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Synchronization of fractional-order hyperchaotic financial systems using impulsive control

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Abstract

This study examines the synchronization of fractional-order hyperchaotic financial systems using state feedback and impulsive control methods. The main objective is to make one financial system follow the behaviour of another system over time. Impulsive control represents sudden management actions or economic policies applied at specific time intervals to improve system performance. Conditions for achieving synchronization are developed and tested through a numerical example. The required solutions are obtained

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using MATLAB's LMI toolbox, and the results show that the proposed method works effectively. Unlike many previous studies, this work does not assume that the activation function satisfies the Lipschitz condition. The findings show that synchronization can still be achieved under more general nonlinear conditions.

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Keywords: *Fractional-order derivative, Mittag-leffler, Hyper-chaotic financial system, Synchronization, Impulsive control.*

1. Introduction

Financial markets are complex systems that involve many factors such as profit margins, interest rates, stock prices, and investment activities. Because of their complicated and changing nature, researchers have used hyperchaotic financial models to better understand their behaviour. Earlier studies have shown that mathematical methods can be useful in analysing economic systems and financial dynamics [2]. Fractional calculus has become an important tool for studying nonlinear systems in mathematics, physics, engineering, and economics. Compared with traditional integer order models, fractional order models can better represent systems that have memory and past effects. For this reason, many researchers have applied fractional calculus to financial systems and have developed methods for synchronization, chaos control, and system stabilization [5]. Impulsive control is another useful technique that has been widely used in economics management [3], [14]. It represents sudden actions or policy decisions applied at specific moments to influence the behaviour of a system. Previous studies have used impulsive control to achieve synchronization and stabilization in chaotic financial systems. However, most of these results were developed for integer order systems and cannot be directly applied to fractional order hyperchaotic financial models [4]. Fractional order systems have different mathematical properties and require special stability conditions. In particular, Mittag-Leffler stability is commonly used when studying fractional-order systems. Both the Caputo and Riemann-Liouville fractional derivatives have received considerable attention in the literature because of their usefulness in modelling real-world processes.

To improve the study of fractional hyperchaotic financial systems, this work develops a suitable controller together with an impulsive control strategy to achieve synchronization between drive and response systems. This synchronization is important because it allows one financial system to follow the behaviour of another under appropriate control actions and economic management measures. The mathematical analysis of fractional order systems is often more challenging than that of integer order systems [9]. Several useful inequality results have been reported in previous studies, but some important quadratic inequalities for fractional systems have not been fully established [13]. Therefore, this work provides the necessary mathematical proof required for the development of the proposed synchronization conditions.

Many studies on synchronization and stability have focused on fractional neural networks where activation functions are usually assumed to satisfy the Lipschitz condition [8], [12]. However, the activation functions considered in this work are nonlinear and do not satisfy this condition. As a result, many existing results cannot be directly used, making the synchronization analysis more difficult. Since financial variables such as stock prices, interest rates, and other market indicators often exhibit memory effects, researchers have increasingly studied fractional-order chaotic financial systems. Motivated by these observations, this paper investigates the synchronization of a fractional-order hyperchaotic financial systems [6]. A new synchronization condition based on linear matrix inequalities is developed, allowing MATLAB to efficiently test and verify the proposed results.

2. Preliminaries

Riemann-Liouville fractional derivatives, Grunwald-Letnikov fractional derivatives, and Caputo fractional derivatives are the three main fractional derivatives.

The Caputo fractional derivative is taken into consideration in this paper.

