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Independent domination bondage number in graphs

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Abstract

We study the independent domination bondage number in simple graphs. An independent dominating set S of a graph G is characterized by two conditions: its vertices are not adjacent to each other, and all vertices outside S are adjacent to at least one vertex within S. The independent domination number, $\gamma_i(G)$, represents the smallest possible size of such a set. Our focus is on the independent domination bondage number, which quantifies the minimum edge removals needed to alter $\gamma_i(G)$. This research establishes several bounds for this parameter and computes its value for specific graph operations involving two graphs.

Subject Classification: *05C05, 05C69.*

Keywords: *Dominating set, Independent domination number, Independent domination bondage number, Operation.*

1. Introduction

Graph theory offers a powerful framework for modeling relationships between discrete objects. A simple graph $G = (V, E)$ consists of a finite set of vertices V and a set of edges E connecting pairs of these vertices. To understand the structure of a graph, we often examine the connections around its vertices. The open neighborhood of a vertex v, denoted $N(v)$, includes all vertices directly connected to v. When we include v itself, we refer to it as the closed neighborhood, $N[v]$. For a set of vertices S, $N(S)$ and $N[S]$ extend these definitions to encompass the neighborhoods of all vertices within S. A unique

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concept for a vertex v within a set S is its private neighborhood, $pn(v,S)$, which contains vertices exclusively connected to v (and no other vertex in S). The degree of a vertex, $deg(v)$, simply counts its direct connections, equivalent to the size of its open neighborhood. A vertex with a degree of one in a tree is commonly called a leaf.

A fundamental concept in graph theory is domination. A dominating set S in a graph G is a subset of vertices such that every vertex not in S is connected to at least one vertex in S. The domination number, $\gamma(G)$, represents the smallest possible size of such a dominating set. It's a measure of how efficiently you can control or cover the entire graph from a small number of points. Finding the domination number is a difficult problem for general graphs. It's a classic NP-complete problem, meaning there is no known efficient algorithm to solve it for all graphs. The field of dominating sets is vast, with over 70 known types and corresponding domination numbers [1, 2].

The stability of domination is also of interest. A domination-critical vertex is one whose removal from a graph decreases its domination number [3, 4]. Interestingly, adding an edge to a graph can only decrease or maintain the domination number, meaning $\gamma(G) - 1 \leq \gamma(G + e) \leq \gamma(G)$ for any edge e not already present. A graph is considered domination stable, or γ-stable, if adding any edge to its complement does not change its domination number [5, 6]. Various aspects of domination stability have been explored for different types of dominating sets [7, 8, 9, 10, 11, 12].

The bondage number, on the other hand, is a measure of the graph's vulnerability in relation to its domination number. Imagine you have a network and you want to know how many connections (edges) you need to sever to increase the number of control points you need. The bondage number, $b(G)$, is the minimum number of edges whose removal from a graph G results in a new graph, G', with a strictly larger domination number, i.e., $\gamma(G') > \gamma(G)$. It quantifies the robustness of a graph's dominating set structure against edge failures. A high bondage number suggests the graph is more resilient to losing connections without requiring a larger dominating set. A low bondage number, like 1, means the removal of just one edge can increase the minimum number of vertices needed to dominate the graph.

This paper focuses specifically on independent dominating sets. An independent set is a collection of vertices where no two vertices are connected by an edge. An independent dominating set combines these two properties: it is a dominating set where all its members are pairwise non-adjacent.

Imagine you are designing a security system for a large facility, like a warehouse or a university campus, which can be modeled as a graph where vertices are locations and edges are lines of sight. You need to place cameras or sensors to monitor every single location. However, for a variety of reasonsâ€”like avoiding electrical interference or simplifying maintenanceâ€”you want to ensure that no two surveillance devices are placed in locations directly adjacent to each other. The independent domination number of this graph gives you the smallest number of cameras or sensors you need to achieve this goal.More precisely, the

independent domination number, $\gamma_i(G)$, is the smallest possible size of an independent dominating set. Closely related is the independence number, $\alpha(G)$, which is the maximum size of any independent set. By definition, the relationships $\gamma(G) \leq \gamma_i(G) \leq \alpha(G)$ always hold. We refer to a dominating set of minimum size as a γ-set, and an independent dominating set of minimum size as an γ_i-set.

