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A mathematical framework for medical image security using DWT-based LL sub-band coefficient analysis and robust watermark embedding

Malvika Gupta *
Parma Nand †
Department of Computer Science & Engineering
Sharda University
Greater Noida 201310
Uttar Pradesh
India

Ankur Choudhary §
Accenture
Noida 201301
Uttar Pradesh
India

Abstract

With the advancement of telemedicine, it is very important to preserve the authenticity of the medical image, shared remotely. Watermarking is commonly used to ensure the legitimacy of medical image over the imaging like CT Scan, MRI and Ultrasound. In this work a strategy has been proposed through Discrete Wavelet Transform to determine the authenticity of the medical image. The performance of DWT using LSB has been evaluated over PSNR and SSIM. After applying different types of attacks it has been observed that MRI image among the other two images CT Scan and Ultrasound has shown significant reluctance to attacks and are imperceptible. The PSNR for MRI is 48.210dB & SSIM is 0.9955 which means that imperceptible.

Subject Classification: *Primary 68U10, Secondary 94A08, 94A12.*

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* *E-mail:* 2020507706.malvika@dr.sharda.ac.in (Corresponding Author)
† *E-mail:* parma.nand@sharda.ac.in
§ *E-mail:* ankur.tomer@gmail.com

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1. Introduction

Medical images are the most sensitive images on the basis of these images, issues are diagnosed and then a line of treatment is suggested by the doctor. With the rapid advancement in the field of telemedicine, it has become a frequent practice that a person sitting far can take advice from a doctor by sharing the medical reports and images. Due to this technological advancement, it has become more important that the images received at the doctors' end are coming from an authentic source and also are not attacked or modified by any noises or signals during transmission [9, 10]. In this article, watermarking to preserve authenticity of medical images Discrete Wavelet Transform is used which has shown significant effect in case of robustness against different attacks and also imperceptibility in the structure of the medical image. This section is divided into four sections: Section-1 defines the Discrete Wavelet Transform Section-2 explains advantages of DWT in watermarking and Section-3 defines the performance metrics based on which effectiveness of the approach is justified.

1.1 *Discrete Wavelet Transform*

It is a mathematical technique that is used to split an image into multiple frequency subbands [11]. An image is broken into approximation and detail coefficients, which provide both spatial and frequency information. DWT is extensively used in image processing due to its ability to provide multi-resolution analysis, which is valuable for watermarking.

1.1.1 Working of DWT

It decomposes the medical image into four bands [3]:

- **LL (Low-Low):** Represents the approximation of the image (low-frequency components).
- **LH (Low-High):** Horizontal details (high-frequency components in the horizontal direction).
- **HL (High-Low):** Vertical details (high-frequency components in the vertical direction).
- **HH (High-High):** Diagonal details (high-frequency components in both horizontal and vertical directions).

The DWT is often applied iteratively to obtain higher-level approximations, which can be useful in embedding a watermark.

1.2.2 Formula of DWT

For 1D signal the DWT is represented by

$$x(t) = a_n \cdot \psi(t - n) \tag{1}$$

where,
x(t) is the signal
an is the wavelet coefficient
$\psi(t - n)$ is the mother wavelet

1.2 *Advantages of DWT in Watermarking*

Table 1 compares DWT with other image-processing techniques based on different features listed in the Table.

Table 1
Comparison of Image Processing Techniques

Feature	DWT	DCT	DFT	DWT-SVD (Hybrid)
Robustness to Noise	High	Low	Moderate	Very High
Compression Resistance	Strong	Weak	Strong	Strong
Computational Complexity	Moderate	Low	High	High
Resilience to Attacks	Strong	Weak	Moderate	Very Strong
Image Quality Retention	High	Moderate	Moderate	Very High

1.3 *Performance Metrics*

There are two performance metrics based on which the watermarking is evaluated:

- **PSNR(Peak Signal-to-Noise Ratio:** It compares the watermarked image with the original image and measures quality of watermarked image [2]. It is defined as:

$$\text{PSNR} = 10 \cdot \log\left(\frac{\text{MAX}^2}{\text{MSE}}\right) \tag{2}$$

  where,

  MAX is the maximum possible pixel value of the image (e.g., 255 for 8-bit images)

  MSE(Mean Squared Error) is the average squared difference between the original image and the watermarked image

  The higher the PSNR, the better the image quality, which means the watermarking process has less impact on the image.