Definition 1: [15] Let γ be the order of the fractional integral function $u(t) \in \mathbb{C}[[0,\infty),\mathbb{R}]$ is

$$D^{-\gamma}u(t) = \frac{1}{\Gamma(\gamma)}\int_0^t (t-s)^{\gamma-1}u(s)ds,$$

Where $\gamma > 0, and\ \Gamma(\gamma) = \int_0^\infty e^{-t}\,t^{\gamma-1}dt.$

Definition 2: [15] The Caputo fractional derivative of order γ for a function $u(t) \in \mathbb{C}^{n+1}[[0,\infty),\mathbb{R}]$ the set of all n order continuous differentiable functions on $[0,\infty)$ is defined as

$$D^{\gamma}u(t) = \frac{1}{\Gamma(n-\gamma)}\int_0^t (t-s)^{n-\gamma-1}u^n(s)ds,$$

Where $\gamma > 0, n > \gamma,$; n is the first integer that is, $n-1 < \gamma < n,$ and the Laplace transform of $D^{-\gamma}u(t)$ as

$$\mathcal{L}\{D^{\gamma}u(t)\} = s^{\gamma}\mathcal{H}(s) - \sum_{k=0}^{n-1} s^{\gamma-k-1}u^k(0),$$

Where $\mathcal{H}(s) = \mathcal{L}\{u(t)\}$ stands for the Laplace transform of $u(t)$. In particular, When $0 < \gamma < 1$ one obtains that

$$D^{\gamma}u(t) = \frac{1}{\Gamma(1-\gamma)}\int_0^t (t-s)^{-\gamma}u'(s)ds,$$

$$\mathcal{L}\{D^{\gamma}u(t)\} = s^{\gamma}\mathcal{H}(s) - s^{\gamma-1}u(0).$$

Definition 3: [15] The Mittag-Leffler functions with one and two parameters are described as

$$E_{\gamma}(v) = \sum_{k=0}^{\infty}\frac{v^k}{\Gamma(\gamma k+1)},\ \gamma > 0, v \in \mathbb{C},$$

$$E_{\gamma,\beta}(v) = \sum_{k=0}^{\infty} \frac{v^k}{\Gamma(\gamma k+\beta)},\ \gamma > 0, \beta > 0, v \in \mathbb{C},$$

respectively, and also the two-parameter Laplace transform of the Mittag-Leffler function is

$$\mathcal{L}\left[t^{\beta-1}E_{\gamma,\beta}(-\chi t^{\gamma})\right] = \frac{s^{\gamma-\beta}}{s^{\gamma}+\chi'},\ t \geq 0, Re(s) > |\chi|^{\frac{1}{\gamma}},$$

where $\chi \in \mathbb{R}$ and Re(s) denote the real part of s.

The drive system could be defined as the fractional-order hyper-chaotic financial system that follows:

$$\begin{aligned} D^{\alpha}x_1 &= {x_3}^2 + (x_2 - a)x_1 + x_4\,, \\ D^{\alpha}x_2 &= 1 - bx_2 - {x_1}^2, \\ D^{\alpha}x_3 &= -x_2x_1 - cx_3\,, \\ D^{\alpha}x_4 &= -d{x_3}^2x_1 - kx_4 \end{aligned} \tag{1}$$

It can be compressed into a single sentence.

$$D^{\alpha}X(t) = AX(t) + g\big(X(t)\big), t \geq 0, \tag{2}$$

where $X = (x_1, x_2, x_3, x_4)^T$,

$$A = \begin{bmatrix} -a & 0 & 0 & 1 \\ 0 & -b & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{bmatrix}, g(X) = (g_1(X), g_2(X), g_3(X), g_4(X))^T = \begin{pmatrix} {x_3}^2 + x_1x_2 \\ -{x_1}^2 + 1 \\ -x_2x_1 \\ -dx_1x_2 \end{pmatrix} \tag{3}$$

the initial value of

$$X(0) = X_0 \tag{4}$$

where a stands for savings, b for the cost of the unit investment, and c for the commodity demand elasticity. The feedback coefficients d and k are linear and nonlinear, respectively. The price index is represented by x_3, the average profit margin by x_4, the interest rate by x_1, and the investment demand by x_2.

