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A new class of diffusion-based hyperbolic model for removing additive noise

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Abstract

This paper introduces a novel nonlinear diffusion-based hyperbolic model and assesses its effectiveness in image denoising. A set of images with different noise levels is processed

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using a diffusion-based hyperbolic model and implemented to preserve the image details. While the hyperbolic term is intended to maintain the important features of the image, like edges, the parabolic term is utilized to minimize the noise. Furthermore, we want to prove the weak solution of our model. The denoising model is discretized by the finite difference methods and results are evaluated by PSNR and SSIM metrics.

Subject Classification: *35L70, 65M06, 76R50, 68U10.*

Keywords: *Partial differential equation, Diffusion model, Diffusion-based hyperbolic model, Weak solution, Finite difference method, Image processing.*

1. Introduction

In 1990, Perona and Malik [1] introduced a diffusion equation with a diffusivity function, which may preserve the edges in image denoising and edge detection without causing any damage. An anisotropic diffusion-based model was proposed by Catte et al. [2], has been put forth since the ground-breaking study of [1]. They used the Gaussian convolution function with a Gaussian filter in the PM model to reduce the staircase effect along the edges and perform well for the image denoising process. Wang and Pang [3] explored the formulation of an anisotropic mean curvature model aimed at enhancing image denoising, focusing on the intricate balance between preserving edges and minimizing noise, and also introduced an anisotropic mean curvature (AMC) model for the image denoising challenge by integrating a weighted matrix with the gradient operator. The classic augmented Lagrangian method is employed to break down the original problem into multiple subproblems that are more manageable to solve. Comprehensive experiments conducted on standard test images demonstrate the robustness of the approach in comparison to various existing denoising models. Maiseli [4] investigated the difficulties associated with nonlinear anisotropic diffusion techniques in the context of image denoising. The research addresses concerns such as sensitivity to parameters, computational demands, and the preservation of edges. Qi et al. [5] introduced a weighted diffusion model with multiple scales. The simulation results and visual comparisons indicate that the model significantly outperforms existing anisotropic diffusion-based models in reducing edge-preserving and staircase artifacts. There are numerous methods for image restoration using nonlinear diffusion models that have been proposed by many researchers [6, 7, 8, 9].

Non-linear hyperbolic diffusion equations perform well in highly noisy regions and greatly reduce the denoising process's intended effect. Ratner and Zeevi [10] presented a hyperbolic-based telegraph model for removing additive noise. Cao et al. [11] developed a telegraph hyperbolic parabolic PDE-based model with a Gaussian kernel to eliminate the noise. To remove additive noise in areas with high levels of noise. Li and Zhi [12] presented an image denoising and deblurring method grounded in a nonlinear hyperbolic model, with an emphasis on directional diffusion. Halim, Wira, and Hadi [13] introduced a new modified nonlinear second-order hyperbolic-parabolic model for better image

denoising. They introduced three modified diffusion functions to enhance the image quality and edge preservation. Majee, Ray and Majee, [14] introduced a coupled PDE-based hyperbolic-parabolic model for speckle noise while keeping important edges in the image. To remove the speckle noise, they introduced adaptive diffusion coefficients that depended on the local image gradient for the smoothing in flat regions and near the edges.

In this paper, a smooth hyperbolic-parabolic non-linear PDE-based diffusion model is presented. First, we presented the abstract of the paper. Section 1 contains the introduction of the paper. After that, an image-denoising algorithm is given in Section 2. The weak solution of the hyperbolic-parabolic model is discussed in the third section. In Section 4, a discrete scheme for the proposed model. The numerical experiment and results in Table 1 and Figures 1-3 in Section 5. The last part of the paper contains the conclusion.

