

Creep analysis of varying thickness on a rotating composite disk with different particle size by using Sherby's law

Nishi Gupta *

Harpreet Kaur †

Gargee Chakraborty §

Department of Mathematics

University Institute of Sciences (UIS)

Chandigarh University

Mohali 140413

Punjab

India

Harjot Kaur ‡

Department of Engineering

Gulzar Group of Institutes (GGI)

Libra

Khanna 141401

Punjab

India

Abstract

The objective of this study is to clarify how different thicknesses affected a revolving composite disk made of AlSiC_p with different particle sizes. Using mathematical modeling, the impact of different particle sizes and thicknesses on a revolving composite disk is calculated in terms of both tangential and radial stress. In comparison to various particle sizes with varied thickness long term deformation occurs. The results are displayed visually demonstrating how creep deformation decreases with changing particle size and thickness.

Subject Classification: 74S05, 74S20, 74F05.

Keywords: Creep, Varying thickness, Particle size, Stresses, Strain rates.

* E-mail: nishi.gupta.phd81@gmail.com (Corresponding Author)

† E-mail: harpreet2678@yahoo.com

§ E-mail: gargee.e17094@cumail.in

‡ E-mail: harjotmangat030@gmail.com

1. Introduction

Creep is a deformation process in which a solid material slowly deforms with time and permanently, sometimes it called cold flow because the material deforms slowly under the influence of stresses. The creep deformation increases with increases in temperature and contact to heat for a long time. Strong stresses are applied for the extended period, which produces deformation which is significantly less than the strength that material yields [1]. As shown in the research, increasing the temperature reduces the material's strength. The creep rate increases when a material is subject to heat for an extended time. The temperature when approaching the melting point, the creep rates climbed steadily. Creep rates are primarily increased by the migration of atoms at high temperatures. Under the influence of stresses, a solid material has a tendency to move slowly or deform permanently. It is developing while having long-term disclosure of stress levels below the material's yield strength. Creep occurs when a material is heated to a temperature close to the melting point over a long period of time. Different materials have different temperature ranges where creep deformation can occur [2-3]. Composite material rotating disks are used in a variety of mechanical and electronic equipment. Because rotating disks are so widely used in rotating machinery such as compressors, pumps, flywheels, gas turbine rotors, computer disk drives, and automotive baking systems, they provide a vast area of research [4-8]. In current study the stable state rotating disk made from AlSiC_p can be taken for different particle size at constant temperature with varying thickness. Consider case first taken particle size $P = 1.7\mu\text{m}$, second case $P = 14.6\mu\text{m}$ and for third case $P = 45.4\mu\text{m}$ with constant temperature 638K having thickness $h_{\text{inner}} = 2.52\text{mm}$ and $h_{\text{outer}} = 0.52\text{mm}$ with volume content 10% where $V_{\text{maximum}} = 22\%$ and $V_{\text{minimum}} = 12\%$ with angular speed $\omega = 15000$. Also, inner and outer radius is $c = 30\text{ mm}$ and $d = 160\text{ mm}$.

2. Mathematical Modeling

2.1 linearly varying thickness

Consider the composite disk thickness h , which varies linearly in the form [9]

$$h(r) = h_{\text{outer}} + 2\phi(c - d) \quad (1)$$

where h_{inner} is thickness at inner also h_{outer} is thickness at outer radii, respectively, and $\phi = \frac{(h_{\text{inner}} - h_{\text{outer}})}{2(c - d)}$ is the disk's slope.

The disk's moment of inertia at inner radius c is I , and at outer radius r and d , it is I_{outer} moment of inertia. Assume that area of the disk cross-section is A by inner radius c and A_0 by outer radius d respectively.

$$I = \int_c^r hr^2 dr, I_{\text{outer}} = \int_c^d hr^2 dr \quad (2)$$

$$\text{and} \quad A = \int_c^r h dr, A_{\text{outer}} = \int_c^d h dr \quad (3)$$

Average tangential stress is,

$$\sigma_{\theta\text{avg}} = \frac{1}{A_{\text{outer}}} \int_c^d h \sigma_{\theta} dr \quad (4)$$

2.2 Distribution of reinforcement in the disk

In the following, the volume percent of SiC shows a change in composition with radial distance, the $V(r)$,

$$V(r) = F - Nr, c < r < d \quad (5)$$

Where,

$$F = V_{maximum} + cN \quad (6)$$

$$N = \frac{V_{maximum} - V_{minimum}}{d - c} \quad (7)$$

where $V_{maximum}$ represents the maximum particle content and $V_{minimum}$ represents the minimum particle content at disk's inner & outer radii.

