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An integrated inventory model considering product display location selection

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Abstract

The selection of product display location will affect the sales volume of convenience store products. The purpose of this research is to get the best inventory policy from the vendor to the buyer considering product display location selection. This inventory system is derived from the concept of the cooperative pricing mechanism for the selection of shelf location and integrated inventory system. This paper derives the annual profit function of the

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integrated inventory system as the objective function. Explore the mathematical properties of this objective function and find the optimal solution of the objective function according to the properties. Finally, numerical examples are used to verify the feasibility of this solution procedure.

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Keywords: *Display location selection, Integrated inventory system, Annual profit.*

1. Introduction

In convenience stores, reducing total costs is the most widespread problem in inventory management. To run a profitable system in inventory management, it must be efficient enough to reduce overall costs. As we know that a convenience store has limited space, display racks for display products are one of the important areas in a convenience store. Therefore, shelf display management is considered as one of the most important things for a convenience store in making decisions. By properly managing the display shelves, convenience stores can increase their profits as well as lower total costs. Managing the proper allocation of shelf space and product displays can increase sales by attracting consumers' attention and encouraging further purchases. With a good inventory management, the policy can improve product quality, increase competitiveness, reduce inventory handling costs, enhance services, and increase operational flexibility by pulling the system. Inventory has an especially significant role in the supply chain system. The traditional economic order quantity model is an important part in the field of inventory system research, but most of its traditional inventory models only consider the single interests of buyers or suppliers. In the current supply chain management, considering the common position of vendors and buyers, and use long-term procurement contracts to stipulate a small number of multiple delivery methods for suppliers, reducing the cost of inventory for both parties. The joint cost can be optimized overall, and both parties can share the best long-term benefits. Vendors and buyers usually set up long-term production and purchase agreements, and maximize their common interests in a cooperative manner. Integrated inventory models often have the advantage of reducing total inventory costs or increasing total profits. Considering the cooperation perspective of vendors and buyers, develop an inventory model with product display position selection of merchandise to explore the integrated inventory model.

Schwaller [18] extended EOQ model to make it more consistent with a real-world inventory environment by adding the assumption that a certain percentage of the products are defective. Tersine [19] explained that inventory management is important in the progress of a company. By managing inventory properly, it will provide many benefits for the company, including reducing total costs, maximizing profits, and increasing customer satisfaction. Eduardo, Ali, and Sunil [6] stated that the main goal of inventory models is to maximize retailer's profit by understanding the optimum order quantity and inventory level. Goyal and Gupta [8] reviewed the literature that discusses the integrated inventory model. Goyal [7] modified the previous paper by considering that the model is based on the latest model and is derived based on assumptions according to the order size. However, Lu [17] suggested in the Goyal model that it is better to find this integrated inventory model using procedural solutions so that optimal policies can be found. Ha and Kim [9] found an integrated inventory model by determining the optimal policy of the integration model. The optimal policy that has been determined has a significant effect on the total cost. Hill [11] determined the integration model by considering policies with one vendor and one buyer. Based on this model, it has increased the number of sales and production. Huang [12] developed an integrated inventory model from vendors to buyers by determining the optimal policy. The aim of this integration model is to reduce the annual related costs. The solution procedure of this integration model is derived, the optimal solution is obtained. Chen et al. [5] showed that the integrated inventory model can optimize sales and control the amount of inventory so that total costs can be reduced. This integration model usually has constraints on controlling inventory and placing items. This obstacle can be prevented by determining product criteria based on product popularity and product price. Agrawal and Susheel [2] found that there are several mathematical models and schemes to maximize the integrated inventory profitability and determine the optimal production-inventory and pricing decisions for an integrated supply chain consisting of one buyer-several buyers where each buyer faces the same demand depends on the price. Lin et al. [16] explored the optimal inventory strategy for perishable goods using a two-stage pricing mechanism in the sales process. Chen [4] considered an integrated inventory model involving one manufacturer and multiple suppliers, allowing the payments which are permissible delay is considered in the proposed model's framework. The objective function of the model is

derived, and a decision support system is derived from the properties of the objective function.

