

The reflection of planewaves in a generalized microstretch thermoelastic medium considering initial stress and two temperature effects

Jiten Sharma *

Sangeeta Kumari †

Department of Mathematics

Chandigarh University

Mohali 140413

Punjab

India

Abstract

The propagation of plane waves in a two-dimensional microstretch thermoelastic medium with two temperatures and initial stress is examined in this study. The study makes use of microstretch theory, which incorporates two temperature considerations, rotation, microstretch effects, and initial stress at the microscale. The Coupled Transverse Displacement (CTD) wave, the Coupled Microrotational Displacement (CMD) wave, and three other waves---the Coupled Longitudinal Displacement (CLD) wave, the Coupled Thermal (CT) wave, and the Coupled Transverse Microrotational (CTM) wave---are found when the model's basic equations are solved for the plane waves. Using the proper boundary conditions, for the five waves the reflection coefficients are calculated, and the graphical results are displayed.

Subject Classification: 74F05, 74J20, 74G60, 74B05.

Keywords: Two-temperature, Microstretch, Plane wave, Reflection coefficient, Initial stress.

1. Introduction

The theory of generalized thermoelasticity serves as a modified form of the classical uncoupled and coupled theories of thermoelasticity [1]. Initial stresses can emerge from processes like manufacturing, temperature variations, mechanical loading, or geological occurrences, which can create residual states in a solid even before any new forces are exerted. To address the importance of initial stress in the real world there were many studies related to them, starting from Biot [2] in which it was studied how initial stress effects elastic waves. Tang

* E-mail: jitensharma1922@gmail.com (Corresponding Author)

† E-mail: sangwan.sangeeta.ss@gmail.com

[3] studied how initial stress affects the propagation of elastic waves in solids. It showed that initial stresses alter the rigidness of the material, causing changes in wave speed compared to an unstressed medium. In 1977 and 1979, Dey and Addy [4] presented two studies on reflection and refraction of plane waves under the influence of initial stresses at a boundary free surface and at an interface respectively. Ogden and Sotiropoulos [5] presented reflection of plane waves under stress conditions in incompressible elastic solids. Othman and Song [6] studied the reflection of plane waves from an elastic solid half-space, taking into account hydrostatic initial stress and the condition of no energy dissipation. Similar studies on initial stress were done by many researchers such as [7, 8].

The analysis of plane wave reflection in elastic solids is rooted in classical elasticity and wave theory, with foundational mathematical framework for elastic wave propagation formalized. The concept of elasticity and plasticity was first introduced by Bingham [9] in 1924. The mathematical theory of elasticity was given by Love [10]. The microstretch theory in elasticity was introduced by Eringen [11], who developed the theory of microstretch elastic solids as a generalization of earlier elasticity theories. Singh and Kumar [12] analysed the behaviour of plane wave propagation within a generalised thermo-microstretch elastic solid and also explored the reflection of plane waves from the free surface of this type of solid. Reflection phenomena of a plane harmonic wave at an interface of a thermo-microstretch elastic half space was discussed by Song, Othman, and Zhao [13]. This analysis explores the features of plane waves in a two-dimensional thermoelastic solid half-space, which incorporates both micro-rotational and microstretch effects in a generalized thermoelastic framework featuring two temperatures. Thermoelasticity in a micro-stretch non-local medium submerged in non-viscous fluid under temperature-controlled conditions was analyzed by Othman, Eraki, and Ismail [14]. Propagation of waves in microstretch thermoelastic solids using two-temperature theory, which transforms heat conduction using two distinct temperature fields, was studied by Kaur et al. [15]. Plane Wave reflection in hygro-thermoelastic half-space with hydrostatic initial stress, rotation and magnetic field was studied by Kumar, Nagar, and Kumari [16].

This research formulates the mathematical problem in non-dimensional terms and employs the Helmholtz principle to streamline displacement potentials. Plane harmonic solutions are utilized to establish relationships pertaining to wave behavior. Explicit equations for reflection coefficients are derived and subsequently solved. The findings are illustrated graphically.

2. Modeling of The Problem

We investigated a two-dimensional half-space configuration where the medium is aligned with the x - y plane and with the y -axis pointing vertically downwards. This configuration establishes a semi-infinite domain that extends infinitely in the positive y -direction from the boundary surface at $y = 0$, as illustrated in Figure 1. Following [17] and [18], the governing equations characterizing generalized isotropic thermo-microstretch elastic solids based on G-N theory incorporating two temperatures are expressed as follows.