According to rule of mixture, density variation $\rho(r)$ in a composite disk can be represented as

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \frac{V(r)}{100} \quad (8)$$

Where density of the aluminum matrix is $\rho_m = "3210kg/m^3"$, while density of the dispersed silicon carbide particles is $\rho_d = "2698.4kg/m^3"$.

By substituting value of $V(r)$ from eqⁿ (5) into eqⁿ (8), we get,

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \left(\frac{F - Nr}{100} \right) = R_\rho - Q_\rho \quad (9)$$

Where,

$$R_\rho = \rho_m + \left(\frac{(\rho_d - \rho_m)F}{100} \right) \quad (10)$$

$$Q_\rho = \frac{N(\rho_d - \rho_m)}{100} \quad (11)$$

2.3 Creep Law

A composite with a stable state creep response has been described by the Sherby's law [10]

$$\dot{\epsilon} = [M(\bar{\sigma} - \sigma_0)]^n \quad (12)$$

With $n = 5$, $\bar{\sigma}$ represents effective stress, σ_0 threshold stress, and $\dot{\epsilon}$ represents effective strain rates.

The creep specification is known through:

$$M = \frac{1}{E} \left[\frac{AD_L \lambda^3}{|\vec{b}_r|^5} \right]^{1/n} \quad (13)$$

where D_L is lattice diffusivity, E Young's modulus, $|\vec{b}_r|$ is the magnitude of Burger's vector, A is constant, M is material creep constant. As a function of particle size, the following regression equation fixes M and σ_0 , the values of creep requirements that have been acquired from the creep findings report for $Al - SiC_p$ composite [11]. $P = 14.6\mu m$, $P = 45.4\mu m$, and $P = 1.7\mu m$ V is the volume, while T is the temperature [12].

$$\ln(M(r)) = (-34.91) + 0.2112\ln P + 4.89\ln T - 0.5891\ln V(r) \quad (14)$$

$$\sigma_0(r) = -0.02050P + 0.0378T + 1.033V(r) - 4.9695 \quad (15)$$

3. Mathematical Formulation

Analyze a disk of $Al - SiC_p$ with an external radius of d and an internal radius of c that rotates at an angular speed with different particle sizes.

For an isotropic revolving disk, the generalized constitutive equations are as follows:

$$\dot{\epsilon}_r(r) = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_r(r) - (\sigma_\theta(r) + \sigma_z(r))] \quad (16)$$

$$\dot{\epsilon}_\theta(r) = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_\theta(r) - (\sigma_z(r) + \sigma_r(r))] \quad (17)$$

$$\dot{\epsilon}_z(r) = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_z(r) - (\sigma_r(r) + \sigma_\theta(r))] \quad (18)$$

where the strain rates are $\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$, $\dot{\epsilon}_z$ and the subscripts show that the strain rates σ_r , σ_θ and σ_z correspond in the directions r, θ, z . Regarding the biaxial stress situation ($\sigma_z = 0$). For the biaxial condition of stress, effective stress $\bar{\sigma}$ is provided below, where it is assumed that the effective stress depends on von Mises' criterion [13-15].

$$\bar{\sigma} = \frac{1}{\sqrt{2}} [\sigma_r^2(r) + \sigma_\theta^2(r) + (\sigma_r(r) - \sigma_\theta(r))^2]^{\frac{1}{2}} \quad (19)$$

Equations (18) & (13) are used in equations (16) and (17) to obtain,

$$\dot{\epsilon}_r(r) = \frac{[M(r)(\bar{\sigma} - \sigma_0(r))]^n (2\gamma(r) - 1)}{2[\gamma^2(r) - \gamma(r) + 1]^{\frac{1}{2}}} \quad (20)$$

$$\dot{\epsilon}_\theta(r) = \frac{[M(r)(\bar{\sigma} - \sigma_0(r))]^n (2 - \gamma(r))}{2[\gamma^2(r) - \gamma(r) + 1]^{\frac{1}{2}}} \quad (21)$$

$$\dot{\epsilon}_z(r) = -(\dot{\epsilon}_r(r) + \dot{\epsilon}_\theta(r)) \quad (22)$$

