

Coincidence point results for three mappings via $(\theta, \alpha, \Upsilon)$ -contractions in S-metric spaces

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Abstract

In the present paper we study coincidence point results for three self-mappings in partially ordered S-metric spaces by using the concept of $(\theta, \alpha, \Upsilon)$ -type contractions. The result produced, within this more extended setting, unify and generalize several previous works in fixed point theory applying them to the framework of S-metric structures equipped with a partial order. A nontrivial example is provided to illustrate the applicability and resilience of the key theorems, which verifies the uniqueness and shows the theoretical significance of the results.

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1. Introduction

Over the last several decades, scientists and engineers have devoted considerable attention to developing analytical approaches for handling complex mathematical problems. In this direction, Fixed Point Theory has become an essential branch of nonlinear analysis because it offers effective techniques for studying the existence and stability behaviour of solutions in many mathematical models. Owing to its broad applicability, the subject has attracted researchers from different areas of mathematics, engineering, and applied sciences. In the last century, most of the work was motivated by the classical work of Geraghty [1], where researchers attempted to obtain more flexible contractive frameworks and generalized distance structures [2, 3, 4]. As a consequence, several generalized metric notions were introduced, including S -metric spaces by Sedghi, Shobe, and Aliouche [5] and in 2018 derived some common fixed-point theorems for four self-maps satisfying generalised contractive conditions in the same metric [6]. Later results were established by assuming conditions on maps such as admissibility, weak compatibility, continuity, and compatibility conditions in generalized metric spaces [7, 8]. In spite of these developments, research concerning more than two mapping pairs has remained limited. In 2019, Mlaiki and Taş [9] proved some fixed-point theorems using the set of simulation functions on an S -metric space. In 2024, Chauhan, Tyagi, and Mani [10] investigated rational contraction mappings in F -modular b -metric spaces and proved fixed-point theorems for such mappings, thereby enriching the theory of modular metric spaces and broadening the applicability of rational contractive conditions. Recently in [11] authors have proved common fixed-point theorems (for mappings not necessarily continuous) in Extended Parametric Sb -Metric Spaces. Some authors [12, 13, 14] further explored some fixed-point results in generalized metric type spaces and highlighted their theoretical and practical significance. Motivated by this gap, various researchers proposed generalized rational contraction techniques and derived several common fixed-point theorems together with applications to integral equations and related nonlinear problems.

Motivated by the preceding discussions, this paper aims to extend and refine the concept of coincidence point theorems for three self-mappings endowed with a partial order in S -metric spaces through the application of $(\theta, \alpha, \Upsilon)$ -type contractive conditions.

For the development of the main results, we first introduce several preliminary definitions and notations that will be used throughout this work. In particular, we denote $D = [0, 1)$, $E = [0, \infty)$.

Definition 1.1: [1] A function $\alpha \in \mathcal{F} = \{\alpha | \alpha: E \rightarrow D\}$ if it satisfies:

$$\text{whenever } \alpha(\delta_\beta) \rightarrow 1 \text{ then } \delta_\beta \rightarrow 0.$$

Definition 1.2: [15] For a set (partially ordered), two self-mappings T, R are referred as weakly increasing if

$$R\rho \preceq TR\rho \text{ as well as } T\rho \preceq RT\rho \quad \forall \rho \in \mathbb{X}.$$

Definition 1.3: For an S -metric space and given two self-mappings R, T on $\mathbb{X}$, the pair (R, T) is said to be compatible if

$$\lim_{\beta \rightarrow \infty} S(RT\rho_\beta, RT\rho_\beta, TR\rho_\beta) = 0,$$

whenever $\{\rho_\beta\}$ is a sequence in $\mathbb{X}$ such that

$$\lim_{\beta \rightarrow \infty} R\rho_\beta = \lim_{\beta \rightarrow \infty} T\rho_\beta = t \text{ for some } t \in \mathbb{X}.$$

Lemma 1.4: [5] An S -metric space is referred to be symmetric, if

$$S(\rho, \rho, \mu) = S(\mu, \mu, \rho) \quad \forall \rho, \mu \in \mathbb{X}.$$

2. Main Results

Let's begin with our main result.