Independent domination stability is a metric for a graph's robustness. It's defined as the fewest number of vertices you need to delete to force the graph's independent domination number to change. This means a more stable graph can lose more vertices before its domination number changes. The authors in [13] explored the characteristics of this stability measure and calculated a number of upper and lower bounds for it. They found the independent domination stability for certain types of graph. They also examined how the id-stability changes when we combine two graphs in specific ways.

A graph G is deemed independent domination critical, or γ_i-critical, if removing any single vertex from the graph changes its independent domination number; effectively, no vertex can be removed from an γ_i-set without increasing the total number of vertices needed for independent domination. Research on independent domination and γ_i-critical graphs has been well-documented in the literature [7, 12, 14, 15].

This paper is structured as follows. In Section 2, we introduce the concept of the independent domination bondage number (id-bondage number) of graphs and discuss some of its fundamental properties. Section 3 presents various bounds for the independent domination bondage number. Finally, in Section 4, we investigate the independent domination bondage number for certain operations involving two graphs.

2. id-bondage number of specific graphs

Here, we will begin by examining the independent domination bondage number, also known as the id-bondage number, of graph G, denoted as $b_{id}(G)$, which is defined as the minimum number of edges that need to be removed from the graph such that the independent domination number of the graph changes. Formally, the independent domination bondage number of a graph G is given by $b_{id}(G) = \min\{|E'|: \gamma_i(G - E') \neq \gamma_i(G)\}$.

Now we determine the independent domination bondage number for specific classes of graphs. To facilitate this, we will start by presenting some initial observations. The following observation is easy to obtain:

Observation 2.1:

(i) If $G = K_{1,n}$ is a star, then $b_{id}(K_{1,n}) = 1$.

(ii) In a complete bipartite graph $K_{m,n}$, the independent domination bondage number is simply the minimum value between m and n.

Imagine taking n separate triangles and gluing them together at just one corner; that is a friendship graph F_n.

Interestingly, the Friendship Theorem (by Erdös, Rényi and Sós [16]) tells us something unique: if every two vertices in a graph have exactly one neighbor in common, then that graph is always a friendship graph. Extending this idea, a generalized friendship graph ($F_{q,n}$) is similar but uses n cycles of any length q (as long as q is 3 or more) all sharing a single vertex (Figure 1).

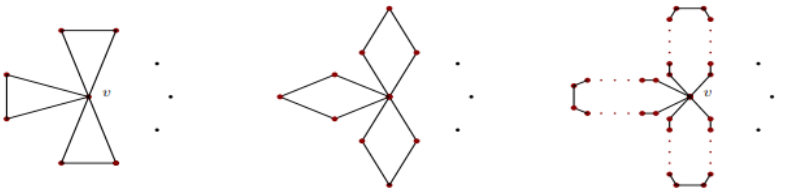


Figure 1
The generalized friendship graphs.

The n-book graph B_n $(n \geq 2)$ is constructed by taking the Cartesian product of $K_{1,n}$ and P_2 (Figure 2). Think of it as n square pages, each sharing a common edge, v_1v_2. We determine the independent domination number for both friendship graphs and book graphs using the next observation.

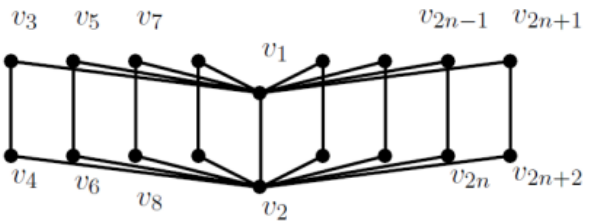


Figure 2
The graph B_n.

Observation 2.2:

(i) The independent domination number of a friendship graph F_n is 1.