(i) Equation of Motion

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (1)$$

where,

$$\sigma_{ij} = (\lambda_2 \Phi^* + \lambda \nabla \cdot u_i) - p \delta_{ij} + (\mu + k) \nabla u_i + \mu (\nabla u_i) - k (\nabla \times \vec{\Phi}) - \hat{\gamma} \Theta \delta_{ij}.$$

(ii) Microrotation and Microstretch Equations

$$(\alpha_1 + \alpha_2 + \alpha_3) \nabla (\nabla \cdot \vec{\Phi}) - \alpha_3 \nabla \times (\nabla \times \vec{\Phi}) + k (\nabla \times u_i) - 2k \vec{\Phi} = j^* \rho \ddot{\vec{\Phi}}, \quad (2)$$

$$\lambda_1 \Delta \Phi^* - \frac{1}{3} \lambda_3 \Phi^* - \frac{1}{3} \lambda_2 \nabla u_i + \frac{1}{3} \hat{\gamma}_1 \Theta = \frac{3}{2} \rho j^* \ddot{\Phi}^*. \quad (3)$$

(iii) Heat Conduction Equation

$$K \nabla^2 \Theta = \rho C_E \ddot{\Theta} + \hat{\gamma} T_0 \nabla \cdot \ddot{u}_i + \hat{\gamma}_1 T_0 \ddot{\Phi}^*, \quad (4)$$

$$\mathbf{m} = \alpha_1 \nabla \cdot \vec{\Phi} \delta_{ij} + \alpha_2 \nabla \vec{\Phi} + \alpha_3 (\nabla \vec{\Phi}). \quad (5)$$

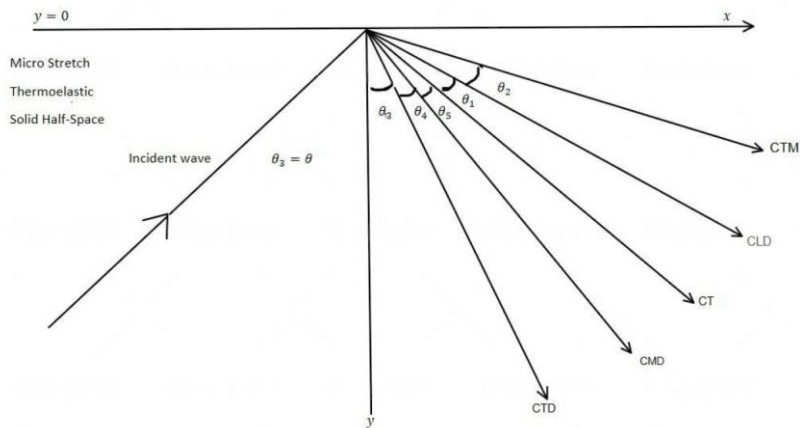


Figure 1
Geometry of the Problem

Here Φ^* is the microstretch scalar, δ_{ij} signifies the Kronecker delta, $\vec{\Phi}$ is the micro-rotational vector, σ_{ij} is the stress tensor, $\mathbf{m}$ is the coupled stress, $k, \alpha_1, \alpha_2, \alpha_3$ are the micropolar constants, λ and μ are Lamé's coefficients, the displacement vector is represented as u_i , $\lambda_1, \lambda_2, \lambda_3$ are the microstretch elastic constants, C_E is the specific body heat, $\hat{\gamma} = (3\lambda + k + 2\mu)\alpha$, $\hat{\gamma}_1 = (k + 2\mu + 3\lambda)\beta$ are material parameters, α is a scalar material parameter affecting the coupling between thermal and mechanical fields, β is another scalar material parameter influencing the coupling between temperature and microstretch variables. $\Theta = T - T_0$ where Θ indicates a minimal temperature increment; T is the absolute temperature of the medium, T_0 indicates the standard uniform temperature of the body, and the reference temperature T_0 is chosen such that $|\Theta/T_0| = 1$. The two-temperature parameter is denoted by a^* , K denotes the

thermal conductivity, p is the initial stress, ρ denotes the density, and j^* is the micro-inertia moment.

The problem is considered such that the medium is parallel to the x - y plane as shown in Figure 1, with displacement vector $u_1 = u_1(x, y, t)$, $u_2 = u_2(x, y, t)$, $u_3 = 0$, and the micro-rotation vector taken as $\vec{\phi} = (0, 0, \phi_3)$ [19]. With the help of the Helmholtz principle, the displacement components are expressed in terms of potential functions ψ_1 and ψ_2 as [20]:

$$u_1 = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}, \quad u_2 = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}. \quad (6)$$