Examine the corresponding response system in the manner described below:

$$\begin{cases} D^{\alpha}Y(t) = AY(t) + g\big(Y(t)\big) + U(t), t \geq 0, t \neq t_k \\ Y(t_k^+) = Y(t_k^-) + h\big(Y(t_k) - X(t_k)\big), k = 1,2, \ldots \\ Y(0) = Y_0 \end{cases} \tag{5}$$

where $U(t) = (u_1(t), u_2(t), u_3(t), u_4(t))^T$, and each $u_i(t)$ is a controller to be designed.

$0 < t_1 < t_2 < \cdots$ and each $t_k (k \in \mathbb{Z}_+)$ represents a fixed impulsive instant, and

$Y(t_k^-) = \lim_{t\to t_k^-} Y(t) = Y(t_k), Y(t_k^+) = \lim_{t\to t_k^+} Y(t), \forall\, k = 1,2, \ldots and\ Y = (y_1, y_2, y_3, y_4)$.

$e(t) = Y(t) - X(t)$, synchronization error model is of the system (1) and (5).

$$\begin{cases} D^\alpha e(t) = Ae(t) + G\big(e(t)\big) + U(t), t \geq 0, t \neq t_k \\ e(t_k^+) = e(t_k^-) + h\big(e(t_k)\big), k = 1,2, \ldots \\ e(0) = Y_0 - X_0 \end{cases} \tag{6}$$

where

$$G(e) = g(Y) - g(X) \tag{7}$$

Design the status feedback controller shown below:

$$U(t) = N\big(X(t) - Y(t)\big), \tag{8}$$

where N is a real time state feedback gain matrix.

The following assumptions are helpful in our study:
(L1) $|x_i(t)| \leq n_i$, and $y_i(t) \leq n_i$ for all $i = 1,2$.
(L2) $h(s) = Js, where\ s \in \mathbb{R}^4$ and J is a constant matrix.

Interest is limited because both the market economy and human society's resources are finite. The investment demand and the rate are indeed constrained as well. Actually, the authors of [16] used a Laplacian semigroup to show that the dynamics of a chaotic financial system are bounded. Consequently, the assumption of boundedness (L1) is justifiable.

Global Mittag-Leffler synchronization between System (5) and System (2) is observed. If there are two positive constants $0 < K_1, K_2 < 1$ such that for all initial values $X_0 = (x_{01}, x_{02}, x_{03}, x_{04})$ and $Y_0 = (y_{01}, y_{02}, y_{03,} y_{04})$ with $|x_{0i}(t)| \leq K_i n_i$ and $|y_{0i}(t)| \leq K_i n_i, i = 1,2$. The null solution of system (6) exhibits global Mittag-Leffler stability.

The subsequent Lemma pertains to the Caputo fractional-order derivative involving a single-dimensional variable, is frequent in various research works [1, 7].

Lemma 1: *Let* $\emptyset(t) \in \mathbb{R}^1$ *be a continuous and derivable function. Then, for any* $t \geq 0$,

$$\frac{1}{2} D^\alpha \emptyset^2(t) \leq \emptyset(t) D^\alpha \emptyset(t), \forall \alpha \in (0,1).$$

3. Main Results

In this part, we introduce the Linear Matrix Inequality LMI-based synchronization condition for chaotic financial systems:

Theorem 1: *Suppose that the conditions (L1) - (L2) are satisfied. If there is a symmetric positive definite matrix P and a positive real number* ε_1 *such that*

$$PA + A^T P - PN - N^T P + \varepsilon_1^{-1} + \varepsilon_1 L^T L < 0, \tag{9}$$

$$(I + J)^T P(I + J) < P \tag{10}$$

Finally, system (5) and system (2), where I is the identity matrix, reach global Mittag-Leffler synchronization.

$$L = \begin{pmatrix} n_2 & n_1 & 0 & 0 \\ n_1 + n_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ dn_2 & dn_1 & 0 & 0 \end{pmatrix}$$

Proof: It follows from (L1) and (3) that

$$|G(e(t)| \leq L|e(t)|,$$

Consider the Lyapunov function as follows,

$$V\big(t, e(t)\big) = e^T(t) P e(t),$$

where P is a symmetric matrix that is positive definite. Below, to achieve a shared outcome. Concerning Caputo fractional derivatives and quadratic forms, we can provisionally consider that P is a symmetric positive definite matrix of n-dimensions, and e represents a vector in n-dimensions.

