

A novel fractal method for visualizing positive data set

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Abstract

This article aims to introduce a novel fractal method for approximating and visualizing the positive sets of data with the help of trigonometric fractal interpolation function, named as Rational-Quadratic Trigonometric Fractal Interpolation Function (RQTFIFs). The theorems, which show the presence and uniqueness of the proposed method of RQTFIF, are also derived. The constraints derived on the vertical scaling factors (VSFs) as well as on the shape parameters for retaining the shape and positivity of the given data sets are also established. The proposed approach of RQTFIFs with both vertical scaling factors (VSF) and also the shape parameters, provide the flexibility and smoothness which makes it applicable to diverse domains of science and engineering.

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1. Introduction

In the traditional numerical theory, various interpolation techniques are present to demonstrate the smooth interpolants for a given set of data. However, in several real-world conditions we deal with the problems that are complex in nature. Barnsley [1] introduced these problems by demonstrating fractal interpolation functions (FIFs) using IFS theory given by the Hutchinson [2]. He defined this natural or experimental data set through the construction of FIFs as graphs of the attractors of certain IFSs. The interpolation technique using fractal concepts has been evolved as a powerful tool for approximating highly irregular data or functions which fail to be differentiable at the dense subset within the interval of interpolation. The core of this study lies in the recursive problem, which is capable in the production of the mathematical structures, which are complicated. The concept of FIF has been studied in multiple forms like multivariable, recurrent variables, hidden variables etc. In the last three decades, various properties of FIFs like Holder continuity, stability, convergence, smoothness, perturbation error and determination of scaling parameters for best fitting etc, have been demonstrated in the literature (for details, see [1, 3–7] and several references thereof).

In an efficient interpolation method, there is a concern for the qualitative properties of the given data set. A technique by which we explore sufficiently smooth function capable of preserving the properties responsible for the shape of the given set of data is stated as the shape preserving interpolation. The conditions of monotonicity, positivity and convexity represent the properties like shape parameters are mathematically presented in the given dataset. The interpolants with these properties have important role to play in the problems related to acoustics, approximation theory, cartography, computer graphics, computer aided geometric design, image analysis, medical sciences, signal processing, weather forecasting and many other areas [8, 9]. Thus, in such cases, some degree of smoothness and certain shape preserving characteristic like positivity, monotonicity etc, are required for visualizing the given data set. Schoenberg [10] introduced spline functions which have been proved to be of prime importance in smooth curve characterizations of the discrete sets of data.

Schweikert [11] developed interpolants with the help of exponential splines, to preserve the shape. Thereafter, many authors studied cubic spline interpolating functions along with the shape preserving properties using tension parameters [12]. Fritsch and Carlson [13] constructed a monotonic piecewise cubic polynomial by demonstrating an algorithm known as the two pass algorithm, for a given data set, which was also monotonic. Hussain, Hussain, and Waseem [14] studied shape preserving quadratic trigonometric functions consisting of two shape parameters. Hussain and Saleem [15] addressed C^1 rational quadratic trigonometric spline using quadratic rational functions. Bashir and Ali [16] used the trigonometric spline which gives the insights on the positive dataset.

Barnsley and Harrington [3] generalized the concept of classical spline interpolation to approximate the smooth functions in the form of spline fractal interpolation by introducing spline while constructing iterated function system

(IFS). Ilyass, Mani, and Shukla [17] further extended it by generalizing the boundary conditions. Prasad, Katiyar, and Singh [18] addressed quadratic trigonometric FIF and data approximation. Thus, the technique of smooth FIFs approximation and its several extensions like interpolation surfaces using fractal theory, further hidden variable FIFs, are explicitly addressed in the literature [5, 6, 17, 20]. Because of their versatility, simplicity, and flexibility, fractal splines can generalize most classical splines. In the case of fractal splines of C^n -continuity, the n th derivative of the interpolants may become non-differentiable, on the points of the denser subset having the interpolated generated splines, whereas under the classical interpolated generated splines, the splines are generally infinitely differentiable apart from the finite knots present. Motivating from the rational quadratic trigonometric spline interpolation methodology of [15] and the fractal interpolation technique of [1], we demonstrate a new type of FIFs called Rational Quadratic Trigonometric FIF (RQTFIF) with the usage of quadratic trigonometric rational functions. The conditions that are sufficient to maintain the parameters for the retaining of the positive results of RQTFIF for the given positive data set are obtained. Further, the existence and uniqueness theorems for RQTFIF are established and the positivity conditions are shown to be preserved when the parameters satisfy certain restrictions. Our approach to RQTFIF with trigonometric functions, vertical scaling factor (VSF) and shape parameters provides more flexibility and smoothness that can be applied to various areas of science and engineering, such as coding theory, image compression, signal processing, and financial series. The proposed RQTFIF may be useful in cases where the existing fractal interpolants are less satisfactory. The flow of this paper goes as follows; after introduction, the basic definition and concepts are followed by the construction of RQTFIF in subsection 2.1. Then, we develop the shape preserving positivity aspects to construct the positive RQTFIF and derive the concerned result followed by an illustrative example in subsection 2.2. At the end, in Section 3, we present the conclusion of this study.