Using the above 2-D problem, equations (1)-(5) reduce to the following:

$$(\lambda + \mu) \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_2}{\partial x \partial y} \right) + \left(\mu + k - \frac{p}{2} \right) \nabla^2 u_1 - k \frac{\partial \phi_3}{\partial y} + \lambda_2 \frac{\partial \Phi^*}{\partial x} - \hat{\gamma} \frac{\partial T}{\partial x} = (1 - e^2 \nabla^2) \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (7)$$

$$(\lambda + \mu) \left(\frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_1}{\partial x \partial y} \right) + \left(\mu + k + \frac{p}{2} \right) \nabla^2 u_2 - k \frac{\partial \phi_3}{\partial x} + \lambda_2 \frac{\partial \phi_3}{\partial y} - \hat{\gamma} \frac{\partial T}{\partial y} = (1 - e^2 \nabla^2) \rho \frac{\partial^2 u_2}{\partial t^2}, \quad (8)$$

$$\nabla^2 \phi_3 + k \left(\frac{\partial u_1}{\partial y} - \frac{\partial u_2}{\partial x} \right) - 2k \phi_3 = j^* \rho \frac{\partial^2 \phi_3}{\partial t^2}, \quad (9)$$

$$\lambda_1 \nabla^2 \Phi^* - \frac{1}{3} \lambda_3 \Phi^* - \frac{1}{3} \lambda_2 \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} \right) + \frac{1}{3} \hat{\gamma}_1 T = \frac{3}{2} \rho j^* \ddot{\Phi}^*, \quad (10)$$

$$K \nabla^2 T = \rho C_E \ddot{T} + \hat{\gamma} T_0 \frac{\partial^2}{\partial t^2} \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} \right) + \hat{\gamma}_1 T_0 \ddot{\Phi}^*. \quad (11)$$

Now we introduce the following non-dimensional quantities:

$$y' = \frac{\omega^*}{c_1} y, \quad u_{1'} = \frac{\rho c_1 \omega^*}{\hat{\gamma} T_0} u_1, \quad u_{2'} = \frac{\rho c_1 \omega^*}{\hat{\gamma} T_0} u_2, \quad t' = \omega^* t, \quad T' = \frac{T}{T_0}, \quad x' = \frac{\omega^*}{c_1} x, \\ \phi_{3'} = \frac{\phi_3}{\hat{\gamma} T_0}, \quad \Phi^* = \frac{\rho C_E^2}{\hat{\gamma} T_0^2}, \quad c_1^2 = \frac{\mu}{\rho}, \quad \omega^* = \frac{\rho C_E^2}{\tau_0}, \quad p' = \frac{p}{\lambda + 2\mu}. \quad (12)$$

Using equation (12) and the above dimensionless quantities in (7)-(11), we obtain:

$$[\nabla^2 - M_2] \psi_1 - M_2 (1 - a^* \nabla^2) T + M_3 \Phi^* = 0, \quad (13)$$

$$[\nabla^2 - M_4] \psi_2 - M_5 \phi_3 = 0, \quad (14)$$

$$[\nabla^2 - 2M_6 - M_7 \frac{\partial^2}{\partial t^2}] \phi_3 + M_6 \nabla^2 \psi_2 = 0, \quad (15)$$

$$[M_8 \nabla^2 - M_9] \Phi^* - M_{10} \nabla^2 \psi_1 + M_{11} (1 - a^* \nabla^2) T = 0, \quad (16)$$

$$[\varepsilon_2 \nabla^2 (1 - a^* \nabla^2)] T = (1 - a^* \nabla^2) T + \varepsilon_1 \frac{\partial^2}{\partial x^2} \nabla^2 \psi_1 + \varepsilon_4 \Phi^*, \quad (17)$$

where,

$$c_2^2 = \frac{2\lambda_1}{3\rho j^*}, \quad c_3^2 = \frac{2\lambda_3}{9\rho j^*}, \quad c_4^2 = \frac{2\lambda_2}{9\rho j^*}, \quad \varepsilon_1 = \frac{\hat{\gamma}^2 T_0}{\rho^2 C_E c_1^2}, \quad \varepsilon_2 = \frac{K}{\rho C_E c_1^2}, \quad \varepsilon_4 = \frac{\hat{\gamma} \hat{\gamma}_1 T_0}{\rho^2 C_E \omega^* c_1^2}, \\ c_0^2 = \frac{\lambda + 2\mu + k}{\rho}, \quad M_2 = \frac{c_1^2}{c_0^2}, \quad M_3 = \frac{\lambda_2}{\lambda + 2\mu + k}, \quad M_4 = \frac{\rho c_1^2}{\mu + k}, \quad M_5 = \frac{k}{\mu + k},$$

$$M_6 = \frac{kc_1^2}{\alpha_3\omega^{*2}}, \quad M_7 = \frac{\rho J^* c_1^2}{\alpha_3}, \quad M_8 = \frac{c_2^2}{c_1^2}, \quad M_9 = \frac{c_3^2}{\omega^{*2}}, \quad M_{10} = \frac{c_4^2}{\omega^{*2}}, \quad M_{11} = \frac{2\gamma_1^2 c_1^2}{9\gamma_j^* \omega^{*2}}.$$

3. Solution of the Problem

We consider the plane harmonic solution as [6]:

$$[\psi_1, \psi_2, \phi_3, \Phi^*, T](x, y, t) = [\bar{\psi}_1, \bar{\psi}_2, \bar{\phi}_3, \bar{\Phi}^*, \bar{T}] \exp(i\xi(x \sin \theta + y \cos \theta - vt)), \quad (18)$$

where $\xi v = \omega$, v represents the complex phase velocity, ξ represents the wave number, and ω represents the frequency.