Hwang, Bum, and Lee [13] explained that one of the really important things in a retail environment is the display racks where the products are displayed. Therefore, shelf display management is an important decision for retailers. Hariga, Abdulrahman, and Abdel [10] explained that the way a product is displayed on a shelf in a store can have a significant effect on sales levels. Effective and efficient shelf display management can reduce total costs and also increase customer satisfaction. Kamasak [15] discussed that the display height of the shelves in a retail store can affect the sales rate of a product. The statistical analysis states that shelf display height has a significant effect on sales levels. Ammar and Gulraiz [3] expressed that shelf display in retail stores can be influences sales because it can attract the attention and make the products more attractive to customer senses. The most profitable of items on the shelf-display between belt and eye level. That level is the best-selling height on a shelf display, at that level, it can place slow moving items and put items with low prices the furthest out of reach of the customer. The main area of the shelf-display is always used for high-demand, seasonal, impulsive, or promotional items. Abouali, Nermine, and Nashat [1] discussed that the height of the display shelf has an influence on customer decision making in buying a product. When the product is on the shelf at eye level, the sales level of the product will be high, while the product that is on a different shelf height, the sales level of the product is not too significant. Zhao, Zhou, and Mahab [20] explained that higher customer demand and keep costs as low as possible makes for efficient retail management. The allocation of shelf space and the location of the display area affects the cost and demand of products. On the shelf, placing products in the right place can attract customer's attention and encourage them to buy the products, but it also leads to higher display costs. Managing display shelf space for a particular product can provide benefits in the form of increased sales and reduced inventory costs so that properly managing the display location of shelf space will provide maximum profit. Jeganathan et al. [14] explored a dual-server queuing inventory system consisting of two parallel queues and high priority customers and low priority customers. They established priority service rules to minimize churn among low priority customers. The variation patterns of some important system indicators and appropriate cost functions were derived.

This paper considers the selection of product display location in the integrated inventory system. Using the concepts of cooperative pricing

mechanism of the product display location selection, the objective function of an integrated inventory system is derived and the mathematical properties of this function are explored. A simple solution process is proposed to obtain the optimal policy by the properties of the objective function.

2. Notation and assumptions

To derive the objective function, the following notation and assumptions are used.

Notation

i	position i of the product display location selection
Q_i	quantity per shipment of position i
D_i	annual demand of position i
Li	listing fees per order of location i
P	production rate
c	product unit cost
R	product selling price
ni	transportation frequency per order of the product display location i
Q	the order quantity, $Q = n_iQ_i$
F	transportation cost per shipment
S_b	ordering cost per order
S_v	setup cost for each batch of production
h_b	inventory carrying cost per unit per year for the buyer
h_v	inventory carrying cost per unit per year for the vendor
$TR(L_i)$	annual income
$TC\ (n_i, Q_i, L_i)$	annual relevant inventory cost
$TAP\ (n_i, Q_i, L_i)$	annual profit

Assumptions

(1) Productivity is constant.

(2) The time planning period is infinite.

(3) Consider a single product with only one vendor and one buyer.

(4) Schedule sequential deliveries so that the next batch reaches the buyer just as the stock of previous shipment batch runs out.

3. Mathematical Model

This paper discusses the integrated inventory system with the choice of product display location in convenience store. Using the concepts of cooperative pricing mechanism of the product display location, the objective function of the integrated inventory system is derived and the mathematical properties of this function are explored. Based on the previous assumptions, Figure 1 shows the inventory levels between the vendor and the buyer.

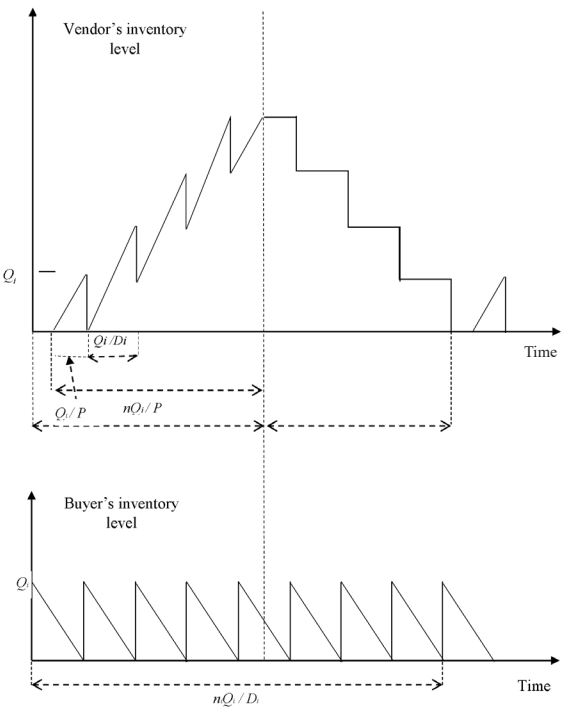


Figure 1

Inventory level trend for both vendor and buyer with time

The purpose of inventory management is to make optimal decisions while still minimizing costs so as to get maximum profit. In this section the optimal solution is sought for the inventory model that considers the position of the shelf display that can maximize profits. To determine the solution of this model, several equations are derived. The location of the shelf display has a product display location (*i*) independent location. The rental fee of placing the product on the *i*th position is L_i, and the annual demand (or sales volume) per year is D_i when the products are displayed at the position *i*. If you choose a position with better sales volume, you need to pay a higher rental cost. Further, for convenience, we assume that the rental fee is $L_1 \le L_2 \le L_3 \le ... \le L_n$, and the demand rate is $D_1 \le D_2 \le D_3 \le ... \le D_n$.