Theorem 2.1: Let $V, R, T: \mathbb{X} \rightarrow \mathbb{X}$ are continuous selfmappings, defined on a complete- S -metric space $\mathbb{X}$, having elements are closed under partial order $\leq$ with the property that $T\mathbb{X} \subseteq V\mathbb{X}$, $R\mathbb{X} \subseteq V\mathbb{X}$, and are such that for all comparable elements $\rho, \mu, \tau \in \mathbb{X}$, it satisfies

$$\theta(S(R\rho, R\mu, T\tau)) \leq \alpha(S(V\rho, V\mu, V\tau)) \Upsilon(S(V\rho, V\mu, V\tau)), \quad (1)$$

where $\alpha \in \mathcal{F}$, $\theta \in \Theta$ and $\Upsilon: \mathbb{X} \rightarrow \mathbb{X}$ is a continuous function with condition

$$\theta(\eta) > \Upsilon(\eta) > 0 \quad \forall \eta > 0. \quad (2)$$

Assume further that pairs (T, V) and (R, V) are compatible with T and R are weakly increasing with respect to V . Moreover, if $V\rho$ and $V\mu$ are comparable, then V, R , and T have a coincidence point $v \in \mathbb{X}$. Additionally, for $(\rho, \mu) \in \mathbb{X} \times \mathbb{X}$, $\exists v_0 \in \mathbb{X}$ such that $R\rho \preceq Rv_0$ and $R\mu \preceq Rv_0$, then V, R , and T have a unique common fixed point.

Proof: Since $T\mathbb{X} \subseteq V\mathbb{X}$, $R\mathbb{X} \subseteq V\mathbb{X}$ and let $\rho_0 \in \mathbb{X}$ be arbitrary, therefore $\exists \rho_1, \rho_2 \in \mathbb{X}$ such that

$$R\rho_0 = V\rho_1 \text{ and } T\rho_1 = V\rho_2.$$

By repeating the above steps, we generate the sequences $\{\rho_\beta\}$ and $\{\mu_\beta\}$ in $\mathbb{X}$ defined by

$$V\rho_{2\beta+1} = R\rho_{2\beta} = \mu_{2\beta}, V\rho_{2\beta+2} = T\rho_{2\beta+1} = \mu_{2\beta+1}, \quad \forall \beta \in \mathbb{N}. \quad (3)$$

Using condition of weak compatibility, we have

$$V\rho_1 = R\rho_0 \preceq T\rho_1 = V\rho_2, V\rho_2 = R\rho_1 \preceq T\rho_2 = V\rho_3,$$

and continuing this way, we obtain

$$V\rho_1 \preceq V\rho_2 \preceq V\rho_3 \preceq \cdots \preceq V\rho_{2\beta+1} \preceq V\rho_{2\beta+2} \preceq \cdots,$$

which implies

$$\mu_0 \preceq \mu_1 \preceq \mu_2 \preceq \cdots.$$

Suppose that $\mu_{2\beta} \neq \mu_{2\beta+1}$ for every integer n . Let us assume $z_\beta = S(\mu_\beta, \mu_\beta, \mu_{\beta+1})$.

Substituting $\rho = \mu = \rho_{2\beta+2}$ and $\tau = \rho_{2\beta+1}$ in Eq.(1), we get

$$\begin{aligned}\theta(S(\mu_{2\beta+2}, \mu_{2\beta+2}, \mu_{2\beta+1})) &= \theta(S(V\rho_{2\beta+3}, V\rho_{2\beta+3}, V\rho_{2\beta+2})) \\ &= \theta(S(R\rho_{2\beta+2}, R\rho_{2\beta+2}, T\rho_{2\beta+1})) \\ &\leq \alpha(S(\mu_{2\beta+1}, \mu_{2\beta+1}, \mu_{2\beta})) \Upsilon(S(\mu_{2\beta+1}, \mu_{2\beta+1}, \mu_{2\beta})).\end{aligned}$$

Using Eq.(2) together with properties of θ (refer Definition from [14]) and $\alpha \in \mathcal{F}$, we get

$$S(\mu_{2\beta+2}, \mu_{2\beta+2}, \mu_{2\beta+1}) \leq S(\mu_{2\beta+1}, \mu_{2\beta+1}, \mu_{2\beta}). \quad (4)$$

Again substituting $\rho = \mu = \rho_{2\beta+1}$ and $\tau = \rho_{2\beta}$ in Eq.(1), we get

$$\theta(S(\mu_{2\beta+1}, \mu_{2\beta+1}, \mu_{2\beta})) \leq \alpha(S(\mu_{2\beta}, \mu_{2\beta}, \mu_{2\beta-1})) \Upsilon(S(\mu_{2\beta}, \mu_{2\beta}, \mu_{2\beta-1})). \quad (5)$$

By using Eq.(2) together with properties of θ and $\alpha \in \mathcal{F}$, we get

$$S(\mu_{2\beta+1}, \mu_{2\beta+1}, \mu_{2\beta}) \leq S(\mu_{2\beta}, \mu_{2\beta}, \mu_{2\beta-1}). \quad (6)$$