Using equation (18) in (13)-(17), we have:

$$(-M_2 - \xi^2)\psi_1 - M_2(1 + a^*\xi^2)T + M_3\Phi^* = 0, \quad (19)$$

$$(-\xi^2 - M_4)\psi_2 - M_5\phi_3 = 0, \quad (20)$$

$$[M_7\xi^2v^2 - 2M_6 - \xi^2]\phi_3 - \xi^2M_6\psi_2 = 0, \quad (21)$$

$$[M_8\xi^2 - M_9]\Phi^* + M_{10}\xi^2\psi_1 + M_{11}[1 + a^*\xi^2]T = 0, \quad (22)$$

$$[-\varepsilon_2\xi^2(1 + a^*\xi^2) - (1 + a^*\xi^2)iv\xi]T - \varepsilon_1\xi^4v^2\psi_1 - \varepsilon_4\Phi^* = 0. \quad (23)$$

Solving equations (20) and (21) together, and then (19), (22) and (23), we get:

$$Av^4 + Bv^2 - C = 0, \quad (24)$$

$$Pv^6 + Qv^4 + Rv^2 - S = 0, \quad (25)$$

where,

$$A = M_4M_7\omega^2 - 2M_6M_4,$$

$$B = \omega^4M_7 - 2M_6\omega^2 - M_4\omega^2 - M_5M_6\omega^2, \quad C = \omega^4,$$

$$P = M_2M_{11}\varepsilon_4,$$

$$Q = M_2M_{11}\varepsilon_4\omega^2a^* - M_2M_8\omega^3i - M_2M_9i\omega^3a^* + M_{11}\varepsilon_4\omega^2 - M_{10}\omega^2M_2\varepsilon_4 \\ + M_3M_{10}i\omega - M_2M_9\omega^4\varepsilon_4 + M_3M_{11}\varepsilon_1\omega^4,$$

$$R = M_2M_8\omega^6\varepsilon_1 - M_2M_9\varepsilon_1a^* + M_3M_{11}a^*\omega^4\varepsilon_1 - M_2M_8\omega^4\varepsilon_2 + M_{11}a^*\omega^4\varepsilon_4 \\ - M_8\omega^5i - M_9i\omega^5a^* - M_2M_{10}\omega^4a^*\varepsilon_4 - M_3M_{10}\varepsilon_2\omega^4 - M_3M_{10}a^*i\omega^5,$$

$$S = M_2M_8\omega^8a^*\varepsilon_1 - M_3M_{10}\omega^6\varepsilon_2a^* - M_2M_8\omega^6\varepsilon_2a^* - M_8\omega^6\varepsilon_2 - M_8\omega^6\varepsilon_2a^*.$$

Following [21], the formulations of phase velocity (V_i), specific loss (S_i), and attenuation coefficient (Q_i) are expressed as:

$$P_i = \frac{|\Re(n_i)| + |\Im(n_i)|^2}{|\Re(n_i)|}, \quad (26)$$

$$S_i = \left(\frac{\Delta W}{W}\right)_i = 4\pi \frac{|\Im(n_i)|}{|\Re(n_i)|}, \quad (27)$$

$$Q_i = \frac{\omega \Im(n_i)}{|\Re(n_i)|^2 + |\Im(V_i)|^2}, \quad (28)$$

where $\Re(n_i)$ and $\Im(n_i)$ represent the real and imaginary parts of n_i , with $i = 1, 2, 3, 4, 5$, respectively.

4. Mathematical Representation of the Reflection Coefficient

Taking into account the following boundary conditions at $y = 0$:

- Stress free surface:

$$\sigma_{xx} = 0, \quad \sigma_{xy} = 0 \quad \text{at } y = 0. \quad (29)$$

- Tangential stress vanishes at $y = 0$:

$$m_{yz} = 0. \quad (30)$$

- Thermally insulated surface:

$$\frac{\partial T}{\partial y} = 0. \quad (31)$$

- Microstretch parameter condition:

$$\frac{\partial \Phi^*}{\partial y} = 0. \quad (32)$$

4.2 For the Incident CLD-Wave

The displacement, rotational, temperature and microstretch potentials $\psi_1, \psi_2, \psi_3, T, \Phi^*$ are taken in the form [22]:

$$\phi_3 = B_0 e^{[i\xi_1(x\sin\theta + y\cos\theta) - i\omega t]} + \sum_{k=1}^2 B_k e^{[i\xi_k(x\sin\theta_k - y\cos\theta_k) - i\omega t]}, \quad (33)$$