Next, a congruent transformation exists that

$$Q^{-1} P Q = \begin{pmatrix} \tau_1 & 0 & \cdots & 0 & 0 \\ 0 & \tau_2 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \tau_n \end{pmatrix},$$

where Q represents the n-dimensional matrix, and $Q^T = Q^{-1}$, along with every τ_i is a positive real number for$i = 1,2 \ldots$ n. In this context, each $\tau_i > 0$ represents one of the eigenvalues of the matrix P, since an n-dimensional symmetric matrix is required to possess n real eigenvalues, and in addition, An n-dimensional symmetric matrix that is positive definite must possess n positive eigenvalues. Consequently, there exists the standard orthogonal matrix Q for which the matrix P compresses with $diag(\tau_1, \tau_2 \tau_3 \ldots \tau_n)$.

Hence,

$$P = Q \begin{pmatrix} \tau_1 & 0 & \cdots & 0 & 0 \\ 0 & \tau_2 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \tau_n \end{pmatrix} Q^{-1}$$

$$= Q \begin{pmatrix} \sqrt{\tau_1} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\tau_2} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \sqrt{\tau_3} \end{pmatrix} Q^T Q \begin{pmatrix} \sqrt{\tau_1} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\tau_2} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \sqrt{\tau_3} \end{pmatrix} Q^T$$

$$= C^T C,$$

where C is positive definite symmetric matrix with

$$C = Q\begin{pmatrix} \sqrt{\tau_1} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\tau_2} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \sqrt{\tau_3} \end{pmatrix} Q^T.$$

Let $v = Ce$, then

$$V\big(t, e(t)\big) = e^T(t)Pe(t) = v^T(t)v(t),$$

where

$$V\big(t, e(t)\big) = \big(v_1(t), v_2(t), \ldots v_n(t)\big)^T$$

Lemma 1 and $v = Ce$ yield

$$\begin{aligned} D^\alpha V\big(t, e(t)\big) &= D^\alpha\big(\textstyle\sum_{i=1}^n v_i^2(t)\big) \\ &= \tfrac{1}{\Gamma(1-\alpha)}\int_0^t \tfrac{(\sum_{i=1}^n v_i^2(t))'}{(t-s)^\alpha} \\ &= \textstyle\sum_{i=1}^n \tfrac{1}{\Gamma(1-\alpha)}\int_0^t \tfrac{(v_i^2(t))'}{(t-s)^\alpha} \\ &= \textstyle\sum_{i=1}^n D^\alpha(v_i^2(t)) \\ &\le 2e^T(t)PD^\alpha e(t). \end{aligned}$$

then

$$\begin{aligned} D^\alpha V\big(t, e(t)\big) &\le e^T(t)(PA + A^TP - PN - N^TP)e(t) + [e^TG\big(e(t)\big) \\ &\quad + G^T(e(t)e(t)] \\ &\le e^T(t)(PA + A^TP - PN - N^TP + \varepsilon_1^{-1} + \varepsilon_1 L^TL)e(t) \\ &\le qV\big(t, e(t)\big), t \ge 0, t \ne t_k, \end{aligned} \tag{11}$$

where

$$q = \tau_{max}(PA + A^TP - PN - N^TP + \varepsilon_1^{-1} + \varepsilon_1 L^TL) < 0$$

There exists a function $v\big(t, e(t)\big) = D^\alpha V\big(t, e(t)\big)$, and so there is a function $\mu(t) \ge 0$ such that

$$\mu(t) + v\big(t, e(t)\big) = qV\big(t, e(t)\big), t \ge 0, t \ne t_k, \tag{12}$$

where $\mu(t) = qV\big(t, e(t)\big) - D^\alpha v\big(t, e(t)\big) \ge 0$.