Combining Eq.(4) and Eq.(6), we get

$$S(\mu_{2\beta+2}, \mu_{2\beta+2}, \mu_{2\beta+1}) \leq S(\mu_{2\beta+1}, \mu_{2\beta+1}, \mu_{2\beta}) \leq S(\mu_{2\beta}, \mu_{2\beta}, \mu_{2\beta-1}).$$

Thus, for all $\beta \geq 2$, we can conclude that

$$S(\mu_{\beta+2}, \mu_{\beta+2}, \mu_{\beta+1}) \leq S(\mu_{\beta+1}, \mu_{\beta+1}, \mu_{\beta}) \leq S(\mu_{\beta}, \mu_{\beta}, \mu_{\beta-1}).$$

Thus $\{z_{\beta}\}$ is monotonically decreasing, so $\exists \eta \geq 0$ with $\lim_{\beta \rightarrow \infty} z_{\beta} = \eta$. If $\eta > 0$, taking limit $\beta \rightarrow \infty$ in Eq. (5) gives

$$\theta(\eta) \leq \alpha(\eta) \Upsilon(\eta).$$

By using properties of α and θ , we get $\theta(\eta) < \Upsilon(\eta)$, which contradicts Eq. (2). Hence $\eta = 0$, i.e.

$$\lim_{\beta \rightarrow \infty} z_{\beta} = \lim_{\beta \rightarrow \infty} S(\mu_{\beta}, \mu_{\beta}, \mu_{\beta+1}) = 0. \quad (7)$$

Claim: $\{V\rho_{\beta}\}$ is Cauchy Sequence. We prove it by considering $\{V\rho_{2\beta}\}$ is not Cauchy Sequence. Then $\exists \varepsilon > 0$ and subsequences $\{\beta_k\}$, $\{m_k\}$ with $\beta_k > m_k > k$, where $k \in \mathbb{N}$ such that

$$S(V\rho_{2m_k}, V\rho_{2m_k}, V\rho_{2\beta_k}) > \varepsilon, \quad S(V\rho_{2m_k}, V\rho_{2m_k}, V\rho_{2\beta_{k-1}}) \leq \varepsilon. \quad (8)$$

From Eq.(8) and using triangle inequality,

$$\begin{aligned}\varepsilon &< S(V\rho_{2m_k}, V\rho_{2m_k}, V\rho_{2\beta_k}) \leq 2S(V\rho_{2m_k}, V\rho_{2m_k}, V\rho_{2m_{k-1}}) \\ &\quad + S(V\rho_{2\beta_k}, V\rho_{2\beta_k}, V\rho_{2m_{k-1}}).\end{aligned}$$

Taking $k \rightarrow \infty$ and using Eq.(7) we obtain

$$\lim_{k \rightarrow \infty} S(V\rho_{2m_k}, V\rho_{2m_k}, V\rho_{2\beta_k}) = \varepsilon. \quad (9)$$

Since

$$\begin{aligned}S(V\rho_{2\beta_k}, V\rho_{2\beta_k}, V\rho_{2m_k}) &\leq 2S(V\rho_{2\beta_k}, V\rho_{2\beta_k}, V\rho_{2\beta_{k+1}}) \\ &\quad + S(V\rho_{2\beta_{k+1}}, V\rho_{2\beta_{k+1}}, V\rho_{2m_k}) \\ &= 2S(V\rho_{2\beta_k}, V\rho_{2\beta_k}, V\rho_{2\beta_{k+1}}) + S(R\rho_{2\beta_k}, R\rho_{2\beta_k}, T\rho_{2m_{k-1}}).\end{aligned}$$

By taking $k \rightarrow \infty$, using Eq.(9) along with the continuity of $\theta \in \Theta$, we obtain

$$\theta(\varepsilon) \leq \lim_{k \rightarrow \infty} \theta(S(R\rho_{2\beta_k}, R\rho_{2\beta_k}, T\rho_{2m_{k-1}})). \quad (10)$$

Now, from Eq.(1), we have

$$\theta(S(R\rho_{2\beta_k}, R\rho_{2\beta_k}, T\rho_{2m_{k-1}})) \leq \alpha(S(V\rho_{2\beta_k}, V\rho_{2\beta_k}, V\rho_{2m_{k-1}})) \\ Y(S(V\rho_{2\beta_k}, V\rho_{2\beta_k}, V\rho_{2m_{k-1}})).$$