$$\psi_2 = \eta_1 B_0 e^{[i\xi_1(x\sin\theta + y\cos\theta) - i\omega t]} + \sum_{k=1}^2 B_k e^{[i\xi_k(x\sin\theta_k - y\cos\theta_k) - i\omega t]}, \quad (34)$$

$$\psi_1 = A_0 F_0 e^{[i\xi_1(x\sin\theta + y\cos\theta) - i\omega t]} + \sum_{r=3}^5 A_r F_r e^{[i\xi_r(x\sin\theta_r - y\cos\theta_r) - i\omega t]}, \quad (35)$$

$$T = A_0 F_0^* e^{[i\xi_1(x\sin\theta + y\cos\theta) - i\omega t]} + \sum_{r=3}^5 A_r F_r^* e^{[i\xi_r(x\sin\theta_r - y\cos\theta_r) - i\omega t]}, \quad (36)$$

$$\Phi^* = A_0 e^{[i\xi_1(x\sin\theta + y\cos\theta) - i\omega t]} + \sum_{r=3}^5 A_r e^{[i\xi_r(x\sin\theta_r - y\cos\theta_r) - i\omega t]}, \quad (37)$$

where the values of F_r, F_r^* , and η_1 are:

$$F_r = \frac{L_{11}F_r^* + L_8\xi_r^2 + L_9 + \omega^2}{L_{10}\xi_r^2}, \quad F_r^* = \frac{\xi_r^2 - L_2\omega^2 - L_3}{L_2}, \quad \eta_1 = \frac{\xi_k^2 + L_4\omega^2}{L_5}. \quad (38)$$

The reflection coefficient is described as the proportion of the amplitudes of the reflected waves to the amplitudes of the incident waves, P_k/Q_0 (where $k = 1, 2$) and M_r/Q_0 (where $r = 3, 4, 5$). The angles θ_k ($k = 1, 2$) and θ_r ($r = 3, 4, 5$) and respective wave numbers are connected via Snell's law [23]:

$$\xi \sin \theta = \xi_1 \sin \theta_1 = \xi_2 \sin \theta_2 = \xi_3 \sin \theta_3 = \xi_4 \sin \theta_4 = \xi_5 \sin \theta_5, \quad \xi_i v_i = \omega. \quad (39)$$

Using equations (33)-(37) and Snell's law (39) in the boundary conditions (29)-(32), we obtain:

$$\frac{P_1}{Q_0} j_{11} + \frac{P_2}{Q_0} j_{12} + \frac{M_3}{Q_0} j_{13} + \frac{M_4}{Q_0} j_{14} + \frac{M_5}{Q_0} j_{15} = H_1, \quad (40)$$

$$\frac{P_1}{Q_0} j_{21} + \frac{P_2}{Q_0} j_{22} + \frac{M_3}{Q_0} j_{23} + \frac{M_4}{Q_0} j_{24} + \frac{M_5}{Q_0} j_{25} = H_2, \quad (41)$$

$$\frac{P_1}{Q_0} j_{31} + \frac{P_2}{Q_0} j_{32} + \frac{M_3}{Q_0} j_{33} + \frac{M_4}{Q_0} j_{34} + \frac{M_5}{Q_0} j_{35} = H_3, \quad (42)$$

$$\frac{P_1}{Q_0} j_{41} + \frac{P_2}{Q_0} j_{42} + \frac{M_3}{Q_0} j_{43} + \frac{M_4}{Q_0} j_{44} + \frac{M_5}{Q_0} j_{45} = H_4, \quad (43)$$

$$\frac{P_1}{Q_0} j_{51} + \frac{P_2}{Q_0} j_{52} + \frac{M_3}{Q_0} j_{53} + \frac{M_4}{Q_0} j_{54} + \frac{A_5}{Q_0} j_{55} = H_5. \quad (44)$$

The values of all the reflection coefficients $j_{1r}, j_{2r}, j_{3r}, j_{4r}, j_{5r}$ are given as, where $k = 1, 2$ and $r = 3, 4, 5$:

$$\begin{aligned} j_{1k} &= -2\mu\xi_k^2 \sin\theta_k \cos\theta_k, \quad j_{1r} = -\lambda\xi_j^2 F_j - 2\mu\xi_j^2 \sin^2\theta_j - B_j F_j, \\ j_{2k} &= -\eta_1 \mu\xi_k^2 (\cos^2\theta_k - \sin^2\theta_k), \quad a_{2r} = -\mu F_j \xi_j^2 \sin\theta_j \cos\theta_j, \\ j_{3k} &= 0, \quad j_{3r} = -F_j t \xi_j \cos\theta_j, \quad j_{4k} = 0, \quad j_{4r} = -F_j^* A_j t \xi_j \cos\theta_j, \\ j_{5k} &= 0, \quad j_{5r} = -B_j t \xi_j \cos\theta_j, \quad \text{where } k = 1, 2, \quad r = 3, 4, 5, \\ H_1 &= -a_{11}, \quad H_2 = -a_{21}, \quad H_3 = -a_{31}, \quad H_4 = -a_{41}, \quad H_5 = -a_{51}. \end{aligned}$$

5. Numerical Results and Discussions

To demonstrate the effect of two temperatures, we take the physical properties of magnesium crystal, listed in Table 1, as given in [24].