The annual integrated related cost consists of the purchase cost, ordering costs and holding costs. After adding the shelf display choice, the annual relevant inventory cost of the model is

$$TC\left(n_i, Q\right) = cD_i + \left\{\frac{\left(S_v + S_b\right)D_i}{n_i Q} + \frac{FD_i}{Q}\right\} + \frac{Q}{2}\left\{\left(n_i - 1\right) - \frac{\left(n_i - 2\right)D_i}{P}\right\}h_v + \frac{Q}{2}h_b + L_i \tag{1}$$

The annual revenue of the system is

$$TR\left(L_i\right) = D_i \times R \tag{2}$$

Combining equations (1) and (2), the annual profit is

$$TAP\left(n_i, Q_i, L_i\right) = TR\left(L_i\right) - TC\left(n_i, Q_i, L_i\right).$$

The annual profit is given by

$$TAP\left(n_i, Q_i, L_i\right)$$
$$= D_i \times (R - c) - \left\{\frac{\left(s_v + s_b\right)D_i}{n_i Q} + \frac{FD_i}{Q}\right\} - \frac{Q}{2}\left\{\left(n_i - 1\right) - \frac{\left(n_i - 2\right)D_i}{P}\right\}h_v - \frac{Q}{2}h_b - L_i \tag{3}$$

For given L_i, if the total number of shipments per lot n_i is fixed, clearly $TAP(n_i, Q_i) = TAP(n_i, Q_i, L_i)$ is a concave function in Q_i for $Q_i > 0$. Therefore, one convenient way to find the maximum is to use the necessary condition that the slope of the curve is zero at the maximum. Let $\frac{dTAP(Q_i)}{dQ_i} = 0$, that is, the optimal size is

$$Q_i^* = \sqrt{\frac{2D_i\left[\frac{s_v+s_b}{n}+F\right]}{\left\{(n-1)-\frac{(n-2)D_i}{P}\right\}h_v+h_b}} \tag{4}$$

Substituting equation (4) into (3), and the result leads to

$$TAP\left(n_i, Q_i^*, L_i\right) = D_i\times(R-c)-\sqrt{2D_i\left(\frac{s_v+s_b}{n_i}+F\right)\Gamma}-L_i \tag{5}$$

where $\Gamma=\left[(n_i-1)-\frac{(n_i-2)D_i}{P}\right]h_v+h_b$

Ignoring the constant D_iR-L_i and terms independent of n_i, it reduces to

$$K\left(n_i\right)=n_i\left(1-\frac{D_i}{P}\right)h_vF+\frac{\left(s_v+s_b\right)\left\{\left(\frac{2D_i}{P}-1\right)h_v+h_b\right\}}{n_i} \tag{6}$$

Maximizing $TAP(n_i, Q_i, L_i)$ is equivalent to minimizing $K(n_i)$. $K(n_i)$ is a convex function on $n_i>0$, and n_i is a positive integer. Let n_i* be the minimum value of this function $K(n_i)$. Use the $K(n_i$*) is less than or equal to $K(n_i$*–1) and $K(n_i$*) is less than or equal to $K(n_i$*+1), direct computation verifies that n_i* satisfies the following condition

$$\left(n_i^*-1\right)n_i^* \le \frac{\left(s_v+s_b\right)\left\{\left(\frac{2D_i}{P}-1\right)h_v+h_b\right\}}{\left(1-\frac{D_i}{P}\right)Fh_v} \le n_i^*\left(n_i^*+1\right) \tag{7}$$

According to equation (7), we get the value n_i*. Substituting for $n_i = n_i$* into equation (4), the optimal value of Q_i^* is calculated. The optimal value Q^* and the maximum annual profit are determined. Thus, the following solution algorithm determines the optimal n^*, Q^* and L^*.

1. For each L_i, compute $\Delta=\frac{(s_v+s_b)\left\{\left(\frac{2D_i}{P}-1\right)h_v+h_b\right\}}{\left(1-\frac{D_i}{P}\right)Fh_v}$, and n_i^* is obtained by equation (7).

2. From equation (4) and $n_i = n_i^*$, calculate the value of Q_i^*.
3. Compute $TAP(n_i^*, Q_i^*, L_i)$, for all $i = 1, 2, ..., n$.
4. Let $TAP(n^*, Q^*, L^*) = \max_i TAP(n_i^*, Q_i^*, L_i)$, then the (n^*, Q^*, L^*) is the optimal solution of the objective function.

3. Numerical Illustration

The analytical solution procedure proposed in this paper is used to obtain th optimal ordering strategy for the numerical example. The following is a numerical example for each parameter used in the model. Let product price R = \$5 /unit, unit variable cost c = \$2, production size per year P = 160000 units, carrying cost for the vendor h_v = \$2/unit/year, carrying cost for the buyer h_b = \$5/unit/year, setup cost per production batch S_v = \$300, ordering cost per order S_b = \$100, transportation cost per shipment F = \$25.