2. Rational Quadratic Trigonometric FIF (RQTFIF)

The approach of trigonometric function is used to construct an interpolating function with fractal properties. Due to the presence of the trigonometric function, parameters of scaling and shape, the proposed RQTFIF method may be used to generalize both the fractal and classical interpolation methods.

Fractal Interpolation Function (FIF): Assume that $\{(\tilde{u}_k, \tilde{v}_k) \in I \times R : k = 1 \text{ to } N\}$ be the mentioned set of the data points with $\tilde{u}_1 < \tilde{u}_2 < \tilde{u}_3 < \dots < \tilde{u}_N$, where $I = [\tilde{u}_1, \tilde{u}_N]$. Assume $I_k = [\tilde{u}_k, \tilde{u}_{k+1}]$ and $l_k: I \rightarrow I_k, k = 1 \text{ to } N-1$ be the contraction mappings given by

$$l_k(\tilde{u}) = \frac{(\tilde{u}_{k+1} - \tilde{u}_k)\tilde{u} + \tilde{u}_N \tilde{u}_k - \tilde{u}_1 \tilde{u}_{k+1}}{\tilde{u}_N - \tilde{u}_1}$$

Take a function (continuous) $F_k: I \times R \rightarrow R, k = 1 \text{ to } N-1$, such that

$$F_k(\tilde{u}, \tilde{v}) = \alpha_k \tilde{v} + q_k(\tilde{u}) \quad (1)$$

satisfying

$$F_k(\tilde{u}_1, \tilde{v}_1) = v_k, F_k(\tilde{u}_N, \tilde{v}_N) = v_{k+1} \quad (2)$$

where $|\alpha_k| < 1$, $\forall k = 1$ to $N-1$ and $q_k(\tilde{u})$ denotes a continuous function, which is appropriate on I . Next, define $w_k: I \times R \rightarrow I_k \times R \quad \forall k = 1$ to $N-1$ & $(\tilde{u}, \tilde{v}) \in I \times R$, such that

$$w_k(\tilde{u}, \tilde{v}) = (l_k(\tilde{u}), F_k(\tilde{u}, \tilde{v})) \quad (3)$$

Then $\{I \times R, w_k(\tilde{u}, \tilde{v}), k = 1$ to $N-1\}$ is stated as an IFS for given interpolating set of data.

The result of Barnsley [Th. 1, 1] guarantees the existence of a unique attractor $G = \{(\tilde{u}, f(\tilde{u})): \tilde{u} \in I\}$ for the IFS, where $f: I \rightarrow R$ is a continuous function such that $f(\tilde{u}_k) = \tilde{v}_k \quad \forall k = 1$ to N .

The RQTFIF functions are obtained by substituting $q_k(\tilde{u})$ in (2) by a suitable trigonometric function, say $S_k(\tilde{u}) \quad \forall k = 1$ to $N-1$. In the present study, we take the below rational quadratic trigonometric function of Hussain and Saleem [15].

$$S_k(\tilde{u}) = \frac{(1 - \sin \Theta)^2 A_k + \sin \Theta (1 - \sin \Theta) B_k + \cos \Theta (1 - \cos \Theta) C_k + (1 - \cos \Theta)^2 D_k}{(1 - \sin \Theta)^2 + \sin \Theta (1 - \sin \Theta) \beta_k + \cos \Theta (1 - \cos \Theta) \gamma_k + (1 - \cos \Theta)^2}, \quad (7)$$

where $\Theta = \frac{\pi(\tilde{u} - \tilde{u}_1)}{2(\tilde{u}_N - \tilde{u}_1)}$, A_k, B_k, C_k, D_k are suitable real parameters and β_k, γ_k denotes the design parameters of the shape and also the tension parameters respectively, in each sub-interval $I_k, k = 1$ to $N-1$.

