those of the recommended estimator.

Remark 2.3: Also, we can delineate a product-type estimator for $\bar{V}$ as

$$\ell^* = \bar{v}\left(\frac{a+b\bar{u}+cs_u^2}{a+b\bar{U}+cS_u^2}\right), \tag{2.8}$$

The exact *Bias (BS)* and MSE of ℓ^* are correspondingly given by

$$BS(\ell^*) = \eta S_v[\Omega_1\rho C_u + N/(N-2)\Omega_2\lambda_{12}], \tag{2.9}$$

$$\begin{aligned} MSE(\ell^*) = \bar{V}^2[&E(\tau_0^2)+\Omega_1^2E(\tau_1^2)+\Omega_2^2E(\tau_2^2)+2\Omega_1E(\tau_0\tau_1)+2\Omega_2E(\tau_0\tau_2)\\ &+2\Omega_1\Omega_2E(\tau_1\tau_2)+\{\Omega_1^2E(\tau_0^2\tau_1^2)+\Omega_2^2E(\tau_0^2\tau_1^2)+2\Omega_1E(\tau_0^2\tau_1)+2\Omega_2E(\tau_0^2\tau_2)\\ &+2\Omega_1^2E(\tau_0\tau_1^2)+4\Omega_1\Omega_2E(\tau_0\tau_1\tau_2)+2\Omega_2^2E(\tau_0\tau_2^2)+2\Omega_1\Omega_2E(\tau_0^2\tau_1\tau_2)\}] \end{aligned} \tag{2.10}$$

As pointed out by Murthy (1967, pp380-381), by using finite population correction (*FPC*), we find the *MSE* of the product type estimator ℓ^* as:

$$MSE(\ell^*) = \eta\bar{V}^2(C_v^2+\Omega_1^2C_u^2+\Omega_2^2\lambda_{04}^*+2\Omega_1\Omega_2\lambda_{03}C_u+2\Omega_1KC_u^2+2\Omega_2\lambda_{12}C_v), \tag{2.11}$$

which is minimized for

$$\left.\begin{aligned} \Omega_1 &= \frac{(\lambda_{03}\lambda_{12}-\lambda_{04}^*KC_u)C_v}{(\lambda_{04}^*-\lambda_{03}^2)C_u} = \Omega_{1(opt)}^*\\ \Omega_2 &= \frac{(\lambda_{03}\rho-\lambda_{12})C_v}{(\lambda_{04}^*-\lambda_{03}^2)} = \Omega_{2(opt)}^* \end{aligned}\right\}, \text{ (say).} \tag{2.12}$$

Here the lowest amount MSE of developed estimator ℓ^* up to *FDA* is given by

$$MSE_{\min}(\ell^*) = \eta\bar{V}^2\left[C_v^2 - K^2C_u^2 - \frac{(\lambda_{03}KC_u-\lambda_{12}C_v)^2}{(\lambda_{04}^*-\lambda_{03}^2)}\right]. \tag{2.13}$$

which equals the lowest amount of MSE of the ratio estimator ℓ.

Remark 2.4: Similarly one can also define a new type of efficient estimator of S_v^2 of v in *SRS* as:

$$P = s_v^2\left(\frac{a+b\overline{X}+cS_u^2}{a+b\overline{x}+cs_u^2}\right) \tag{2.14}$$

3. Efficiency comparison

We have prepared some theoretical comparisons between newly developed estimators and conventional estimators.

From (1.2), (1.4), (1.6), (1.12), and (2.7), we have

$$Var(\overline{v})-MSE_{\min}(\ell)=\eta\overline{V}^2\left[K^2C_u^2+\frac{(\lambda_{03}KC_u-\lambda_{12}C_v)^2}{(\lambda_{04}^*-\lambda_{03}^2)}\right]\geq 0, \tag{3.1}$$

provided $\lambda_{03}\rho\neq\lambda_{12}$.

$$MSE(t_R)-MSE_{\min}(\ell)=\eta\overline{V}^2\left[C_u^2(1-K)^2+\frac{(\lambda_{03}KC_u-\lambda_{12}C_v)^2}{(\lambda_{04}^*-\lambda_{03}^2)}\right]\geq 0, \tag{3.2}$$

provided $K\neq 1$ and $\lambda_{03}\rho\neq\lambda_{12}$.