2. Hyperbolic-Parabolic Model for Image Denoising Algorithm

The optimized energy functional considering the static constraint mean and variance is proposed by Welk et al. [15]:

$$f(u) = \int_{\Omega} \rho\,(|\nabla u|^2)dx\,dy + \frac{\lambda}{2}\int_{\Omega} (u - u_0)^2\,dx\,dy,$$

The first term in the R.H.S. is the regularization term and the second term with Lagrange multiplier λ is the data fidelity term. Using Euler-Lagrange and gradient descent methods, we reached:

$$\frac{\partial u}{\partial t} = \nabla \cdot (\gamma(|\nabla u|^2)\nabla u) - \lambda(u - u_0), \tag{2.1}$$

whereas the initial boundary conditions are:

$$u(x, y, 0) = u_0(x, y),$$

$\frac{\partial u}{\partial t}(\mathrm{x}, \mathrm{y}, 0) = 0, (\mathrm{x}, \mathrm{y}) \in \Omega$ is bounded domain of R^2

Here, the initial guess $u_0(x, y)$ is noisy, $\frac{\partial \mathrm{u}}{\partial \vec{n}} = 0$ on the boundary of $\partial\Omega$ represents homogeneous Neumann boundary conditions and the diffusivity $\gamma(s^2)$ is linked to $\rho'(s^2)$.

The hyperbolic PDE-based model achieves superior outcomes in highly noisy regions without sacrificing the fine features of low-frequency signals [16-20]. This technique is very effective in preserving better edges and removing additive noise:

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = \nabla \cdot (\gamma(|\nabla u|^2)\nabla u) - \lambda(u - u_0). \tag{2.2}$$

Our approach for removing additive Gaussian white noise is created by applying a priori smoothness to the model (2.2). It is defined as

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = \nabla \cdot (\gamma(|\nabla \eta_\sigma * u|^2)\nabla \eta_\sigma * u) - \lambda(\eta_\sigma * u - u_0). \tag{2.3}$$

This model is known as the diffusion-based hyperbolic model with the same condition as above.

Witkin [21] observed that the heat equation may be solved using the signal at the initial datum by (signal $*$ Gaussian) at each scale, and $*$ represents a convolution. The gradient of the parabolic equation solution at time , using the initial datum $u_0(x,y)$, is represented by the quantity $(\eta_\sigma * \nabla u)(x,y,t)$ which is equals to $(\nabla\eta_\sigma * u)(x,y,t)$. The function η_σ serves as a low-pass filter [2, 22, 23].

For the numerical experiment, we employ Charbonnier diffusivity, where $\gamma(s)$ is taken as $\frac{1}{\sqrt{1+\left(\frac{|s|^2}{P^2}\right)}}$ and it is coupled to the convex regularizer $\rho(s^2) = \sqrt{P^4 + P^2 s^2 - P^2}$, see reference [24, 25].

This model eliminates the diffusion effect close to the image edges, producing considerably sharper edges and greater details. If we still write $\eta_\sigma * u$ as u, (2.3) and (2.2) have the same formulation. In the next section, a unique weak solution is proposed for the presented model (2.3).

3. Uniqueness of Weak Solution

The well-posedness of the suggested smooth-hyperbolic diffusion model is examined [26]. For $1 \le p \le \infty$, It is denoted by $(L^p, \|.\|_{L_p})$ the standard spaces of the p-th order integrable function Ω. Let us set $H = L^2(\Omega), W = H^2(\Omega) \cap H_0^1(\Omega)$ and W is the compact subspace of H, where $H^2(\Omega)$ and $H_0^1(\Omega)$ represent standard Sobolev spaces. A unique weak solution of (2.3) is a function u: $[0,T] \times \Omega \to R$ if the following condition holds:

$$u \in L^\infty(0,T;W),\ u_t \in L^\infty(0,T;W),\ u_{tt} \in L^\infty(0,T;H) \tag{3.1}$$

$$\int_\Omega u_{tt}\,\phi\,dx\,dy + \int_\Omega u_t\,\phi\,dx\,dy = \int_\Omega \gamma(u).\nabla u\,.\nabla\phi\,dx\,dy - \lambda(u-u_0)\,dx\,dy, \forall\ \phi \in H^1\,. \tag{3.2}$$

To establish the weak solution, we have the assumption:

D_1. $\gamma(u)$ is the diffusivity function, which is Lipschitz continuous, bounded and decreasing, with Lipschitz constant $\frac{C}{P^2}$.