The below result satisfies the existence of RQTFIF.

Theorem 2.1: Suppose that $\{(\tilde{u}_k, \tilde{v}_k) \in I \times R: k = 1$ to $N\}$ is the given data set points with $\tilde{u}_1 < \tilde{u}_2 < \tilde{u}_3 < \dots < \tilde{u}_N$. Also, assume that $l_k(\tilde{u}) = \frac{(\tilde{u}_{k+1} - \tilde{u}_k)\tilde{u} + \tilde{u}_N \tilde{u}_k - \tilde{u}_1 \tilde{u}_{k+1}}{\tilde{u}_N - \tilde{u}_1}$, $a_k = l'_k = \frac{(\tilde{u}_{k+1} - \tilde{u}_k)}{(\tilde{u}_N - \tilde{u}_1)}$ and $F_k(\tilde{u}, \tilde{v}) = \alpha_k \tilde{v} + S_k(\tilde{u})$ for $k = 1$ to $N-1$. Assume for some integer $n \geq 0$, $|\alpha_k| < \alpha_k^n$, $S_k \in C^n[\tilde{u}_1, \tilde{u}_N], k = 1$ to $N-1$. Suppose $F_{k,j}(\tilde{u}, \tilde{v}) = \frac{\alpha_k \tilde{v} + S_k^{(j)}(\tilde{u})}{a_k^j}$, where $S_k^{(j)}(\tilde{u})$ is j^{th} derivative of $S_k(\tilde{u})$ corresponding to $\tilde{u}$ and $\tilde{v}_{1,j} = \frac{S_1^{(j)}(\tilde{u}_1)}{a_1^j - \alpha_1}$, $\tilde{v}_{N,j} = \frac{S_{N-1}^{(j)}(\tilde{u}_N)}{a_{N-1}^j - \alpha_{N-1}}, j = 1$ to n .

If $F_{k,j}(\tilde{u}_N, \tilde{v}_{N,j}) = F_{k+1,j}(\tilde{u}_1, \tilde{v}_{1,j}), \quad \forall k = 1$ to $N-1$ & $j = 1$ to n , then $\{(l_k(\tilde{u}), F_k(\tilde{u}, \tilde{v})), k = 1$ to $N-1\}$ establishes a trigonometric FIF $f \in C^n[\tilde{u}_1, \tilde{u}_N]$, and $f^{(j)}$ is the rational trigonometric FIF established by $\{(l_k(u), F_{k,j}(\tilde{u}, \tilde{v})), k = 1$ to $N-1\}, j = 1$ to n .

Proof: It follows immediately from Barnsley and Harington [3, Th. 2] by taking $q_k(\tilde{u}) = S_k(\tilde{u})$, a rational quadratic trigonometric function.

Theorem 2.2: Assume that $\{(\tilde{u}_k, \tilde{v}_k) \in I \times R: k = 1$ to $N\}$ be the given set of data satisfying $\tilde{u}_1 < \tilde{u}_2 < \tilde{u}_3 < \dots < \tilde{u}_N$ and d_k is the derived value at k^{th} knot for $k = 1$ to N . Take an IFS $\{C; w_k(\tilde{u}, \tilde{v}) = (l_k(\tilde{u}), F_k(\tilde{u}, \tilde{v})), k = 1$ to $N-1\}$ with

$l_k(\tilde{u}) = \frac{(\tilde{u}_{k+1} - \tilde{u}_k)\tilde{u} + \tilde{u}_N u_k - \tilde{u}_1 \tilde{u}_{k+1}}{\tilde{u}_N - \tilde{u}_1}$, and $F_k(\tilde{u}, \tilde{v}) = \alpha_k \tilde{v}_k + S_k(\tilde{u}_k)$, where α_k be a scaling factor satisfying $|\alpha_k| < 1$ and $S_k(\tilde{u})$ is given by (5) for $k = 1$ to $N-1$.

Let $F_{k,1}(\tilde{u}_k, \tilde{v}_k)$ be the derivative of $F_k(\tilde{u}_k, \tilde{v}_k)$ corresponding to $\tilde{u}_k$ such that

$F_k(\tilde{u}_1, \tilde{v}_1) = \tilde{v}_k, F_k(\tilde{u}_N, \tilde{v}_N) = \tilde{v}_{k+1}, F_{k,1}(\tilde{u}_1, \tilde{v}_1) = d_k, F_{k,1}(\tilde{u}_N, \tilde{v}_N) = d_{k+1}$ then, the attractor of the IFS is the graph of a C^1 -RQTFIF.