Table 1
Material Parameters

Parameter	Value	Unit
k	1	$\text{kg m}^{-1} \text{s}^{-2}$
λ_2	0.5	$\text{kg m}^{-1} \text{s}^{-2}$
λ	9.4	$\text{kg m}^{-1} \text{s}^{-2}$
ρ	1.74×10^3	kg m^{-3}
α_3	0.779	kg m s^{-2}
j^*	0.2	m^3
λ_1	0.779	$\text{kg m}^{-1} \text{s}^{-2}$
K^*	0.1	$\text{W m}^{-1} \text{K}^{-1}$
λ_3	0.1	$\text{kg m}^{-1} \text{s}^{-2}$
T_0	293	K
C_E	1.04	$\text{J kg}^{-1} \text{°C}$
β	2.48	$\text{kg m}^{-1} \text{s}^{-2}$
K_0	1.3	$\text{W m}^{-1} \text{K}^{-1}$
μ	4	$\text{kg m}^{-1} \text{s}^{-2}$

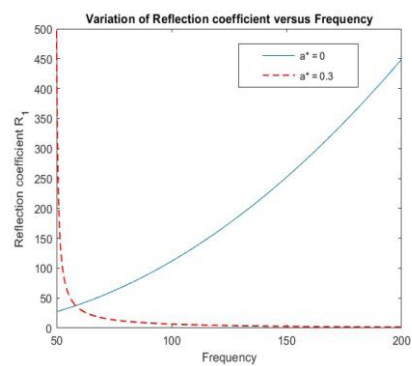


Figure 2
Reflection Coefficient R_1 vs Frequency

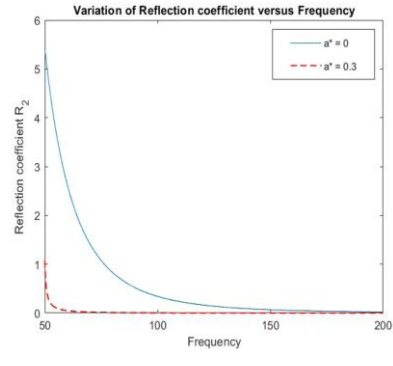


Figure 3
Reflection Coefficient R_2 vs Frequency

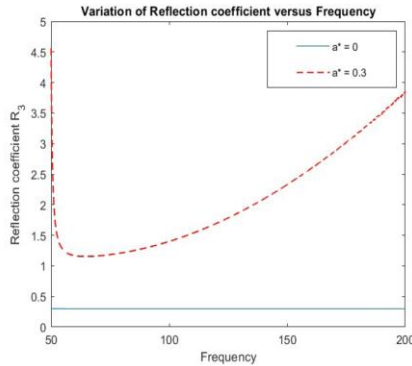


Figure 4
Reflection Coefficient R_3 vs Frequency

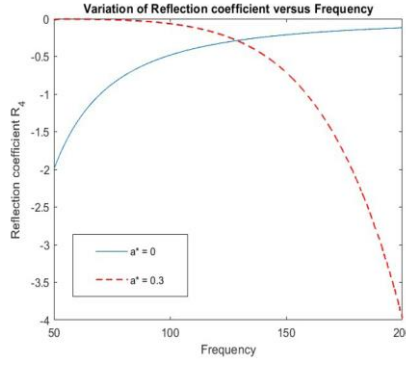


Figure 5
Reflection Coefficient R_4 vs Frequency

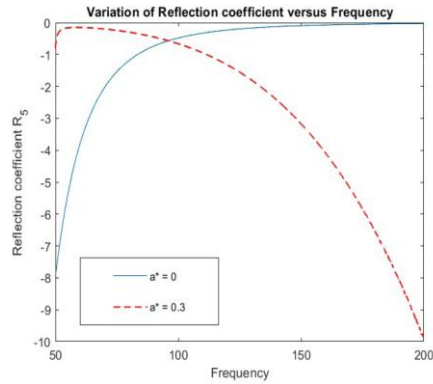


Figure 6
Reflection Coefficient R_5 vs Frequency

The coefficients of reflection, R_1 and R_2 , shown in Figures 2 and 3, correspond to the Coupled Transverse Displacement (CTD) and Coupled Microrotational Displacement (CMD) waves, respectively. These coefficients exhibit distinct frequency-dependent behaviors under the influence of two-temperature theory and initial stress conditions. For reflection coefficient R_1 , Figure 2 reveals that the two-temperature parameter significantly enhances energy reflection at lower frequencies, with the coefficient showing an increasing trend with frequency. This behavior contrasts with the case when the temperature parameter is not applied, where the reflection remains relatively constant. The physical interpretation suggests that the dual temperature formulation creates additional thermal coupling mechanisms that redistribute incident wave energy back toward the boundary. Under two-temperature conditions, the reflection coefficient R_2 , as depicted in Figure 3, shows a stabilizing effect. When the two-temperature parameter is activated, the microrotational displacement wave exhibits little frequency variation, suggesting that the microstretch-thermal coupling produces a damping mechanism that prevents excessive boundary reactions. For applications involving materials with intricate microstructures, where energy dissipation is essential, this behaviour is especially significant. Both coefficients are made more complex by the initial stress parameter. When combined with two-temperature effects, initial stress tends to amplify the frequency dependence of R_1 while maintaining the stability characteristics of R_2 , suggesting that initial conditions enhance the selectivity of wave reflection based on frequency content.

The reflection coefficients R_3 , R_4 , and R_5 , illustrated in Figures 4, 5, and 6 respectively, are linked to the Coupled Longitudinal Displacement (CLD), Coupled Thermal (CT), and Coupled Transverse Microrotational (CTM) waves. These coefficients exhibit more complex frequency-dependent behaviors under two-temperature conditions. As shown in Figure 4, reflection coefficient R_3 shows a pronounced reduction with increasing frequency when two-temperature effects are included, particularly in the mid-frequency range. This behavior indicates that the longitudinal displacement coupling with dual