- **SSIM (Structural Similarity Index):** It analyzes how visually similar two images are in terms of luminance, contrast and structure. It goes from -1 to 1, where a value of 1 means that the two photos are identical. [7]. SSIM is calculated as:

$$\text{SSIM}(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{3}$$

  where,

  μ_x, μ_y are the means of the two images.

  σ_x^2, σ_y^2 are the variances of the two images.

  σ_{xy} is the covariance between the two images.

  C_1, C_2 are small constants to stabilize the division when the denominator is close to zero.

1.3.1 Comparison and Result

These measurements ensure that the proposed watermarking technology is invisible and robust, maintaining the quality of medical images while embedding the watermark.

- Images with higher PSNR values are of superior quality and have less distortion.
- SSIM values close to 1 suggest that the structural integrity of the medical image is well preserved after watermarking.

2. Literature Review

The study [4] uses Discrete Cosine Transform and a modified ResNet50 for feature extraction. In this the watermark is embedded into the extracted features using Arnold mapping and Logistic mapping to enhance security. This strategy was evaluated on a publically available dataset on Kaggle which contains X-Ray images of different classifications including Covid-19, Tuberculosis etc. It is analyzed on PSNR and NC performance parameters that the PSNR value is around 20 and NC is 0.86. The study [5] is focused on to ensure the authenticity of the received medical image against the attacks and imperceptibility. The method is evaluated based on PSNR value is 17.32 using LSB which is very less, therefore, the quality of watermarked image is low.

The study [6] discusses about the different watermarking techniques. It shows the comparison among the different techniques and the domains.The study [2] focuses on selection of ROI and watermarking using DWT and SVD. The proposed methodology waorks in two steps, first to identify the ROI and second, to alter the watermark image in time-frequency domain to attain the security and robustness against the various attacks.

The proposed methodology [12] is based on the blind watermarking strategy for the security of medical photographs. We utilize an integrated technique where the retinal picture is subjected to DWT and LL sub-band is subjected to SVD. The watermark is embedded in the LSB of the component formed by the parity of successive coefficients. This approach exhibits the great improvement in imperceptibility and robustness of the watermark pictures. The proposed solution [1] was a combined DWT and 2-D SVD strategy for protecting and authenticating the watermark. “Both the medical and non-medical images have shown significant improvement. It has exhibited the increase in imperceptibility and robustness with mean of 46.95dB and above 0.99 correspondingly.

The approach [3] used different medical images like MRI, CT Scan, X-Ray and Ultrasound DICOM images. In this approach the watermark is first encrypted with the logistic chaotic map and then inserted into the medical image. The insertion is done by using Discrete Wavelet Transform and a set of DWT coefficients are selected using rank-based criterion. This approach is evaluated based on two performance metrics PSNR, 49dB and NCC which has shown significant improvement against different attacks. The methodology [8] discusses the embedding of watermark into a medical image in LL sub-band using the

zigzag scanning method. It provides PSNR of 70dB which signifies the robustness of the method in various attacks.

3. Methodology and Implementation

This section is divided into subsections: proposed methodology and how it is implemented.

3.1 *Methodology*

This section describes the approach to insert and extract watermark in medical photos using Discrete Wavelet Transform (DWT). In this section the technology utilized to embed and extract the watermarks in medical photographs leveraging the Discrete Wavelet Transform (DWT) is presented. Below each of these steps is covered in detail.