(ii) For generalized friendship graphs where each cycle has 4,5, or 6 vertices (i.e., $F_{4,n}, F_{5,n}$ and $F_{6,n}$), their independent domination number is $n + 1$.

(iii) If B_n is a book graph, its independent domination number is n.

Now we obtain the id-bondage number of the friendship and the book graphs:

Proposition 2.3:

(i) $b_{id}(F_n) = 2$.

(ii) For $k = 4,5,6$, $b_{id}(F_{k,n}) = 3$.

(iii) $b_{id}(B_n) = 3$.

Proof:

(i) By removing two edges incident to central vertex, we have:

$$\begin{aligned}\gamma_i(F_n - \{e_1, e_2\}) &= \gamma_i(K_2 \cup F_{n-1}) \\ &= \gamma_i(K_2) + \gamma_i(F_{n-1}) \\ &= 1 + 1 = 2 \neq \gamma_i(F_n).\end{aligned}$$

So this leads to our conclusion.

(ii) By removing three edges from $F_{k,n}$, for $k = 4,5,6$, we have:

$$\begin{aligned}\gamma_i(F_{k,n} - \{e_1, e_2, e_3\}) &= \gamma_i(F_{k,n-1} \cup 2K_2), \\ &= \gamma_i(F_{k,n-1}) + \gamma_i(2K_2) \\ &= n + 2 \neq n + 1.\end{aligned}$$

Thus, we have the result.

(iii) When three adjacent edges $\{e_1, e_2, e_3\}$ from B_n are removed, we have:

$$\begin{aligned}\gamma_i(B_n - \{e_1, e_2, e_3\}) &= \gamma_i(2K_1 \cup B_{n-1}), \\ &= \gamma_i(2K_1) + \gamma_i(B_{n-1}) \\ &= 2 + n - 1 = n + 1 \neq n.\end{aligned}$$

Thus, this leads to our conclusion. +

Theorem 2.4: *There are graphs G and H with $\gamma_i(G) = \gamma_i(H)$, but their independent domination bondage numbers can differ by an arbitrarily large amount.*

Proof: Let $G = K_n$ and $H = K_{1,n-1}$. For $n \geq 3$, $\gamma_i(G) = \gamma_i(H) = 1$, $b_{id}(G) = n - 1$, and $b_{id}(H) = 1$. We have $|b_{id}(G) - b_{id}(H)| = n - 2$, which grows arbitrarily large as $n \to \infty$. Thus, this leads to our conclusion.

To begin our study of the independent domination bondage number for paths and cycles, we will first make the following observation.

Observation 2.5: [14] The independent domination number for a path graph P_n and a cycle graph C_n is given by the ceiling of n divided by 3.

Now, we determine the id-bondage of paths.

Proposition 2.6: The independent domination bondage number for a path graph P_n is 2 when $n \equiv 1(mod3)$. In all other cases, $b_{id}(P_n)$ is 1.

Proof: Here are the cases we consider:

Case 1: $n \equiv 1(mod3)$

For $n = 3k + 1$, for some integer $k \geq 1$. By eliminating edges e_1 and e_2 of P_n, we have $\gamma_i(P_n - \{e_1, e_2\}) = \gamma_i(2K_1 \cup P_{n-2}) = 2 + \gamma_i(P_{n-2}) = k + 2 \neq k + 1 = \gamma_i(P_n)$. Thus, $b_{id}(P_n) = 2$.

Case 2. $n \not\equiv 1(mod\ 3)$

For $n = 3k$ or $n = 3k + 2$, removing any single edge suffices to increase $\gamma_i(P_n)$, so we have $\gamma_i(P_n - e_1) > \gamma_i(P_n)$. Thus, $b_{id}(P_n) = 1$.

Next, we determine the id-bondage number of cycles.

Proposition 2.7: The id-bondage number for a cycle graph C_n is 3 when n is congruent to 1 modulo 3. In all other cases, $b_{id}(C_n)$ is 2.