Let $*$ represent the convolution operator, then it results from applying the Laplace transform and the inverse Laplace transform of (12) which

$$V\big(t, e(t)\big) = V\big(0, e(0)\big)E_\alpha(qt^\alpha) - \mu(s) * \Big(t^{\alpha-1}E_{\alpha,\alpha}(qt^\alpha)\Big), t \ge 0, t \ne t_k, \tag{13}$$

Which together with $\mu \ge 0$ means

$$V\big(t, e(t)\big) \le V\big(0, e(0)\big)E_\alpha(qt^\alpha), \forall t \in (t_{k-1}, t_k]. \tag{14}$$

Conversely, (14) and (10) produce

$$V\big(t_k^+, e(t_k^+)\big) = [(I + J)e(t_k)]^TP[(I + J)e(t_k)]$$

$$= e^T(t_k)(I+J)^T P(I+J)e(t_k) \tag{15}$$
$$\leq e^T Pe(t_k) \leq V\big(0, e(0)\big)E_\alpha(qt_k^\alpha), \forall k = 1,2, \ldots,$$

which, together with (14), suggests

$$\tau_{min}(P)\|e(t)\|^2 \leq V\big(t, e(t)\big) \leq V\big(0, e(0)\big)E_\alpha(qt^\alpha), \forall t \geq 0,$$
$$\|e(t)\| \leq \sqrt{\frac{1}{\tau_{min}(P)}V\big(0, e(0)\big)E_\alpha(qt^\alpha)},\ \forall t \geq 0$$

Given that q < 0, this previous inequality indicates that the trivial solution of System (6) is universally Mittag-Leffler stable. This completes the proof.

Remark 1: The mathematical techniques employed in this study differ from those of [10, 11] because Theorem 1's LMI-based synchronization criterion differs from those of [10], Theorem 1 and [11], and Theorem 3.1. Although our Theorem 1 is appropriate for the solvability of the LMI toolbox, the Matlab LMI toolbox is actually inapplicable to [10], Theorem 1 and [11], Theorem 3.1. In particular, some novel methods are used to derive the fractional-order quadratic inequality formula in the proof of Theorem 1 in order to derive the LMI-based synchronization criterion. These methods include the thorough application of the congruent transformation of matrix, the definition of the fractional-order derivative, and its properties.

4. Numerical simulation

The system (2) and (5) with the data $a = 0.9, b = 0.5, c = 1.5, d = 0.05, k = 0.17\ and\ \alpha = 0.95.$

$$J = \begin{pmatrix} -0.0111 & 0 & 0 & 0 \\ 0 & -0.0112 & 0 & 0 \\ 0 & 0 & -0.0113 & 0 \\ 0 & 0 & 0 & -0.0111 \end{pmatrix},$$

$$N = \begin{pmatrix} 3.1 & 0.0001 & 0 & 0 \\ 0.0001 & 3.2 & 0 & 0 \\ 0 & 0 & 3.3 & 0 \\ 0 & 0 & 0 & 3.1 \end{pmatrix}$$

Let $n_1 = 6, n_2 = 7$

$$A = \begin{bmatrix} -0.9 & 0 & 0 & 1 \\ 0 & -0.5 & 0 & 0 \\ 0 & 0 & -1.5 & 0 \\ 0 & 0 & 0 & -0.17 \end{bmatrix}, g(X) = (g_1(X), g_2(X), g_3(X), g_4(X))^T = \begin{pmatrix} {x_3}^2 + x_1x_2 \\ -{x_1}^2 + 1 \\ -x_2x_1 \\ -0.05x_1x_2 \end{pmatrix}$$

$$L = \begin{pmatrix} 7 & 6 & 0 & 0 \\ 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.35 & 0.3 & 0 & 0 \end{pmatrix}$$