Proposition: Under the assumption D_1, equation (2.3) has a unique weak solution.

Proof: Assume the following sequence $(u_k) \subset L^\infty(0,T;W)$, $(u_k)_t \in L^\infty(0,T;W)$, and $(u_k)_{tt} \in L^\infty(0,T;H)$ The sequence is defined as:

$$\begin{gathered}(u_k)_{tt} + (u_k)_t = \Delta.(\gamma|\nabla u_k|^2\nabla u_k) - \lambda(u_k - u_0)\ on\ (0,T)\times\Omega \\ u_k(x,y,0) = u_0,\ (u_k)_t(x,y,t) = u_1 \text{ on } \Omega \\ u_k(x,y,t) = 0, \frac{\partial u_k}{\partial n}(x,y,t) = 0\ on\ (0,T)\times\ \partial\Omega,\end{gathered} \tag{3.3}$$

where $u_k(x,y,t) = u_k, (u_k)_t(x,y,t) = (u_k)_t$, $(u_k)_{tt}(x,y,t) = (u_k)_{tt}$.

It can be observed that by applying the theorem 1.1 provided in [27], u_{k-1} is a unique weak solution u_k to equation (3.3). To demonstrate the

convergence of u_k to a weak solution u for the suggested model (2.3) must demonstrate that u_k, $(u_k)_t$ are bounded in $L^2(0,T;W)$ and , $(u_k)_{tt}$ is bounded in $L^2(0,T;H)$. Consequently, u_k and $(u_k)_t$ are compact in $L^2(0,T;H)$.

The equation (3.3) is multiplying by u_k and applying the integration in the domain $[0,\mathrm{T}]\times\Omega$, we get

$$\int_\Omega\ u_k\ (u_k)_t\,dx\,dy+\frac{1}{2}\int_\Omega\ u_k^2\,dx\,dy-\int_0^t\int_\Omega(\gamma|\nabla u_{k-1}|^2)\ |\nabla u_k|^2\,dx\,dy\,ds$$
$$=\int_\Omega\ u_0\,u_1\,dx\,dy+\frac{1}{2}\int_\Omega\ |u_0|^2\,dx\,dy+\int_0^t\int_\Omega\ (u_k)_t^2\ dx\,dy\,ds+ \qquad (3.4)$$
$$\lambda\int_0^t\int_\Omega(\,u_k-\,u_0)\ \ u_k\,dx\,dy\,ds.$$

The equation (3.4) yields to

$$\int_\Omega\ u_k^2\,dx\,dy+\int_0^t\int_\Omega|\nabla u_k|^2\,dx\,dy\,ds\le c\ \int_0^t\int_\Omega\ (u_k)_t^2\ dx\,dy\,ds+c \qquad (3.5)$$
$$\forall\ t\in[0,T]\ on\ (0,T)\times\Omega,$$

where c is independent of k and a positive constant. After that, we multiply equation (3.4) by $(u_k)_t$ and integrate it into $(0,T)\times\Omega$ to get the following estimate:

$$\frac{1}{2}\int_\Omega\ \ (u_k)_t^2\,dx\,dy+$$
$$\int_0^t\int_\Omega\ (u_k)_t^2\ dx\,dy\,ds-\int_0^t\int_\Omega(\gamma|\nabla u_{k-1}|^2)\ |\nabla u_k|^2\,dx\,dy\,ds$$
$$=\lambda\int_0^t\int_\Omega(\,u_k-\,u_0)\ \ (u_k)_t\,dx\,dy\,ds+\frac{1}{2}\int_\Omega\ u_1^2\,dx\,dy \qquad (3.6)$$

Finally, multiplying equation (3.4) by ∇u_k and $\nabla(u_k)_t$ and using (3.5) and (3.6) we get;

$$\int_\Omega\ (u_k^2+\ (u_k)_t^2)\,dx\,dy+\int_0^t\int_\Omega|\nabla u_k|^2\,dx\,dy\,ds+$$
$$\int_0^t\int_\Omega|\nabla\,(u_k)_t|^2\,dx\,dy\,ds\le c\ \forall\ [0,T] \qquad (3.7)$$

Consequently u_k, $(u_k)_t$ are bounded in $L^2(0,T;W)$, and $(u_k)_{tt}$ is bounded in $L^2(0,T;H)$, there exists a unique weak solution for model (2.3).