Proof: By using the similar arguments as given in Barnsley [1, Th. 1], it exhibits that the IFS has a unique attractor which is mentioned as the graph of the RQTFIF.

2.1. Construction of RQTFIF

In this Section, the RQTFIF are introduced to construct IFS for fractal interpolation. Motivated by the work of Hussain and Saleem [15], we define RQTFIF and show that the classical rational quadratic trigonometric interpolation defined in [15] is a special case of RQTFIF, when we take the value of parameter $\alpha = 0$.

Following Theorem 2.2 and equation (5), we obtain the below functional relation

$$g(l_k(\tilde{u})) = \alpha_k g(\tilde{u}) + \frac{A_k(1-\sin \xi)^2 + B_k \sin \xi(1-\sin \xi) + C_k \cos \xi(1-\cos \xi) + D_k(1-\cos \xi)^2}{(1-\sin \xi)^2 + \beta_k(1-\sin \xi) \sin \xi + \gamma_k(1-\cos \xi) \cos \xi + (1-\cos \xi)^2}, \quad (5)$$

$$\text{where } \xi = \frac{\pi(\tilde{u} - \tilde{u}_1)}{2(\tilde{u}_N - \tilde{u}_1)}.$$

Using the C^1 -Hermite interpolation conditions, i.e.

$g(\tilde{u}_k) = \tilde{v}_k, g(\tilde{u}_{k+1}) = \tilde{v}_{k+1}, g^{(1)}(\tilde{u}_k) = d_k, g^{(1)}(\tilde{u}_{k+1}) = d_{k+1}$, in equation (6), we get

$$A_k = \tilde{v}_k - \alpha_k \tilde{v}_1, B_k = \beta_k(\tilde{v}_k - \alpha_k \tilde{v}_1) + \frac{2(a_k d_k - \alpha_k d_1)(\tilde{u}_N - \tilde{u}_1)}{\pi},$$

$$C_k = \gamma_k(\tilde{v}_{k+1} - \alpha_k \tilde{v}_N) + \frac{2(\alpha_k d_N - a_k d_{k+1})(\tilde{u}_N - \tilde{u}_1)}{\pi}, D_k = \tilde{v}_{k+1} - \alpha_k \tilde{v}_N. \quad (6)$$

Now, it is simple to check that C^1 -RQTFIF $g \in C^1[\tilde{u}_1, \tilde{u}_N]$ is the fixed point of a C^1 -trigonometric IFS

$$\{I \times R: w_k(\tilde{u}, \tilde{v}) = (l_k(\tilde{u}), F_k(\tilde{u}, \tilde{v})), \forall k = 1 \text{ to } N-1\} \quad (7)$$

with

$$l_k(\tilde{u}) = \frac{(\tilde{u}_{k+1} - \tilde{u}_k)\tilde{u} + \tilde{u}_N u_k - \tilde{u}_1 \tilde{u}_{k+1}}{\tilde{u}_N - \tilde{u}_1} \quad (8)$$

and

$$F_k(\tilde{u}, \tilde{v}) = \alpha_k \tilde{v} + \frac{A_k(1-\sin \xi)^2 + B_k \sin \xi(1-\sin \xi) + C_k \cos \xi(1-\cos \xi) + D_k(1-\cos \xi)^2}{(1-\sin \xi)^2 + \beta_k(1-\sin \xi) \sin \xi + \gamma_k(1-\cos \xi) \cos \xi + (1-\cos \xi)^2}, \quad (9)$$

where A_k, B_k, C_k, D_k are given by equation (7).

Obviously, the fixed point 'g' of IFS (8) is notably unique for fixed values of the scaling parameters. This is the required RQTFIF.

Remark 2.1: If the value of $\alpha_k = 0$ for $k = 1$ to $N-1$, the RQTFIF reduces to the classical rational quadratic trigonometric interpolation of Hussain and Saleem [15].

2.2. Positivity preserving RQTFIF

In this section, we find the constrained fractal interpolation preserving positivity condition. Further, we demonstrate it with the help of an example which ensures the conditions of positivity of a specified positive data by the prescribed RQTFIF.