Where, $\gamma(r) = \frac{\sigma_r(r)}{\sigma_\theta(r)} \quad (23)$

Radial stress & tangential stress ratios by any radius r

Solving eqⁿ (18) & eqⁿ (19) we develop,

$$\sigma_\theta(r) = \frac{u_a^{\frac{1}{n}}}{M(r)} \varphi_1(r) + \varphi_2(r) \quad (24)$$

Where,

$$\varphi_1(r) = \frac{\varphi(r)}{[\gamma^2(r) - \gamma(r) + 1]^{1/2}} \quad (25)$$

$$\varphi_2(r) = \frac{\sigma_0(r)}{[\gamma^2(r) - \gamma(r) + 1]^{1/2}} \quad (26)$$

$$\varphi(r) = \left[\frac{2(\gamma^2(r) - \gamma(r) + 1)^{1/2}}{r(2 - \gamma(r))} \exp \left(\int_c^r \frac{\varnothing(r)}{r} dr \right) \right]^{1/n} \quad (27)$$

Where,

$$\varnothing(r) = \frac{2\gamma(r) - 1}{2 - \gamma(r)} \quad (28)$$

The equation for forces equilibrium could be expressed as,

$$\frac{d}{dr} [r\sigma_r h(r)] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \quad (29)$$

Equation (29) can be integrated to obtain the limit between c and d with boundary conditions “ $\sigma_r = 0$ at $r = c$, $\sigma_r = 0$ at $r = d$.”

$$\int_c^d h(r) \sigma_\theta dr = \omega^2 \left[R_\rho I_{outer} - Q_\rho \left(\frac{(h_{outer} + 2\partial d)(d^4 - c^4)}{4} - \frac{2\partial(d^5 - c^5)}{5} \right) \right] \quad (30)$$

By finding limits between c and d and multiplying the equation (24) by $h(r)dr$, then integrating the resultant eqⁿ, we find

$$\dot{\epsilon}_a^{\frac{1}{n}} = \frac{1}{\int_c^d \frac{h(r) \varphi_1(r) dr}{M(r)}} \left[A_{outer} \sigma_{\theta avg} - \int_c^d h(r) \varphi_2(r) dr \right] \quad (31)$$

Equation (4) is obtained by dividing equation (30) by A_{outer} ,

$$\sigma_{\theta avg} = \frac{1}{A_{outer}} \int_c^d h \sigma_\theta dr = \frac{\omega^2}{A_{outer}} \left[R_\rho I_0 - Q_\rho \left(\frac{(h_{outer} + 2\partial d)(d^4 - c^4)}{4} - \frac{2\partial(d^5 - c^5)}{5} \right) \right] \quad (32)$$

By substituting value of $\dot{\epsilon}_a^{\frac{1}{n}}$ from equ (31) into equ (24) after knowing $\sigma_{\theta avg}$, tangential stresses (σ_θ) are calculated as follows:

$$\sigma_\theta(r) = \frac{\varphi_1(r) \left[A_{outer} \sigma_{\theta avg} - \int_c^d h(r) \varphi_2(r) dr \right]}{M(r) \int_c^d \frac{h(r) \varphi_1(r) dr}{M(r)}} + \varphi_2(r) \quad (33)$$

Radial stresses between the limits c and r, are integrated using equation (29) as follows:

Equation (29) for the integration of radial stresses between limits c and r is,

$$\sigma_r(r) = \frac{1}{r h(r)} \left[\int_c^r h(r) \sigma_\theta dr - \omega^2 R_\rho I - \omega^2 Q_\rho \left(\frac{(h_{outer} + 2\partial d)(r^4 - c^4)}{4} - \frac{2\partial(r^5 - c^5)}{5} \right) \right] \quad (34)$$

4. The graphical representation and discussion

Figures below illustrate effect of different sizes of particles on a revolving composite disk varying in thicknesses:

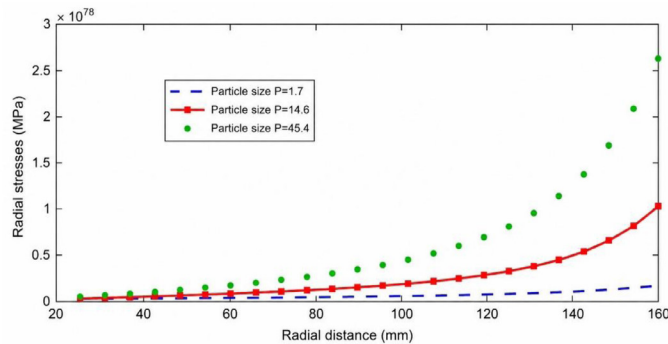


Figure 1
Variation of radial stresses in rotating composite at different particle with variable thickness at constant temperature.