Proof: Here are the cases we consider:

Case 1: $n \not\equiv 1(mod\ 3)$

For $n = 3k$ or $n = 3k + 2$, removing two specific edges suffices to change $\gamma_i(C_n)$. We have $\gamma_i(C_n - \{e_1, e_2\}) = \gamma_i(K_1 \cup P_{n-1}) = 1 + \gamma_i(P_{n-1}) \neq \gamma_i(C_n)$. Thus, $b_{id}(C_n) = 2$.

Case 2: $n \equiv 1(mod 3)$

For $n = 3k + 1$, removing three specific edges change $\gamma_i(C_n)$. We have $\gamma_i(C_n - \{e_1, e_2, e_3\}) = \gamma_i(2K_1 \cup P_{n-3}) = 2 + \gamma_i(P_{n-3}) \neq \gamma_i(C_n)$. Thus, $b_{id}(C_n) = 3$.

The following observation determines the independent domination number of the complete graph K_n.

Observation 2.8: If K_n is a complete graph, then $\gamma_i(K_n) = 1$.

Now, we will compute the id-bondage number of the complete graph K_n.

Proposition 2.9: If K_n is a complete graph of order n, then $b_{id}(K_n) = n - 1$.

Proof: We know that $\gamma_i(K_n) = 1$. Removing at least $n - 1$ edges to change the independent domination number of K_n,

$$\begin{aligned}\gamma_i(K_n - (n-1)e) &= \gamma_i(K_i \cup K_{n-1}) \\ &= \gamma_i(K_1) + \gamma_i(K_{n-1}) \\ &= 1 + 1 = 2 \neq \gamma_i(K_n) = 1.\end{aligned}$$

Thus, $b_{id}(K_n) = n - 1$. Hence, we have the result.

Next, we turn our attention to cactus graphs.

A cactus graph is a connected graph where no single edge belongs to more than one cycle. This means that every block (a maximal 2-connected subgraph) within a cactus graph is either a simple edge or a cycle. If all the cycle blocks in a cactus graph G have the same number of vertices, say m, then the cactus is called m-uniform.

A triangular cactus is a specific type of cactus graph where all its blocks are triangles (cycles of length 3), making it a 3-uniform cactus. A cut-vertex in a triangular cactus is a vertex that is part of two or more triangles. If each triangle in a triangular cactus G shares at most two cut-vertices, and every cut-vertex is part

of exactly two triangles, then G is called a chain triangular cactus (Figure ??). The length of such a chain is defined by the number of triangles it contains. A chain triangular cactus with n triangles has $2n + 1$ vertices and $3n$ edges (see [17]).

If a chain triangular cactus contains more than a single triangle, it will invariably feature two end triangles. Each of these particular triangles is linked to the main structure by just one cut-vertex. This characteristic allows us to specifically refer to a chain triangular cactus of length n as T_n.

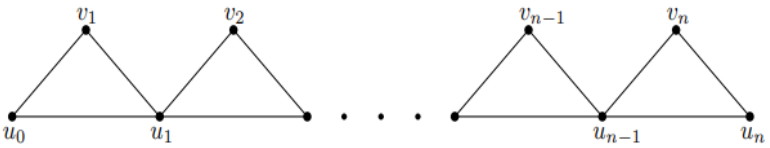


Figure 3
The cactus T_n.

If we replace the triangles in the previous definitions with C_4, we get square cacti, where each block is a C_4.

Figure 2.13 shows an example of a square cactus chain.

The way an internal square connects to its neighbors determines its type:

An ortho-square has its two cut-vertices (the vertices connecting it to other squares) adjacent to each other.

A para-square has its two cut-vertices non-adjacent. The general term for all chain para-square cacti of length n is n is Q_n. More specifically, if all internal squares in a chain of length n are para-squares, it is uniquely referred to as Q_n. Conversely, if all internal squares in a chain of length n are ortho-squares, it is uniquely denoted as O_n (see Figure 5).