$$MSE(t_P)-MSE_{\min}(\ell)=\eta\overline{V}^2\left[C_u^2(1+K)^2+\frac{(\lambda_{03}KC_u-\lambda_{12}C_v)^2}{(\lambda_{04}^*-\lambda_{03}^2)}\right]\geq 0, \tag{3.3}$$

provided $K\neq -1$ and $\lambda_{03}\rho\neq\lambda_{12}$.

$$MSE(t_{\mathrm{Re}g})-MSE_{\min}(\ell)=\eta\overline{V}^2\left[\frac{(\lambda_{03}KC_u-\lambda_{12}C_v)^2}{(\lambda_{04}^*-\lambda_{03}^2)}\right]\geq 0, \tag{3.4}$$

provided $\lambda_{03}\rho\neq\lambda_{12}$.

Expressions (3.1), (3.2),(3.3), and (3.4) are positive. Thus it follows that the newly developed estimator ℓ is a more efficient estimator than the $\overline{v}$, t_R, t_P and the $t_{\mathrm{Re}g}$ at its optimal condition (2.6).

4. Numerical illustrations

For evaluation of the virtues of the newly developed estimator ℓ relative to the other existing estimators of $\overline{V}$ we have considered the population data set as follows:

Source: Murthy (1967).

$N = 34$, $n = 5$, $f = 0.1471$, $\rho = 0.9769$, $\bar{V} = 199.44$, $\bar{U} = 208.89$, $C_v = 0.6187$, $C_u = 0.5664$, $K = 1.0671$, $\lambda_{04} = \beta_2(u) = 2.2246$, $\lambda_{03} = \sqrt{\beta_1(u)} = 0.4949$, $\lambda_{12} = 0.5616$, $S_u = 118.3153$. We have computed PREs of estimators ℓ_i of $\bar{V}$ concerning $\bar{v}$ using the following formula:

$$PRE(\ell_i, \bar{v}) = \frac{MSE(\bar{v})}{MSE(\ell_i)} \times 100, \quad (i = 1, 2, \ldots, 15) \tag{4.1}$$

and results are demonstrated in Table 3.

Table 3

PREs of estimators of $\bar{V}$ concerning $\bar{v}$

Estimator	PRE
$\ell_0 = \bar{v}$	100.00
$\ell_1 = t_R$	2022.64
ℓ_2	2010.03
ℓ_3	2017.00
ℓ_4	1930.44
ℓ_5	2000.72
ℓ_6	1971.66
ℓ_7	1983.43
ℓ_8	2009.73
ℓ_9	2017.22
ℓ_{10}	1799.13
ℓ_{11}	2207.43
ℓ_{12}	2276.85
ℓ_{13}	2194.65

We have also calculated the PREs of the customary linear regression estimator t_{Reg} and newly developed estimator ℓ at their optimum conditions concerning the customary unbiased estimator $\bar{v}$ respectively as:

$$PRE(t_{\mathrm{Reg}}, \bar{v}) = \frac{MSE(\bar{v})}{Var(t_{\mathrm{Reg}})} \times 100 = 2189.79, \tag{4.2}$$

$$PRE(t_{opt}, \overline{v}) = \frac{MSE(\overline{v})}{MSE_{\min}(\ell)} \times 100 = 2535.81. \tag{4.3}$$

From (4.2), (4.3), and the Table-3, we observed that the suggested estimators ℓ_i, (i = 11, 12, and 13) performed better than $\ell_0 = \overline{v}$, $\ell_1 = t_R$, $t_{\mathrm{Re}g}$, ℓ_2 ℓ_3, ℓ_4, ℓ_5, ℓ_6 ℓ_7, ℓ_8 and ℓ_9, ℓ_{10} in the sense of having a bigger gain inefficiency. However, the PRE of the recommended estimator ℓ at their optimum condition is the largest among all the estimators discussed here.

5. Conclusion and Recommendation

In this work, the newly developed estimator's Bias (*BS*) and *MSE* equations have been obtained by the *FDA*. The suggested estimator includes the customary unbiased and ratio estimators, as well as estimators from Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et. al. (2004), Kadilar and Cingi (2006), and Yan and Tian (2010). The existing estimator's attributes are easily accessible via the newly developed estimator's properties. Based on findings, the recommended estimator can effectively combine numerous results in one spot. The newly developed estimator is more efficient than other current population mean estimators both conceptually and experimentally. As a result, we recommend that the newly developed estimator be used in practice. However, due to the small empirical study, this finding cannot be generalized.

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