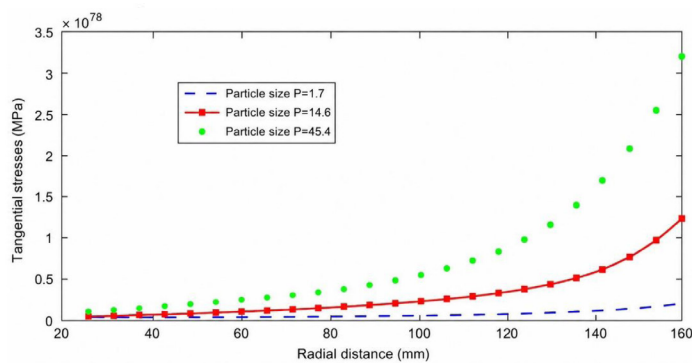


Figure 2
Variation of tangential stresses in rotating composite at different particle with variable thickness at constant temperature.

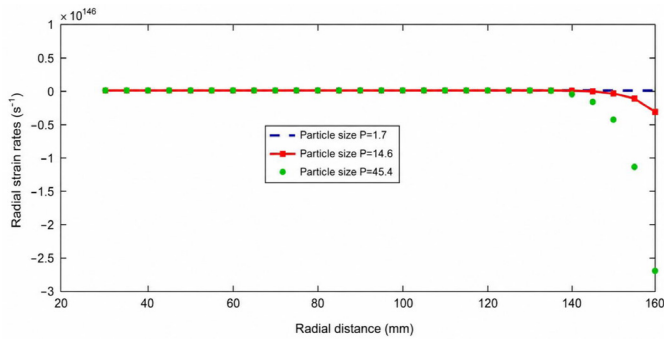


Figure 3
Variation of radial strain rates in rotating composite at different particle with variable thickness at constant temperature.

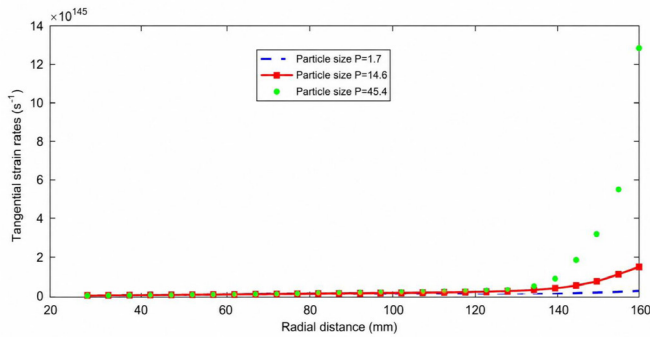


Figure 4
Variation of tangential strain rates in rotating composite at different particle with variable thickness at constant temperature.

The analysis has been carried out for effect different particle sizes on rotating composite disk with variable thickness and constant temperature as shown in Figures 1, 2, 3 and 4. In Figures 1, 2 increase particle size with increases radial distance the radial and tangential stresses start increases but particle size “ $P=1.7\mu\text{m}$ ”, “ $P = 14.6 \mu\text{m}$ ” gives almost similar results as compare to particle size “ $P = 45.4 \mu\text{m}$ ”. In Figure 3 increase particle size with increases radial distance the radial strain rates decreases as presented in Figure but tangential strain rates increase with particles size in Figure 4.

5. Conclusion

Particle size $P=1.7\mu\text{m}$, $P = 14.6 \mu\text{m}$ with varying thickness and constant temperature on rotating composite disk gives almost same results, hence use particle sizes $P=1.7\mu\text{m}$ and $P = 14.6 \mu\text{m}$ to decrease distortion in rotating composite disks. The proposed mathematical model can be extended to more complex rotating systems governed by nonlinear differential equations. Such extensions are supported by the mathematical framework [16]. Also the developed mathematical model provides a systematic approach for evaluating the creep behaviour of rotating composite disks and can be further integrated with optimization techniques for improved structural design [17].