3.1.1 Proposed Methodology

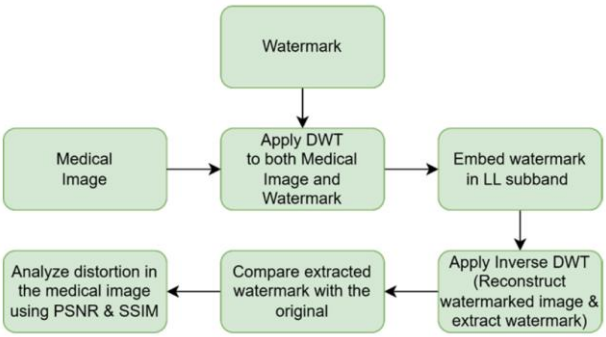


Figure 1
Proposed Methodology

The procedure described in Figure 1 for the watermarking application of DWT is developed as follows in principal stage:

1. **Preprocessing:** The processed medical images are preprocessed to embed a watermark, resizing, normalizing, as well as noise reduction operations for good quality watermark image embedding.
2. **Wavelet Decomposition:** The image is decomposed in subbands : LL, LH, HL, HH. The watermark is placed into the low-frequency LL subband in order to minimize the visual distortion and preserve the diagnostic quality of the image.
3. **Watermark Embedding:** A watermark (e.g. logo, text, encrypted content) is embedded in the LL subband. Changing the wavelet coefficients would introduce the watermark, which were then used to reconstruct the image with the embedded watermark.

4. **Wavelet Reconstruction:** The image is reconstructed from modified subbands. The result is a watermarked image that retains its original quality but has a hidden watermark.
5. **Watermark Extraction:** The inverse DWT is applied on the subbands and the watermark is extracted from the LL subband to extract the watermark.
6. **Performance Evaluation:** The performance assessment of the watermarking method is viewed as an indicator of quality results such as PSNR (Peak Signal to Noise Ratio) and SSIM (Structural Similarity Index). These and other parameters quantify the impact of the watermark embedding on the image quality.

3.2 *Watermark Embedding using DWT*

The watermark embedding process using DWT involves the following steps:

3.2.1 Pre-processing of Medical Images

In the event of embedding watermarking, medical images need to be preprocessed to confirm that the image is in a suitable state for embedding a watermark.

- **Resizing:** The medical image may be resized to fit within a predefined resolution or to ensure that it is of the correct dimensions for processing. It guarantees that the image would be incorporated within the watermark embedding framework. If the original image I(x,y) is resized to dimensions M×N, the resized image $I_r(m, n)$ can be expressed as

$$I_r(m,n) = I\left(\frac{mH}{M}, \frac{nW}{N}\right) \tag{4}$$

where,

  - H X W = original image dimensions
  - M X N = resized image dimensions

- **Normalization:** Image pixels are normalized to a common range, usually between 0 and 1, to enable the embedding and extraction of watermarks. Pixel values are normalized to the range [0,1]:

$$I_n(x,y) = \frac{I(x,y) - I_{\min}}{I_{\max} - I_{\min}} \tag{5}$$

where

  - Imin = minimum pixel intensity
  - I_{max} = maximum pixel intensity

- **Noise Removal:** Medical images may contain noise due to factors like image acquisition and scanning. Preprocessing techniques such as Gaussian filtering or median filtering are employed to remove noise before watermarking.

$$I_f(x,y) = I(x,y) * G(x,y) \tag{6}$$

where * denotes convolution and

$$G(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \quad (7)$$

- **Grayscale Conversion:** In some cases, the image is converted to grayscale, especially if the watermark is to be embedded in a single channel (e.g., intensity).

$$I_g(x,y) = 0.299R(x,y) + 0.587G(x,y) + 0.114B(x,y) \quad (8)$$

where R, G and B represent the red, green and blue channels

3.2.2 Watermark Embedding Procedure

It involves the following steps:

- **Use DWT to decompose the image:** The image is divided into four subbands: high-frequency details (HH), low-frequency details (LL), high-frequency details (LH), and HL. The watermark is inserted in the LL subband, as it contains most of the image's structural information and is less prone to noticeable changes. We use the following formula to apply watermark:

$$LL' = LL + \alpha\ W$$

- **Adjust the Wavelet Coefficients:** o ensure minimal visual impact, the watermark is embedded by altering the coefficients in the LL subband, frequently by applying methods like LSB embedding or by adding undetectable values to the wavelet coefficients.
- **Reconstruct the Watermarked Image:** Inverse DWT (IDWT) is used to reconstruct the image, combining the altered subbands to create the final watermarked image. The watermark is hidden and not visible to the human eye, but it can be extracted at a later time.