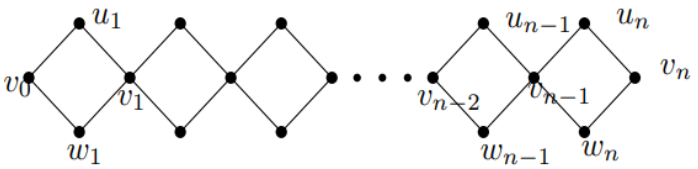


Figure 4
The cactus Q_n.

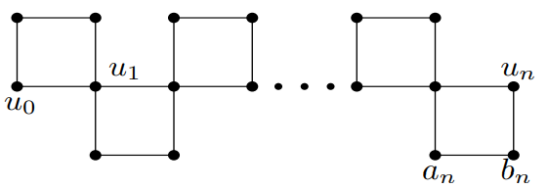


Figure 5
The cactus O_n.

The subsequent observation reveals the independent domination number of cactus graph T_n.

Observation 2.10: If T_n is a chain triangular cactus graph, then $\gamma_i(T_n) = n$.

We compute the id-bondage number of T_n.

Proposition 2.11: If T_n is a chain triangular cactus graph, then $b_{id}(T_n) = 2$.

Proof: By removing one edge from T_n, the independent domination number of T_n dose not change. However, if we remove two edges from T_n, the independent domination number of T_n will change. Since $\gamma_i(T_n) = n$, we have $\gamma_i(T_n - \{e_1, e_2\}) \neq \gamma_i(T_n) = n$. Thus, $b_{id}(T_n) = 2$.

The subsequent observation reveals the independent domination number of cactus graphs Q_n and O_n.

Observation 2.12: For the para-chain cactus graph and ortho-chain square cactus graph, $\gamma_i(Q_n) = \gamma_i(O_n) = \left\lceil \frac{n}{2} \right\rceil$.

We first compute the id-bondage number of the para-chain cactus graph Q_n.

Proposition 2.13: $b_{id}(Q_n) = 2$.

Proof: Removing one edge from Q_n does not increase $\gamma_i(Q_n)$ because each square is still independently dominated by a vertex. However, if we remove two edges from Q_n, the independent domination number will change. We have, $\gamma_i(Q_n - \{e, e_1\}) \neq \gamma_i(Q_n) = \left\lceil \frac{n}{2} \right\rceil$. Thus, $b_{id}(Q_n) = 2$.

Now we shall compute the id-bondage number of the ortho-chain square cactus graph O_n.

Proposition 2.14: If O_n is an ortho-chain square cactus graph, then $b_{id}(O_n) = 2$.

Proof: It is similar to proof of Proposition 2.13.

3. Bounds on the id-bondage number

Here, we will set some bounds for a graph's id-bondage number.

We will examine how the independent domination bondage number changes when an edge is removed from a graph (comparing $b_{id}(G)$ and $b_{id}(G - e)$). We will also establish some maximum possible values for $b_{id}(G)$.

Proposition 3.1: If you have a graph G and an edge e within it, then the independent domination bondage number of G is always less than or equal to the independent domination bondage number of $G - e$ plus one. In other words, $b_{id}(G) \leq b_{id}(G - e) + 1$.

Proof: When removing an edge e from a graph G does not change its independent domination number ($\gamma_i(G) = \gamma_i(G-e)$), it means that G 's independent domination bondage number ($b_{id}(G)$) cannot be much larger than that of the modified graph ($G-e$), specifically, it will be at most $b_{id}(G-e)+1$. And also if $\gamma_i(G) \neq \gamma_i(G-e)$, $b_{id}(G) = 1$ and so $b_{id}(G) \leq b_{id}(G-e)+1$.

Based on Proposition 3.1 and employing the principle of mathematical induction, we can conclude that

$$b_{id}(G) \leq b_{id}(G - e_1 - \cdots - e_s) + s,$$

where $1 \leq s \leq n-2$ and $n = |E(G)|$.