4. Discrete Scheme

This section uses an explicit numerical technique to discretize hyperbolic-parabolic models (2.2) and (2.3). Assume that $(u(x_i\,,y_j\,,t_n\,)$ is the value's closest approximation $u_{i,j}^n$. This gives us a definition of the discrete approximation as $\mathrm{x_i}=\mathrm{i}\Delta\mathrm{x},\mathrm{y_j}=\mathrm{j}\Delta\mathrm{x},\mathrm{i},\mathrm{j}\ =\ 1{,}2{,}3\ldots\mathrm{N}$, where Δx spatial step size and $t_n\ =n\Delta\mathrm{t},\ n\ge1$, where Δt is the time step size and $N\Delta x\ =\ 1$, then

$$\Delta_x^+u_{i,j}^n=\frac{u_{i+1,j}^n-u_{i,j}^n}{\Delta\mathrm{x}},\ \delta u_{i,j}^n=\frac{u_{i,j}^n-u_{i,j}^{n-1}}{\Delta\mathrm{t}},\ \Delta_y^+u_{i,j}^n=\frac{u_{i,j+1}^n-u_{i,j}^n}{\Delta\mathrm{x}},\ \delta^2u_{i,j}^n=\frac{\delta u_{i,j}^n-\delta u_{i,j}^{n-1}}{\Delta\mathrm{t}},$$
$$\Delta_y^-u_{i,j}^n=\frac{u_{i,j-1}^n-u_{i,j}^n}{\Delta\mathrm{x}},\ \Delta_x^-u_{i,j}^n=\frac{u_{i-1,j}^n-u_{i,j}^n}{\Delta\mathrm{x}}.$$

If we still write, $\eta_\sigma * u \; as \; u$. The discrete scheme of the diffusion-based hyperbolic models (2.2) and (2.3) can be written as

$$\delta^2 u_{i,j}^n + \delta u_{i,j}^{n-1} = \frac{1}{2\Delta x}[(\gamma_{i+1,j}^n + \gamma_{i,j}^n)\Delta_x^+ u_{i,j}^n - (\gamma_{i,j}^n + \gamma_{i-1,j}^n)\Delta_x^- u_{i,j}^n] + \frac{1}{2\Delta x}[(\gamma_{i,j+1}^n + \gamma_{i,j}^n)\Delta_y^+ u_{i,j}^n - (\gamma_{i,j}^n + \gamma_{i,j-1}^n)\Delta_y^- u_{i,j}^n] - \lambda\,(u_{i,j}^n - u_{i,j}^0). \quad (3.8)$$

The diffusivity parameter $\gamma(|\nabla u|^2)$ discretization is presented as:

$$\gamma_{i,j}^n = \rho'\left(\left(\frac{u_{i+1,j}^n - u_{i-1,j}^n}{\Delta x}\right)^2 + \left(\frac{u_{i,j+1}^n - u_{i,j-1}^n}{\Delta x}\right)^2\right).$$

We choose the value $\frac{\Delta t}{\Delta x^2} \leq 0.5$, it is stable and convergent, as shown in [28].

5. Numerical Implementation

Many degraded images were used to test the suggested nonlinear hyperbolic PDE-based noise reduction method. By taking into account different values for the σ (variance) parameter, different amounts of Gaussian noise were produced, which damaged the images. The original image contains the pixel values in the range [0, 255]. To add Gaussian white noise in Matlab, we utilize the standard imnoise function. The image's intensities are scaled on a scale from 0 to 1. Using the values from [29, 30], we take P to be 5 and λ to be 0.85. To carry out the numerical experiments, we select $\frac{\Delta t}{\Delta x^2} = 0.4$ for model (2.1) and $\Delta t = 0.01$ for (2.2) and (2.3). The proposed filtering method runs quickly enough because the N iteration count is fairly small.