Theorem 2.3: Suppose that $\{(\tilde{u}_k, \tilde{v}_k, d_k), k = 1 \text{ to } N\}$ be a given positive data set, where d_k is the derived value at the k^{th} knot. Then, the attractor of rational trigonometric IFS is positive if parameter β_k , γ_k and the scaling factor α_k satisfy the conditions given below.

$$\alpha_k \in \left(0, \min \left\{ \frac{\tilde{v}_k}{\tilde{v}_1}, \frac{\tilde{v}_{k+1}}{\tilde{v}_N}, a_k \right\} \right), \quad \beta_k > -\frac{2(a_k d_k - \alpha_k d_1)(\tilde{u}_N - \tilde{u}_1)}{\pi(\tilde{v}_k - \alpha_k \tilde{v}_1)} \text{ and}$$

$$\gamma_k > -\frac{2(\alpha_k d_N - a_k d_{k+1})(\tilde{u}_N - \tilde{u}_1)}{\pi(\tilde{v}_{k+1} - \alpha_k \tilde{v}_N)}, \text{ for } k = 1 \text{ to } N-1.$$

Proof: If $\alpha_k \geq 0$, then from equation (7), it is easy to show that the sufficient condition for $g(l_k(\tilde{u})) > 0$ for all $\tilde{u} \in I$ is $S_k(\tilde{v}) > 0$ for $k = 1$ to $N-1$.

Since $(1 - \sin \xi)^2, (1 - \sin \xi) \sin \xi, (1 - \cos \xi) \cos \xi$ and $(1 - \cos \xi)^2$ are all positive for $\xi \in \left[0, \frac{\pi}{2}\right]$. Therefore, $S_k(\tilde{u}) > 0$ holds if $A_k > 0, B_k > 0, C_k > 0, D_k > 0$ or we have

$$\tilde{v}_k - \alpha_k \tilde{v}_1 > 0, \quad \beta_k(\tilde{v}_k - \alpha_k \tilde{v}_1) + \frac{2(a_k d_k - \alpha_k d_1)(\tilde{u}_N - \tilde{u}_1)}{\pi} > 0$$

$$\gamma_k(\tilde{v}_{k+1} - \alpha_k \tilde{v}_N) + \frac{2(\alpha_k d_N - a_k d_{k+1})(\tilde{u}_N - \tilde{u}_1)}{\pi} > 0, \quad \tilde{v}_{k+1} - \alpha_k \tilde{v}_N > 0 \quad (10)$$

The first and last inequalities for $k = 1$ to $N-1$ of (11) hold, if we take

$$\alpha_k < \min \left\{ \frac{\tilde{v}_k}{\tilde{v}_1}, \frac{\tilde{v}_{k+1}}{\tilde{v}_N}, a_k \right\}.$$

Hence, we have
$$\alpha_k \in \left(0, \min \left\{ \frac{\tilde{v}_k}{\tilde{v}_1}, \frac{\tilde{v}_{k+1}}{\tilde{v}_N}, a_k \right\} \right). \quad (11)$$

Now, we have to find $\beta_k > 0$ and $\delta_k > 0$ satisfying second and third inequalities of (11). For this, we assume $d_k \geq 0$ for all $k = 1$ to $N-1$. Thus, second and third inequalities of (11) hold if we consider

$$\beta_k > -\frac{2(a_k d_k - \alpha_k d_1)(\tilde{u}_N - \tilde{u}_1)}{\pi(\tilde{v}_k - \alpha_k \tilde{v}_1)} \text{ and } \gamma_k > -\frac{2(\alpha_k d_N - a_k d_{k+1})(\tilde{u}_N - \tilde{u}_1)}{\pi(\tilde{v}_{k+1} - \alpha_k \tilde{v}_N)}. \quad (12)$$

In the following example, we have verified that the RQTFIF is more flexible to preserve positivity than the classical rational quadratic trigonometric interpolation.

Example 1: Let $\{(1000, 0.31), (1250, 0.4), (1500, 0.5), (1920, 1.86), (1950, 2.5576), (1980, 4.4509), (2001, 6.167), (2015, 7.2532), (2035, 8.6187), (2050, 9.3764)\}$ be the given interpolation data set. Using arithmetic mean method of [19], the derivatives at the knots are obtained as follows: $[d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}] = [0.0003, 0.0004, 0.0015, 0.0219, 0.0432, 0.0741, 0.0792, 0.0738, 0.0581, 0.0429]$.