Theorem 3.2: *For a graph G containing n vertices, it follows that $b_{id}(G) \leq n+1-2\gamma_i(G)$.*

Proof: Let k represent the id-bondage number of graph G. This means that if we remove any number of edges from G (from 1 up to $k-1$ edges), say $e_1, \ldots, e_i$, the independent domination number of the graph remains unchanged:$\gamma_{id}(G) = \gamma_{id}(G-e_1) = \cdots = \gamma_{id}(G - e_1 - \cdots - e_i)$. We also know that a graph's independent domination number is never more than half its total number of vertices. Therefore, $\gamma_i(G) = \gamma_i(G - e_1 - \cdots - e_{k-1}) \leq \frac{n-k+1}{2}$.

Corollary 3.3: *If a graph G with at least two vertices has an independent domination bondage number of $n-1$, where n is the number of vertices, then its independent domination number must be* 1.

Proof: This result is a direct implication of Theorem 3.2.

4. id-Bondage in graph operations

This section explores the independent domination bondage number for various ways to combine two graphs. We will start with the join operation. The join of two graphs, $G \vee H$, is formed by taking the separate graphs G and H (assuming they do not share any vertices or edges) and adding all possible edges between every vertex of G and every vertex of H.

Observation 4.1: For any non-empty graphs G and H, the independent domination number of their join, $\gamma_i(G \vee H)$, is the smaller of their individual independent domination numbers, $\min\{\gamma_i(G), \gamma_i(H)\}$.

Proof: Any minimum independent dominating set from G (or H) will also serve as a minimum independent dominating set for the combined graph $G \vee H$. +

Following from Observation 4.1, we have:

Theorem 4.2: *If G and H are non-empty graphs, then the independent domination bondage number of their join, $b_{id}(G \vee H)$, is the minimum of their individual independent domination bondage numbers, $min\{b_{id}(G), b_{id}(H)\}$.*

Next, let's define the lexicographic product (also called composition) of two graphs, denoted $G[H]$. Imagine replacing each vertex in graph G with a complete copy of graph H.

Specifically, for two graphs G and H, $G[H]$ has a vertex for every possible pair of a vertex from G and a vertex from H. A vertex (a, a') is adjacent to (b, b') if either a is adjacent to b in G, or if a and b are the same vertex in G and a' is adjacent to b' in H (see e.g. [18]).

The following theorem gives the independent domination number for this product:

Theorem 4.3: *The independent domination number of the lexicographic product of G and H, $\gamma_i(G[H])$, is the product of their individual independent domination numbers, $\gamma_i(G)\gamma_i(H)$.*

Proof: To find a minimum independent dominating set in $G[H]$, you select a minimum independent dominating set in G. Then, for each copy of H (which replaces a vertex in G), you choose a minimum independent dominating set within that H copy.

Based on Theorem 4.3, we can state:

Corollary 4.4: For any two graphs G and H, the independent domination bondage number of their lexicographic product, $b_{id}(G[H])$, is the minimum of their individual independent domination bondage numbers, $\min\{b_{id}(G), b_{id}(H)\}$.

Finally, we determine the independent domination bondage number for the corona of two graphs.

Theorem 4.5: *For any two graphs G and H, the independent domination number of their corona, $\gamma_i(G \circ H)$, is the number of vertices in G multiplied by the independent domination number of H, i.e., so $|V(G)|\gamma_i(H)$.*

Proof: A minimum independent dominating set in $G \circ H$ is formed by finding a minimum independent dominating set within each of the copies of H that are attached to the vertices of G.

Theorem 4.5 leads to the following result:

Corollary 4.6: *For every two graphs G and H, the id-bondage number of their corona, $b_{id}(G \circ H)$, is less than or equal to the number of vertices in G multiplied by the independent domination bondage number of H, so $b_{id}(G \circ H) \leq |V(G)|b_{id}(H)$.*

Proof. Because the independent domination number of $G \circ H$ is directly dependent on $\gamma_i(H)$, removing $b_{id}(H)$ edges from H will change $\gamma_i(G \circ H)$, thus providing this upper bound.

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