Utilizing metrics like the structural peak signal-to-noise ratio (PSNR), we evaluated the effectiveness of our image-enhancing technique. It is given below:

$$\text{PSNR} = 10 log_{10}\left(\frac{R^2}{\frac{1}{mn}\sum_{i,j}^{n}(u(i,j) - u_{new}(i,j))^2}\right), \quad (5.1)$$

where u observed image and u_{new} reconstructed image.



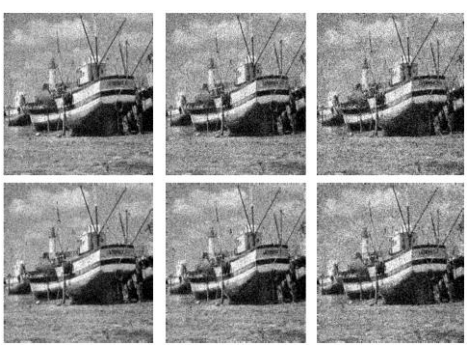
(a) (b)

Figure 1
Left original Lena image and right Boat image.



Figure 2

First row presents Lena images with Gaussian noise at different values of noise variance $\sigma^2 = 0.010, 0.015, 0.020$, respectively; second to fourth row represent the result by models (2.1), (2.2), and (2.3), respectively.



Contd...

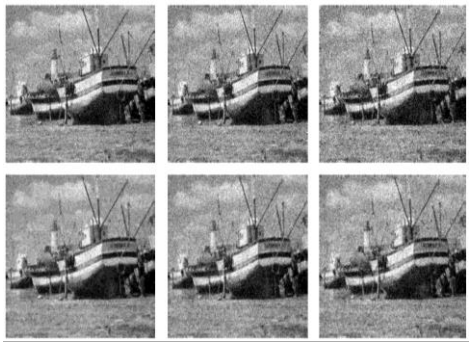


Figure 3

First row presents Boat images with Gaussian noise at different values of noise variance $\sigma^2 = 0.010, 0.015, 0.020$, respectively; second to fourth row represent the result by models (2.1), (2.2), and (2.3), respectively.

Table 1

Quantitative analysis of diffusion model and diffusion-based hyperbolic model using Structural Similarity Index Measure (SSIM) and PSNR as metrics.

		Lena Image		Boat Image	
	Noise Variance	**PSNR**	**SSIM**	**PSNR**	**SSIM**
Noisy image	0.010	20.28	0.3899	20.18	0.4125
	0.015	18.58	0.3286	18.44	0.3462
	0.020	17.45	0.2971	17.24	0.2962
Model (2.1)	0.010	20.59	0.4004	20.38	0.4128
	0.015	18.84	0.3371	18.70	0.3476
	0.020	17.71	0.2948	17.48	0.3017
Model (2.2)	0.010	23.56	0.5274	23.22	0.5386
	0.015	21.53	0.4395	21.32	0.4494
	0.020	20.11	0.3874	19.77	0.3875
Model (2.3)	0.010	26.02	0.6466	25.60	0.6503
	0.015	24.23	0.5632	24.01	0.5648
	0.020	23.02	0.5039	22.79	0.5036
	Iteration count	50		50	

Conclusion

The smooth hyperbolic parabolic diffusion model that is proposed for removing additive Gaussian white noise by incorporating a priori smoothness into the diffusion process. The proposed formulation effectively combines the advantages of hyperbolic and parabolic terms, where the hyperbolic component helps preserve significant image features such as edges, while the parabolic term helps decrease noise. The results in terms of PSNR and SSIM (higher values) confirm that the nonlinear diffusion-based hyperbolic model (2.3) provides a stable, reliable, and efficient framework for image denoising corresponding to old models (2.1) and (2.2).

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