On applying (1) and (2), the maps $l_k(\tilde{u})$ for $k = 1$ to 9 are calculated as
 $l_1(\tilde{u}) = 0.2381\tilde{u} + 761.9$, $l_2(\tilde{u}) = 0.2381\tilde{u} + 1011.9$, $l_3(\tilde{u}) = 0.4\tilde{u} + 1100$,
 $l_4(\tilde{u}) = 0.0286\tilde{u} + 1891.4$, $l_5(\tilde{u}) = 0.0286\tilde{u} + 1921.4$, $l_6(\tilde{u}) = 0.02\tilde{u} + 1960$,
 $l_7(\tilde{u}) = 0.0133\tilde{u} + 1987.7$, $l_8(\tilde{u}) = 0.019\tilde{u} + 1996$, $l_9(\tilde{u}) = 0.0143\tilde{u} + 2020.7$.

Using (10), the parameters for maps $F_k(\tilde{u}, \tilde{v})$ for different choices of α_k and $\beta_k = \gamma_k$, $k = 1$ to 9 are organized in Table 1.

Table 1
Parameters for maps $F_k(\tilde{u}, \tilde{v})$ used in Example 1.

Figure No.	Parameters									
(a)	α_k	0.04	0.05	0.19	0.02	0.02	0.01	0.01	0.01	0.1
	$\beta_k = \gamma_k$	100	100	100	100	100	100	100	100	100
(b)	α_k	0.01	0.02	0.19	0.02	0.02	0.01	0.01	0.01	0.1
	$\beta_k = \gamma_k$	100	100	100	100	100	100	100	100	100
(c)	α_k	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.1
	$\beta_k = \gamma_k$	100	100	100	100	100	100	100	100	100
(d)	α_k	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.1
	$\beta_k = \gamma_k$	1	1	1	1	1	1	1	1	1

The scaling factors for the given data set are governed by the equation (12) and are as follows $\alpha_1 \in [0, 0.0426]$, $\alpha_2 \in [0, 0.0533]$, $\alpha_3 \in [0, 0.1983]$, $\alpha_4 \in [0, 0.0285]$, $\alpha_5 \in [0, 0.0285]$, $\alpha_6 \in [0, 0.02]$, $\alpha_7 \in [0, 0.0133]$, $\alpha_8 \in [0, 0.0190]$, $\alpha_9 \in [0, 0.0142]$. Also, the shape parameters β_k & γ_k are constrained by equation (13) for $k = 1$ to 9. The RQFIFs for these choices of parameters are generated in Figures 1(I)-(IV). It is observed that the positivity is retained on the given data set, if we choose the value of the parameters satisfying the conditions as given in Theorem 2.3 (see Figures 1(I)-(IV)), whereas the graph of the RQTFIF fails to maintain the positivity, in the case when the value of the parameters is chosen otherwise (see Figure 1(IV)).

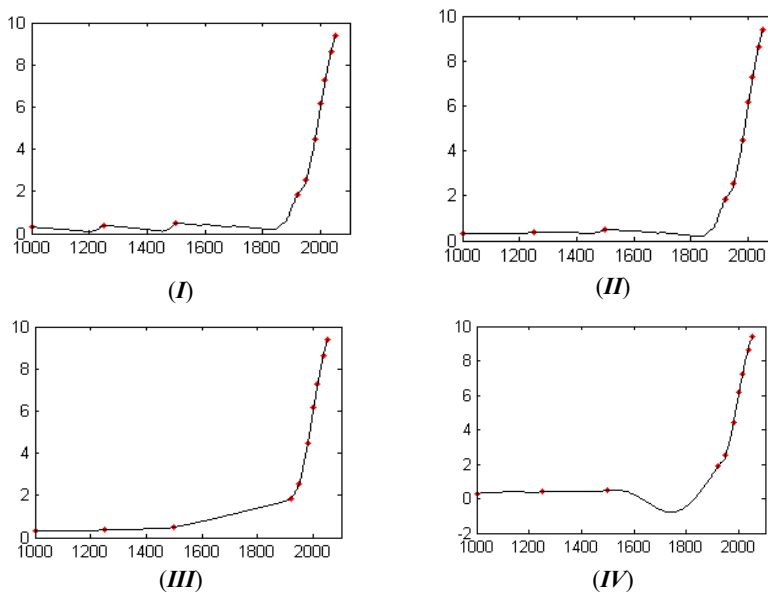


Figure 1
Graphs on RQTFIF

3. Conclusion

An alternative approach generalizing the classical shape preserving interpolation method for visualizing the positive dataset is presented. The required conditions to be satisfied by the vertical scaling factors and the shape parameters to retain the dataset, have been obtained in this study. The local control of vertical scaling factors is illustrated and it is noticed that for some specific selection of vertical scaling factors, the RQTFIF preserves the positivity while classical rational quadratic interpolation function does not.

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