3.3 *Watermark Extraction*

The watermark extraction is important as it will contain the information about the authenticity of the image transferred or received from a source. Following steps are followed:

- **Extract the Watermark:** The watermark components are separated from the LL subband based on the embedding strength factor, ensuring minimal distortion to the medical image.
- **Apply Inverse DWT (IDWT):** To reconstruct the extracted watermark, IDWT is performed to put the extracted watermark back into the spatial domain.
- **Compare with the Original Watermark:** The extracted watermark is then compared to the original watermark to evaluate the accuracy and robustness of the watermarking scheme.

3.4 *Implementation*

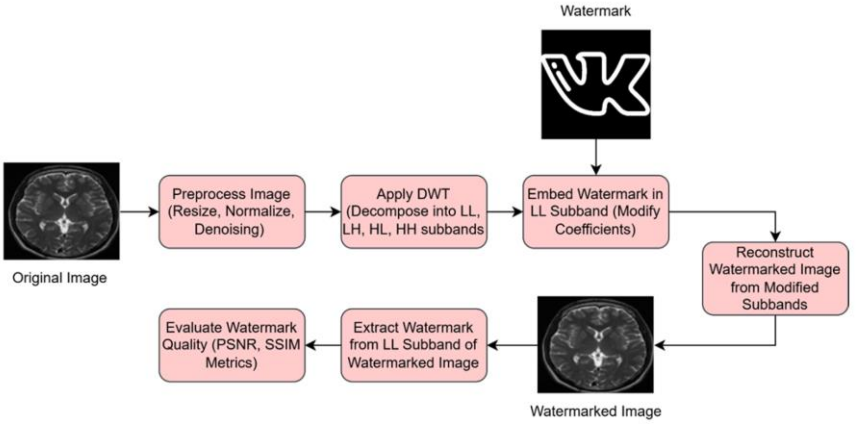


Figure 2
Process of Embedding and Extraction of Watermark into a Medical Image (MRI Image)

The Figure 2 describes the algorithmic steps followed for embedding and extraction of watermark. **Algorithm Embedding and Extraction of Watermark**

(a) The medical image is loaded and converted into grayscale images as it reduces the complexity and focuses on the intensity of the image.

$$(I_{LL}, I_{LH}, I_{HL}, I_{HH}) = \mathrm{DWT}(I) \tag{9}$$

(b) The image is pre-processed by resizing and normalizing to ensure that pixel values are between 0 and 1. If noise reduction is required, then the usage of Gaussian filters or median filters is employed.

$$I_{LL}{}' = I_{LL} + \alpha W \tag{10}$$

(c) Apply DWT to decompose the image into four bands LL, LH, HH, HL. LL sub-band has the most important elements of the image among these.

$$I_w = \mathrm{IDWT}(I_{LL}{}', I_{LH}, I_{HL}, I_{HH}) \tag{11}$$

(d) Convert the watermark to grayscale and resize it to match the dimension of LL sub-band. It is done by modifying wavelet coefficients, which is done using the method Least Significant Bit(LSB) embedding or adding a small value to the coefficients.

$$W_{ext} = \frac{I_{LL}{}' - I_{LL}}{\alpha} \tag{12}$$

(e) After embedding image is reconstructed by applying IDWT to the modified sub-bands.

(f) Once the image is reconstructed the quality is then evaluated using performance metrics, PSNR and SSIM.

$$PSNR = 10\log_{10}\left(\frac{MAX^2}{MSE}\right) \tag{13}$$

$$SSIM(I, I_w) = \frac{(2\mu_I\mu_{I_w}+C_1)(2\sigma_{II_w}+C_2)}{(\mu_I^2+\mu_{I_w}^2+C_1)(\sigma_I^2+\sigma_{I_w}^2+C_2)} \tag{14}$$

(g) Extraction is done by subtracting the value of original LL sub-band from the modified LL sub-band.

(h) At the end PSNR and SSIM is calculated for the original image and is compared with the PSNR and SSIM of watermarked image to check for the distortion.