The following feasibility data is obtained by solving LMI conditions (9) and (10) using the MATLAB LMI toolbox:

$$P = \begin{pmatrix} 14.0082 & 2.8517 & 0 & 1.9269 \\ 2.8517 & 3.0627 & 0 & 0.4274 \\ 0 & 0 & 0.2848 & 0 \\ 1.9269 & 0.4274 & 0 & 1.0081 \end{pmatrix}, \varepsilon_1 = 0.5013$$

4.1 *Phase Portrait Figure*

The three dimensional phase pictures in Figure 1 various state spaces demonstrate the chaotic dynamics of the suggested fractional order financial system.

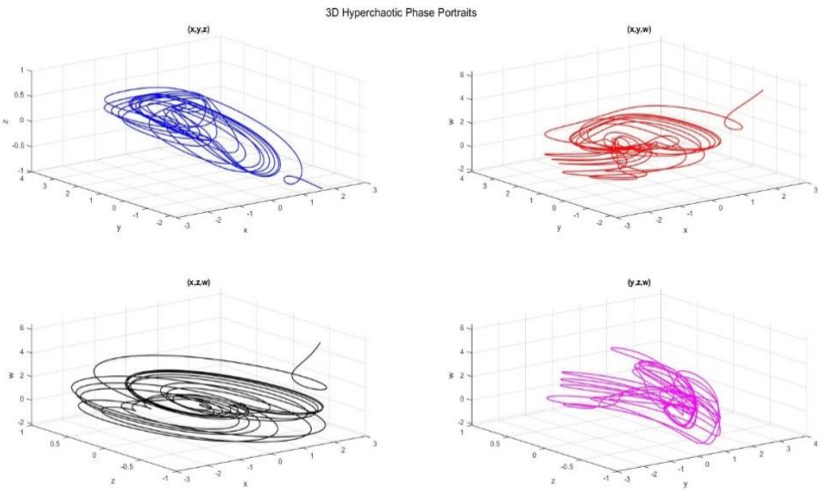


Figure 1
Phase portraits of the fractional-order hyperchaotic financial system

Figure 1 shows phase portraits of the fractional-order hyperchaotic financial system in three dimensions in several state spaces: (a) (x,y,z), (b) (x,y,w), (c) (x,z,w), and (d) (y,z,w). The obtained attractors verify that the suggested financial system exhibits hyperchaotic behaviour.

4.2 *Synchronization Figure*

Figure 2 shows the drive and response systems' synchronization performance for every state variable.

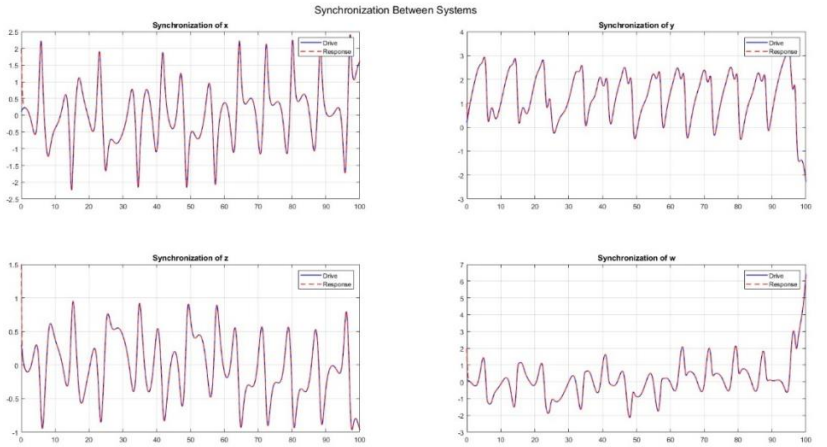


Figure 2
Drive and response hyperchaotic financial systems synchronize

Figure 2 shows how the drive and response fractional-order hyperchaotic financial systems Synchronize for the state variables x, y, z, and w. Under the intended impulsive controller, the overlapping trajectories show good synchronization.