Taking limits and using the Definition 1.1, we get

$$\lim_{k \rightarrow \infty} \theta(S(R\rho_{2\beta_k}, R\rho_{2\beta_k}, T\rho_{2m_{k-1}})) < Y(\varepsilon). \quad (11)$$

Combining Eq.(10) and Eq.(11), we obtain a contradiction. Indeed:

$$\theta(\varepsilon) \leq \lim_{k \rightarrow \infty} \theta(S(R\rho_{2\beta_k}, R\rho_{2\beta_k}, T\rho_{2m_{k-1}})) < Y(\varepsilon),$$

a contradiction to Eq.(2) and so the seq. $\{V\rho_\beta\}$ is a Cauchy sequence. Given that the S -metric is complete, it follows that there exists some $v \in \mathbb{X}$ with

$$\lim_{\beta \rightarrow \infty} V\rho_{2\beta+1} = \lim_{\beta \rightarrow \infty} R\rho_{2\beta} = \lim_{\beta \rightarrow \infty} \mu_{2\beta} = \lim_{\beta \rightarrow \infty} V\rho_{2\beta+2} \\ = \lim_{\beta \rightarrow \infty} T\rho_{2\beta+1} = \lim_{\beta \rightarrow \infty} \mu_{2\beta+1} = \lim_{\beta \rightarrow \infty} V\rho_\beta = v. \quad (12)$$

We now assert that v act as a coincidence-point for three maps V , T , and R . Since, V is continuous

$$\lim_{\beta \rightarrow \infty} V(V\rho_\beta) = Vv. \quad (13)$$

By triangle inequality using twice,

$$S(Vv, Vv, Rv) \leq 2S(Vv, Vv, V(\rho_{2\beta+1})) + 2S(V(R\rho_{2\beta}), V(R\rho_{2\beta}), R(V\rho_{2\beta})) \\ + S(R(V\rho_{2\beta}), R(V\rho_{2\beta}), Rv). \quad (14)$$

By compatibility of (R, V) for any sequence $\{\rho_\beta\}$ in $\mathbb{X}$, we know

$$\lim_{\beta \rightarrow \infty} R\rho_\beta = \lim_{\beta \rightarrow \infty} T\rho_\beta = t \text{ for some } t \in \mathbb{X},$$

and thus

$$\lim_{\beta \rightarrow \infty} S(V(R\rho_{2\beta}), V(R\rho_{2\beta}), R(V\rho_{2\beta})) \rightarrow 0. \quad (15)$$

Taking $\beta \rightarrow \infty$ in Eq.(14) and using Eq.(12)-Eq.(15) we get,

$$S(Vv, Vv, Rv) \leq 0,$$

hence, $Vv = Rv$. Similarly, by the same argument, $Vv = Tv$. Thus, $Vv = Rv = Tv$, so V, T, R has a coincidence point v .

Uniqueness: Suppose $\kappa, \pi \in \mathbb{X}$ are coincidence points of V, R, T , i.e. $V\kappa = R\kappa = T\kappa, V\pi = R\pi = T\pi$. We want to show that

$$V\kappa = V\pi. \quad (16)$$

From above assumed condition there exists $v_0 \in \mathbb{X}$ such that $R\kappa \preceq Rv_0, R\pi \preceq Rv_0$. Now, as in the proof mentioned above, we can define a sequence $\{Vv_\beta\}$ by

$$Vv_{2\beta+1} = Rv_{2\beta}, Vv_{2\beta+2} = Tv_{2\beta+1}, \forall \beta \in \mathbb{N}.$$

Again, we have

$$R\kappa = V\kappa \preceq Vv_\beta, R\pi = V\pi \preceq Vv_\beta, \forall \beta \in \mathbb{N}.$$

Substitute $\rho, \mu = v_{2\beta}$ and $\tau = \kappa$ in Eq.(1). Then

$$\begin{aligned} \theta(S(Vv_{2\beta+1}, Vv_{2\beta+1}, V\kappa)) &= \theta(S(Rv_{2\beta}, Rv_{2\beta}, T\kappa)) \\ &\leq \alpha(S(Vv_{2\beta}, Vv_{2\beta}, V\kappa))Y(S(Vv_{2\beta}, Vv_{2\beta}, V\kappa)). \end{aligned}$$

Since $\alpha \in \mathcal{F}$, we get

$$\theta(S(Vv_{2\beta+1}, Vv_{2\beta+1}, V\kappa)) \leq Y(S(Vv_{2\beta}, Vv_{2\beta}, V\kappa)). \quad (17)$$