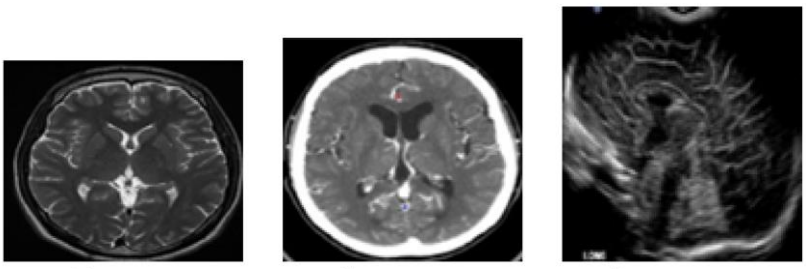
4. Result

This section shows the outcomes of the watermarking approach applied to various medical pictures like MRI, CT-Scan, X-Ray with the help of Discrete Wavelet Transform(DWT). The results are assessed using two performance metrics: PSNR (Peak Signal-to-Noise Ratio) and SSIM (Structural Similarity Index). This section presents the performance of the watermarking approach, experimental setting and comparison between the performance metrics utilizing DWT.

4.1 *Experimental Setup*

The experimental setup for evaluating the performance of the watermarking method involves the following components:

1. Dataset:
   - Medical images (e.g., MRI scans, CT scans, X-ray images) shown in Figure 3 are used as the primary dataset for this experiment.
   - Images are collected from publicly available medical datasets or simulated for testing purposes. All images used are grayscale to simplify the pre-processing and watermarking processes.



(a) MRI Image (b) CT Scan Image (c) Ultrasound Image

Figure 3
Three images side by side

2. Watermark:
   - The watermark used for this experiment is a simple binary watermark (either a logo, text or a random noise pattern). The watermark is placed

in the LL (Low-Low) subband of the DWT decomposition of the image.

3. Watermarking Method:
   - The watermark is incorporated in the LL subband with a little alteration of the wavelet coefficients. To make it imperceptible the watermark is added with a scaling factor.
   - Inverse DWT (IDWT) is used to recreate the watermarked image. The technique of extraction of watermark is similar without the original image.
4. Image Processing Attacks: To test the robustness of the watermarking method, several common image processing attacks are applied to the watermarked image, such as:
   - Compression (e.g., JPEG compression)
   - Noise Addition (e.g, Gaussian Noise)
   - Resizing: scaling image
   - Cropping:removing portions of the image
5. Evaluation Metrics:
   - Performance of the watermarking approach is examined using PSNR (Peak Signal-to-Noise Ratio) and SSIM (Structural Similarity Index) to determine the image quality and efficiency of the watermark.

4.2 *Analysis of Results*

The values of PSNR and SSIM are given in the Table 2 for different medical images.

Table 2

PSNR and SSIM values for different medical images 1

S.No	Image	PSNR	SSIM
0	MRI	48.210	0.9955
1	CTScan	34.250	0.6593
2	Ultrasound	49.253	0.9978

1. MRI:
   - PSNR: 48.22 dB (High) — The high PSNR value indicates that the watermarking has very little effect on the image quality. The image quality is almost preserved.
   - SSIM: 0.9956 (Very High) — A high SSIM value shows that the structural similarity between the original and watermarked images is excellent, meaning the watermark is imperceptible.
2. CT Scan:
   - PSNR: 34.23 dB (Moderate) — The PSNR value is relatively lower than MRI and Ultrasound, indicating some loss of quality due to watermark embedding.

- SSIM: 0.6593 (Moderate) — A significantly lower SSIM value suggests that the watermark has caused a noticeable change in the image structure, which could affect diagnostic interpretation.
    * PSNR: 49.27 dB (Very High) — The high PSNR indicates minimal impact on image quality, similar to MRI.
    * SSIM: 0.9866 (High) — This value suggests that the watermarking process has preserved the structure of the image well, though slightly lower than MRI.

4.3 *PSNR and SSIM comparisons*

These performance metrics indicate the quality of watermarked images which are elaborated below:

- MRI shows the highest PSNR and SSIM, indicating that the watermark has minimal impact on image quality.
- CT Scan has the lowest SSIM and moderate PSNR, suggesting some quality degradation after watermark embedding, which is common for images with more complex structures.
- Ultrasound retains high PSNR and SSIM, showing that the watermarking method preserves quality well, even in detailed medical images.