temperature fields facilitates greater energy penetration into the medium rather than boundary reflection. The physical mechanism involves the enhanced thermal-mechanical coupling that redistributes energy more effectively throughout the material volume. The thermal wave reflection coefficient R_4 , plotted in Figure 5, exhibits the most dramatic frequency dependence, with steep variations indicating strong thermal-mechanical interactions. Under two-temperature conditions, R_4 shows initial stability followed by significant changes at higher frequencies. This behavior suggests that the dual temperature formulation creates competing thermal pathways that affect how thermal energy is reflected from the boundary. The steep transitions observed correspond to resonance frequencies where the thermal and mechanical fields achieve optimal coupling. Reflection coefficient R_5 , associated with the transverse microrotational wave and shown in Figure 6, demonstrates non-monotonic behavior across different parameter combinations. The two-temperature effect tends to smooth high-frequency resonances,

preventing sharp energy accumulations at the boundary. This smoothing effect is particularly beneficial for preventing material damage due to localized energy concentrations.

6. Conclusion

Using two-temperature theory and initial stress, the study examines the propagation and reflection of plane waves in a two-dimensional generalised thermoelastic microstretch medium. Under appropriate boundary conditions, analytical expressions for the reflection coefficients of these coupled waves were obtained and numerically assessed for materials like magnesium crystal.

- The findings show that the frequency-dependent behaviour of wave reflection is significantly influenced by the two-temperature parameter. The two-temperature effect stabilises microrotational wave reflection while reducing energy reflection in transverse displacement waves at lower frequencies, suggesting intricate thermal-mechanical coupling and damping mechanisms.
- The analysis shows that the coupled behaviour of mechanical, rotational, microstretch, and thermal fields is influenced by microstretch effects, which result in more complex boundary value problems and enhanced wave phenomena. Overall, the research provides new insights into the fundamental wave behavior in microstretch thermoelastic solids with two-temperature and initial stress (p) considerations.

References

- [1] R. B. Hetnarski and J. Ignaczak, "Generalized thermoelasticity," *J. Therm. Stresses*, vol. 22, no. 4-5, pp. 451-476 (1999), doi: 10.1080/014957399280832.
- [2] M. A. Biot, "The influence of initial stress on elastic waves," *J. Appl. Phys.*, vol. 11, no. 8, pp. 522-530 (1940), doi: 10.1063/1.1712807.
- [3] S. Tang, "Wave propagation in initially-stressed elastic solids," *Acta Mech.*, vol. 4, no. 1, pp. 92-106 (1967), doi: 10.1007/BF01291091.
- [4] S. Dey and S. K. Addy, "Reflection of plane waves under initial stresses at a free surface," *Int. J. Non-Linear Mech.*, vol. 12, no. 6, pp. 371-381 (1977).
- [5] R. W. Ogden and D. A. Sotiropoulos, "The effect of pre-stress on the propagation and reflection of plane waves in incompressible elastic solids," *IMA J. Appl. Math.*, vol. 59, no. 1, pp. 95-121 (1997), doi: 10.1093/imamat/59.1.95.