References

- [1] E. P. Popov, *Engineering Mechanics of Solids*, 2nd ed. Singapore: Pearson Education (2001).
- [2] M. A. Meyers and K. K. Chawla, *Mechanical Behaviour of Materials*. Cambridge, U.K.: Cambridge Univ. Press (2008).
- [3] V. K. Gupta, H. N. Chandrawat, S. B. Singh, and S. Ray, “Creep behaviour of a rotating functionally graded composite disc operating under thermal gradient,” *Metall. Mater. Trans. A*, vol. 35, no. 4, pp. 1381–1391 (2004).
- [4] S. Chaurasiya and M. Suthar, “A comprehensive study on utilization of steel slag as road construction material,” *Lecture Notes Civ. Eng.*, vol. 527, pp. 1151–1166 (2025).
- [5] M. Kaptan, I. Ustabas, E. Cuce, P. M. Cuce, E. Alvur, A. Saxena, S. Alshahrani, and A. Bouabidi, “Structural and mechanical performance characteristics of construction plasters with different material compositions and advanced geopolymerization,” *Jordan J. Civ. Eng.*, vol. 19, no. 1, pp. 145–163 (2025).
- [6] A. K. Singh, M. H. Shakir, Siddhartha, S. Yadav, M. Yadav, N. M. Tripathi, and I. Fidan, “Investigations on the mechanical characterization of additively manufactured polymer composite materials fab-

- ricated via MEX," in *Advances in Pre- and Post-Additive Manufacturing Processes: Innovations and Applications*. Boca Raton, FL, USA: CRC Press, pp. 64–77 (2024).
- [7] V. Ahuja, S. Mehariya, R. Gurav, and S. K. Bhatia, "Exopolysaccharides based composite materials for industrial applications," in *Microbial Exopolysaccharides: Production and Applications*. Boca Raton, FL, USA: CRC Press, pp. 304–321 (2024).
 - [8] V. Kumar and R. Sharma, "Experimental, characterization and evaluation of various composite materials for automobile components," *Int. J. Mech. Prod. Eng. Res. Dev.*, vol. 8, no. 3, pp. 525–540 (2018).
 - [9] M. Garg, B. S. Salaria, and V. K. Gupta, "Effect of thermal gradient on steady state creep in a rotating disc of variable thickness," *Procedia Eng.*, vol. 55, pp. 542–547 (2013).
 - [10] O. D. Sherby, R. H. Klundt, and A. K. Miller, "Flow stress, subgrain size, and subgrain stability at elevated temperature," *Metall. Trans. A*, vol. 8, no. 6, pp. 843–850 (1977).
 - [11] S. B. Singh and S. Ray, "Newly proposed yield criterion for residual stress and steady state creep in an anisotropic composite rotating disc," *J. Mater. Process. Technol.*, vol. 143, pp. 623–628 (2003).
 - [12] S. B. Singh and S. Ray, "Modeling the anisotropy and creep in orthotropic aluminum–silicon carbide composite rotating disc," *Mech. Mater.*, vol. 34, no. 6, pp. 363–372 (2002).
 - [13] D. Deepak, V. K. Gupta, and A. K. Dham, "Creep modeling in functionally graded rotating disc of variable thickness," *J. Mech. Sci. Technol.*, vol. 24, no. 11, pp. 2221–2232 (2010).
 - [14] M. Garg, B. S. Salaria, and V. K. Gupta, "Modeling creep in a variable thickness rotating FGM disc under varying thermal gradient," *Eng. Comput.*, vol. 32, no. 5, pp. 1230–1250 (2015).
 - [15] V. Gupta and S. B. Singh, "Mathematical modeling of creep in a functionally graded rotating disc with varying thickness," *Regen. Eng. Transl. Med.*, vol. 2, no. 3, pp. 126–140 (2016).
 - [16] S. Sapna, A. Sharma, N. Mani, and R. Shukla, "Vibration control and optimization using circular non-homogeneity parameters," *Appl. Math.*, vol. 6, no. 6, p. 83 (2023).
 - [17] R. Durgadevi, S. Jayakumar, and R. Vijayaragavan, "Flowshop scheduling problem with objective of minimizing TCT," *J. Interdisciplinary Math.*, vol. 26, no. 1, pp. 1–10 (2023).