4.3 *Impulsive Control Figure*

The impulse control operations used at discrete impulsive instants to establish synchronization are depicted in Figure 3.

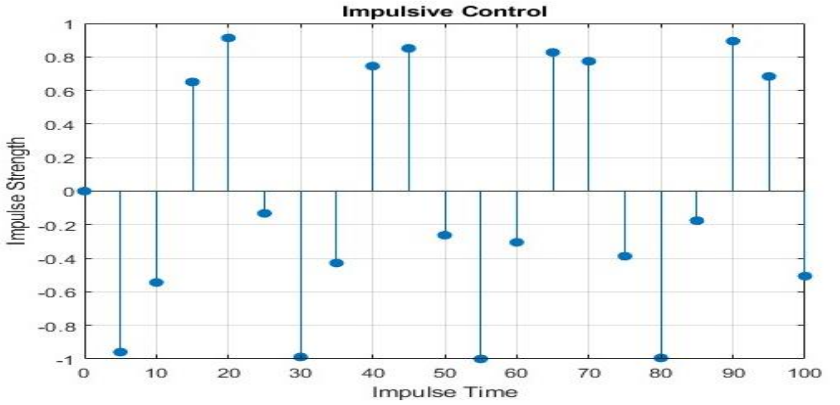


Figure 3
The impulse control operations used at discrete impulsive instants to establish synchronization are depicted

The fractional-order hyperchaotic financial system is subjected to impulsive control signals at discrete impulse instants (Figure 3). The impulses enhance convergence performance and stabilise the synchronization dynamics.

4.4 *Synchronization Error Figure*

Figure 4 shows the convergence of synchronization faults, confirming the efficacy of the suggested control approach.

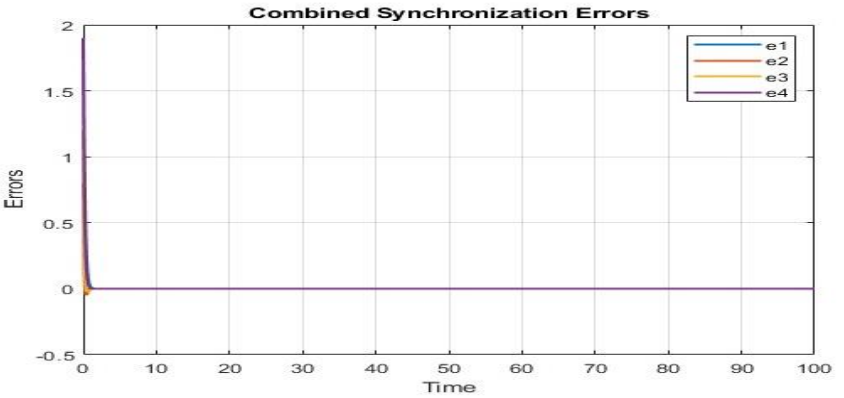


Figure 4
Synchronization of errors

Figure 4 The synchronization faults' time evolution e_1, e_2, e_3, e_4 The global Mittag-Leffler synchronization of the drive and response systems is confirmed by the asymptotic convergence of all error states to zero.

5. Conclusions

This study develops a state feedback controller and impulsive control technique to achieve global Mittag-Leffler synchronisation of fractional-order hyperchaotic financial systems. Numerical simulations and the MATLAB LMI toolkit are used to confirm the efficacy of the suggested criterion. The acquired results demonstrate the successful synchronisation of advanced and backward economic systems through suitable impulsive economic management. The stability of the suggested approach is confirmed by the synchronisation errors' asymptotic convergence to zero. The simulations also show how crucial economic support and regulation are to the stabilisation of intricate financial dynamics. Lastly, the result indicates that future research on impulsive control with time delay and event-triggered processes in macroeconomic models is still worthwhile.

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