Table 1

Listing fees and demand

I	L_i	D_i
1	10000	8000
2	15000	10000
3	17000	12000
4	18000	16000
5	18500	18000

To verify this application, the variation values of i are used, with i = 1, 2, 3, 4, and 5, where the value of i is the height of the shelf display location. So, there are 5 comparison data to find the optimal solution. L_i and D_i are known for each value of i, where L_i is listing fees per order of each location and D_i is annual demand height display of each location. Based on Table 1, it can be seen that for each values of L_i and D_i are different where the greater the value of i, the greater the values of L_i and D_i.

The optimal strategy is determined by utilizing the properties of the objective through the proposed solution procedure. By following the solution procedure, the optimal values of n_i^*, Q_i^* and L_i will be obtained. The following is an applicate of an integrated inventory model using the proposed solution procedure.

For each L_i, use equation (7) to calculate n_i^*, the values of L_i and D_i for each level in the shelf display location can be applied. So, the value n_i^* can be obtained and the results are shown in Table 2.

Table 2

The numerical results of n_i^*

i	L_i	D_i	n_i^*
1	10000	8000	5
2	15000	10000	5
3	17000	12000	6
4	18000	16000	6
5	18500	18000	6

From Table 2, it can be obtained the value of n_i^* for each values of L_i and D_i in the each level of shelf display location. From equation (4), and $n = n_i^*$, compute the value of Q_i^*. Therefore, the results of Q_i^* will be obtained which are shown in the Table 3.

Table 3

The results of Q_i^*

i	L_i	D_i	n_i^*	Q_i^*
1	10000	8000	5	407.844
2	15000	10000	5	503.256
3	17000	12000	6	469.318
4	18000	16000	6	483.779
5	18500	18000	6	490.888

After knowing the value of Q_i^*, the next step is to get the value of $TAP(n_i^*, Q_i^*, L_i)$ by equation (3) for all (n_i^*, Q_i^*, L_i). The results can be obtained in the Table 4.

The last of the solution procedure is to find the highest value of $TAP(n_i^*, Q_i^*, L_i)$, which is set the optimal $TAP(n^*, Q^*, L^*) = \max_i TAP(n_i^*, Q_i^*, L_i)$. After that, the optimal solution (n^*, Q^*, L^*) will be obtained. By applying the solution step above and the Table 4, the optimal solution is obtained the values of $i = 3$, $L_3 = 17000$, $D_3 = 12000$, $n_3^* = 6$, and

Table 4
The results of $TAP(n_i^*, Q_i^*, L_i)$

i	L_i	D_i	n_i^*	Q_i^*	$TAP(n_i^*, Q_i^*, L_i)$
1	10000	8000	5	407.844	16850.972
2	15000	10000	5	503.256	28739.257
3	**17000**	**12000**	**6**	**469.318**	**32359.013**
4	18000	16000	6	483.779	31178.710
5	18500	18000	6	490.888	30590.752

get $Q_3^* = 469.318$. The total integrated annual profit obtained is 32359.013. The results of the solution procedure are shown in the Table 4. From the numerical results of the summarized solution procedures, the medium level of shelf display location has the largest total annual profit, which is 32359.013. The low level of shelf display location has the lowest total annual profit, which is 16850.972. The (32359.013 – 16850.972) = $15508.041 indicates an increase with the choice of the shelf display. It can be shown that the total annual profit is increasing by considering the choice of the shelf display. So that the higher the shelf display is directly proportional to the higher of listing fees and the higher demand does not ensure that the total profit will be higher also. From this research, the maximum total profit is obtained by considering the product display location selection. The product display location selection will affect the sales volume of goods. So retailers need to manage the locations of shelf display to maximize profits. This is appropriate with the statement that facing level is an important variable for product display location selection. Usually the eye level is in the medium level on the shelf display. So, it can be concluded that the largest total annual profit is in the medium level of high shelf display locations.

5. Conclusion

This article incorporated the integrated inventory model and the selection of shelf-display locations. These factors have not been considered in previous studies. Considering the perspective of cooperation between suppliers and retailers, an integrated inventory model of this study was developed with the selection of product shelf positions to explore an integrated inventory model. From the perspective of profit, the objective

function of the integrated inventory system is derived and the mathematical properties of this function are explored. A simple solution process is proposed to obtain the optimal ordering policy by the properties of the objective function. The suggested solution procedure in the article is easy to implement. The results of this study conclude that developing an integrated inventory model requires consideration of the selection of product shelf display location. In the numerical analysis, the results show that the medium level of product shelf display location has the largest annual profit. It can be shown that the total annual profit is increasing significantly by considering the choice of the shelf display.

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