- [6] M. I. A. Othman and Y. Song, "Reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation," *Int. J. Solids Struct.*, vol. 44, no. 17, pp. 5651-5664 (2007), doi: 10.1016/j.ijsolstr.2007.01.022.
- [7] J. Sharma, S. Kumari, and S. Sharma, "Reflection of plane waves with two temperature and initial stress in nonlocal thermo elasticity without energy dissipation," *Grenze Int. J. Eng. Technol.*, vol. 10 (2024).
- [8] S. Kumari, M. Singh, and S. Sharma, "Influence of gravity and hydrostatic initial stress on wave propagation in context of three phase lag thermoelasticity," *Mech. Solids*, vol. 59, no. 4, pp. 1990-2009 (2024), doi: 10.1134/S0025654424602921.
- [9] E. C. Bingham, "Plasticity and elasticity," *J. Franklin Inst.*, vol. 197, no. 1, pp. 99-115 (1924).
- [10] A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity*. Cambridge, U.K.: Cambridge University Press (1927).
- [11] A. C. Eringen, "Microstretch elasticity," in *Microcontinuum Field Theories: I. Foundations and Solids*. New York, USA: Springer, pp. 249-267 (1999).
- [12] B. Singh and R. Kumar, "Wave propagation in a generalized thermo-microstretch elastic solid," *Int. J. Eng. Sci.*, vol. 36, no. 7-8, pp. 891-912 (1998), doi: 10.1016/S0020-7225(97)00099-2.
- [13] Y. Q. Song, M. I. A. Othman, and Z. Zhao, "Reflection of plane waves from a thermo-microstretch elastic solid with temperature dependent elastic properties," *Multidiscip. Model. Mater. Struct.*, vol. 10, no. 2, pp. 228-249 (2014), doi: 10.1108/MMMS-07-2013-0052.
- [14] M. I. A. Othman, E. E. M. Eraki, and M. F. Ismail, "A comparative study of temperature-dependent characteristics and non-local behavior in a submerged microstretch thermoelastic medium using two models," *Sci. Rep.*, vol. 14, p. 27096 (2024), doi: 10.1038/s41598-024-77233-9.
- [15] M. Kaur, R. Kumar, S. Sharma, H. H. Taha, A. Kamel, A. A. El-Bary, and K. Lotfy, "Two temperatures effect on wave propagation in microstretch thermoelastic medium with microtemperatures," *Sci. Rep.*, vol. 15, p. 43816 (2025), doi: 10.1038/s41598-025-31454-8.
- [16] S. Kumar, H. Nagar, and S. Kumari, "Surface waves in hygro-thermoelastic half-space with hydrostatic initial stress, rotation

- & magnetic field," *J. Interdiscip. Math.*, vol. 29, no. 6, pp. 2049-2065 (2026), doi: 10.47974/JIM-2583.
- [17] S. Kumari, Bharti, and B. Singh, "Effects of two-temperature on Rayleigh wave in generalized magneto-thermoelastic media with hydrostatic initial stress," *J. Heat Transfer*, vol. 141, no. 7, p. 072002 (2019), doi: 10.1115/1.4043677.
- [18] P. Ailawalia, Priyanka, M. Marin, and H. Altenbach, "Variable thermal conductivity in context of Green-Naghdi theory of thermo-microstretch solids," *ZAMM - J. Appl. Math. Mech.*, vol. 104, no. 4, p. e202400010 (2024), doi: 10.1002/zamm.202400010.
- [19] V. A. Eremeyev, L. P. Lebedev, and H. Altenbach, *Foundations of Micropolar Mechanics*. Berlin, Germany: Springer (2012), doi: 10.1007/978-3-642-28353-6.
- [20] H. von Helmholtz, "Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen," *J. Reine Angew. Math.*, vol. 55, pp. 25-55 (1858), doi: 10.1515/crll.1858.55.25.
- [21] B. Singh, A. K. Yadav, and D. Gupta, "Reflection of plane waves from a micropolar thermoelastic solid half-space with impedance boundary conditions," *J. Ocean Eng. Sci.*, vol. 4, no. 2, pp. 122-131 (2019), doi: 10.1016/j.joes.2019.02.003.
- [22] S. Kumari, M. Singh, and S. Sharma, "Gravitational effect on piezothermoelasticity in the context of three phase lag model with two temperature," *Int. J. Appl. Comput. Math.*, vol. 9, no. 6, p. 139 (2023), doi: 10.1007/s40819-023-01617-0.
- [23] B. Singh, "Propagation characteristics of plane waves in nonlocal isotropic diffusive materials," *Appl. Math. Model.*, vol. 104, pp. 306-314 (2022), doi: 10.1016/j.apm.2021.11.028.
- [24] M. I. A. Othman, S. Y. Atwa, A. Jahangir, and A. Khan, "Gravitational effect on plane waves in generalized thermo-microstretch elastic solid under Green Naghdi theory," *Appl. Math. Inf. Sci. Lett.*, vol. 1, no. 2, pp. 25-38 (2013).

Received February, 2026