4.4 *Quality of Watermarked Images*

In this study, the quality of the watermarked images is assessed both visually and quantitatively:

(a) **Visual Inspection:** The watermarked images should closely resemble the original, with no visible distortion. The watermark should be imperceptible, ensuring the diagnostic utility of the image is maintained.

(b) **Impact of Compression and Noise:** After applying JPEG compression or adding Gaussian noise, the watermark should still be detectable. While image quality may degrade (lower PSNR and SSIM values), the watermark extraction should remain successful.

(c) **Quality Preservation:** The watermarking process should preserve critical image features (e.g., tissues, organs) necessary for diagnosis. A minor reduction in quality is acceptable as long as it does not affect the image's interpretability.

4.5 *Discussion of Results*

The results of this study using DWT watermarking highlighted several key points, which are discussed below:

(a) **Robustness:** The technique shows resilience against typical image processing threats such noise, scaling, and compression. Even after these manipulations, the watermark remains detectable and recoverable, essential for real-world applications.

(b) **Imperceptibility:** The watermark does not substantially diminish the medical photos' visual quality. The high PSNR (48.22 for MRI) and SSIM

(0.9956 for MRI) values show that the watermark embedding process is efficient, preserving image quality, which is crucial for accurate medical diagnosis.

(c) **Extraction Accuracy:** The watermark extraction remains highly accurate, even with the application of various image processing attacks, demonstrating the methodâ€™s resilience in medical environments.

(d) **Performance Evaluation:** PSNR and SSIM values demonstrate that the watermarking technique maintains image quality while embedding the watermark. A slight decrease in these values is acceptable as long as the watermark remains intact and the image's diagnostic value is not compromised.

(e) **Practical Applications:** This watermarking method can be employed in healthcare systems to ensure secure transmission, storage, and access to medical images. It also facilitates the authentication and traceability of medical images back to their original source, improving security in medical data management.

5. Challenges in DWT- Based Watermarking

DWT faces challenges in balancing imperceptibility, robustness, reversibility, and security for medical images.

(a) **Imperceptibilty Challenge:** Watermarking should not distort medical images, as even minor changes can affect diagnosis accuracy. Embedding in the LL subband reduces visual artifacts, but some distortions may still appear in high-contrast areas. Adaptive embedding techniques can help minimize these effects.

(b) **Robustness vs. Reversibility Trade-off:** Stronger watermarks are less able to fully restore the original image, but they are more resilient to cropping, noise, and compression. One of the biggest challenges is still striking the ideal balance between robustness and reversibility.

(c) **Security Risks and Malicious Attacks:** Medical images are vulnerable to tampering, watermark removal, and forgery attacks. Techniques like noise addition, filtering, or scaling can degrade watermark integrity. Future solutions may involve encryption-based watermarking to improve security.

(d) **Computational Complexity and Storage:** DWT requires multi-level decomposition, increasing processing time and storage needs, especially for high-resolution images (e.g., MRI, CT scans). Hybrid techniques (DWT-SVD, DWT-DCT) can help reduce computational overhead.

6. Conclusion

This study is carried out to evaluate the performance of DWT using LSB for different medical images, MRI, CT-Scan and Ultrasound for watermarking. According to the results obtained on various images, the PSNR for an MRI image is 48.210dB, indicating that it has very little effect on the image quality, and the SSIM is 0.9955, indicating that the watermark is undetectable. In case of CT-

Scan the PSNR is 34.25dB, which is lower than both MRI and Ultrasound 49.2527dB, which shows that after watermarking in the CT-Scan image, the quality is degraded. In case of CT-Scan, SSIM is 0.6593, which is again very less than both MRI and Ultrasound image, 0.9978, which can cause noticeable changes in the images, which could only affect diagnostic interpretation. But overall, the results are satisfactory and to improve for different images Arnold Mapping and Logistic Mapping,, along with different watermarking techniques can be used and also embedding can be done in other sub-bands.

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