Similarly, substituting $\rho, \mu = v_{2\beta+1}$ and $\tau = \kappa$ in Eq.(1), we obtain

$$\theta(S(Vv_{2\beta+2}, Vv_{2\beta+2}, V\kappa)) \leq Y(S(Vv_{2\beta+1}, Vv_{2\beta+1}, V\kappa)). \quad (18)$$

Combining Eq.(17) and Eq.(18), for all $\beta \in \mathbb{N}$, and using the continuity of θ and Y , we get

$$S(Vv_{\beta+1}, Vv_{\beta+1}, V\kappa) \leq S(Vv_\beta, Vv_\beta, V\kappa).$$

Thus, $\exists \eta \geq 0$ such that

$$\lim_{\beta \rightarrow \infty} S(Vv_\beta, Vv_\beta, V\kappa) = \eta. \quad (19)$$

Suppose $\eta > 0$. Then, from Eq.(1),

$$\theta(S(Vv_{2\beta+1}, Vv_{2\beta+1}, V\kappa)) \leq \alpha(S(Vv_{2\beta}, Vv_{2\beta}, V\kappa)) Y(S(Vv_{2\beta}, Vv_{2\beta}, V\kappa)).$$

As $\beta \rightarrow \infty$ and using Eq.(2), we reach at a contradiction. Therefore $\eta = 0$, hence, from Eq.(19)

$$\lim_{\beta \rightarrow \infty} S(Vv_\beta, Vv_\beta, V\kappa) = 0. \quad (20)$$

Similarly, we can show

$$\lim_{\beta \rightarrow \infty} S(Vv_\beta, Vv_\beta, V\pi) = 0. \quad (21)$$

Now, using uniqueness of the limit and Eq.(16)–Eq.(21), we can write

$$\lim_{\beta \rightarrow \infty} Rv_{2\beta} = V\kappa = V\pi, \lim_{\beta \rightarrow \infty} Tv_{2\beta+1} = V\kappa = V\pi.$$

Using compatibility of (R, V) we obtain

$$\lim_{\beta \rightarrow \infty} S(V(Rv_{2\beta}), V(Rv_{2\beta}), R(Vv_{2\beta})) = 0,$$

Let $\tau = V\kappa$. Consider

$$\begin{aligned} S(V\tau, V\tau, R\tau) &\leq 2S(V\tau, V\tau, V(Av_{2\beta})) + 2S(V(Rv_{2\beta}), V(Rv_{2\beta}), R(Vv_{2\beta})) \\ &\quad + S(R(Vv_{2\beta}), R(Vv_{2\beta}), R\tau). \end{aligned}$$

Taking $\beta \rightarrow \infty$ and using continuity of R , we get $V\tau = R\tau$. Similarly, on using Eq.(16), we have

$$\tau = V\kappa = V\tau = R\tau = T\tau.$$

Let more than one fixed point exists. Then $\exists$ second fixed point $\sigma \in \mathbb{X}$ such that $\sigma = V\kappa = V\sigma = R\sigma = T\sigma$. But from Eq.(16) we have $V\sigma = V\tau$. Hence $\sigma = V\sigma = V\tau = \tau$, which contradicts our assumption. Hence, uniqueness is established and the theorem is proved.

Example 2.2: Define:

1. A metric $S: D \times D \times D \rightarrow E$ by $S(\rho, \mu, \tau) = \max\{\rho, \mu, \tau\}, \forall \rho, \mu, \tau \in D$.
2. Maps $V, T, R: D \rightarrow D$, by

$$V(\rho) = \rho, T(\rho) = \frac{\rho}{3} \text{ and } R(\rho) = \frac{\rho}{4}.$$

Clearly, the space (D, S) constitutes a complete S -metric space, and the three maps V, T, R are continuous. Further, define the maps $\alpha: E \rightarrow D$ and $\theta, Y: E \rightarrow E, \forall \eta > 0$ as

$$\alpha(\eta) = \frac{\eta}{10}, \text{ and } \theta(\eta) = \eta > \frac{\eta}{2} = Y(\eta).$$

Thus all the conditions of Theorem 2.1 are fulfilled. Moreover, $V(0) = T(0) = R(0) = 0$, and is unique.

3. Conclusion

- i) This paper proves the coincidence point theorem (Theorem 2.1) for three self-mappings in the context of (θ, α, Y) -type contractions.
- ii) Theorem 2.1 both generalize and unify previously known fixed point theorems [1, 5, 8], extending their validity to the more comprehensive setting endowed with a partial order, in sense of S -metric spaces.
- iii) To verify and highlight the theoretical significance of the findings, an example (Example 2.2) is presented to show how the main results can be applied.

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