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Generalized (Λ, *e*)-closed sets in topological spaces

Wadei Al-Omeri
Department of Mathematics
Faculty of Science
Jadara University
Irbid 21111
Jordan

Abstract

This study introduced and studied a new class of sets, called generalized (Λ, *e*)-closed sets. Moreover, some properties of generalized (Λ, *e*)-closed sets are investigated. Several preservation characterizations concerning of e-regular spaces and e-normal spaces have been obtained and discussed.

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1. Historical background

The notion of generalized closed sets in topological spaces was introduced by Levine [1] in 1970 in the paper with title "Semi-open sets and semi-continuity in topological spaces". Moreover, Levine defined a separation axiom called T_1 between T_0 and T_1. Several authors was extended this principle with many applications, many low separation axioms are introduced, also in 2021, Al-Omeri, Kaviyarasu, and Rajeshwari [9] introduced and defined strong and weak sets in topological spaces. Since then, many variations of generalized closed sets are introduced and investigated. As a modification of generalized closed sets.

Özkoç et al.[5] gave the definition of Λ_e-Sets and $\vee_e$-sets which is the intersection of e-open sets and centered on the basic properties of Λ_e -open sets. (Λ, e)-closure and (Λ, e)-open sets were defined with the use of e-open set and e-closure exponent defined by Ekici [4]. In this article, we define generalized (Λ, e)-closed sets and explore some of their basic properties. Finally, some characterizations of Λ_e-regular spaces and Λ_e-normal spaces are provided."

ORCID-ID: `0000-0002-6872-1838`

E-mail: `wadeimoon1@hotmail.com`

For a subset B of $\mathcal{K}$, the closure, interior as well as complement of B in $(\mathcal{K})$ are denoted by $Cl(B)$, $Int(B)$ and $\mathcal{K}-B$, respectively. I denote the collection of all e-open sets and the collection of all e-closed sets of $(\mathcal{K})$ by $EO(\mathcal{K})$ and $EC(\mathcal{K})$. The collection of all open neighborhoods surrounding the point k of $\mathcal{K}$ is denoted by $\mathcal{U}(k)$ A subset B of a space $\mathcal{K}$ is said to be δ-open [3] if $B = Int(B)$, where $Int_\delta(B) = \{k|(\exists U \in \mathcal{U}(k))(Int(Cl(U)) \subseteq B)\}$. The complement of a δ-open set is referred to as δ-closed [3, 8]. Correspondingly, a subset B of a space $\mathcal{K}$ is said to be δ-closed [3, 7] if $B = Cl(B)$, where $Int_\delta(B) = \{k|(\forall U \in \mathcal{U}(k))(Int(Cl(U)) \cap B \neq \phi)\}$. A subset B of a space $\mathcal{K}$ is said to be e-open [4] (resp. pre-open [2]) if $B \subseteq Cl(Int(B)) \cap Int(Cl(B))$ (resp. $B \subseteq Int(Cl(B))$). The complement of an e-open (resp. pre-open) set is called e-closed (resp. pre-closed). E. Ekici studied several fundamental and interesting properties of e-open sets see also [6, 7].

The e-closure of a set B, represented as $eCl(B)$, is defined as the intersection of all e-closed sets that encompass B. Conversely, the e-interior of a set B, denoted as $eInt(B)$, is characterized as the union of all e-open sets that are contained within B. In this paper, the notation $(\mathcal{K}, \Gamma)$ or simply $\mathcal{K}$ will consistently refer to a topological space.

Definition 1.1: [4] For a topological space $(\mathcal{K}, \Gamma)$. The union of any collection of e-open sets is also e-open, while the intersection of an open set with an e-open set results in an e-open set.

Definition 1.2: [5] Let $(\mathcal{K}, \Gamma)$ be a topological space and $B \subseteq \mathcal{K}$. I denote the subsets B^{Λ_e} and B_{Λ_e} as follows: $B^{\Lambda_e} = \cap \{V|(B \subseteq V)(V \in EO(\mathcal{K}))\}$, and

$$B_{\Lambda_e} = \cup \{U|(U \subseteq B)(U \in EO(\mathcal{K}))\}.$$

Definition 1.3: [5] A subset A of a topological space $(\mathcal{K}, \Gamma)$ is called an e-set if $B = B^{\Lambda_e}$.

Lemma 1.4: [5] For a subsets B, A and $B_\lambda (\lambda \in \Delta)$ of a topological space $(\mathcal{K}, \Gamma)$, the following properties hold:

- $B \subset B^{\lambda_e}$ and $[B^{\lambda_e}]^{\lambda_e} = B^{\lambda_e}$.
- If $B \subseteq A$, then $B^{\lambda_e} \subseteq A^{\lambda_e}$.
- B is λ_e-closed set iff $B = B^{\lambda_e}$.
- $[\cap \{B_\lambda | \lambda \in \Delta\}]^{\lambda_e} = \cap \{B_\lambda^{\lambda_e} | \lambda \in \Delta\}$.
- $[\cup \{B_\lambda | \lambda \in \Delta\}]^{\lambda_e} \supseteq \cup \{B_\lambda^{\lambda_e} | \lambda \in \Delta\}$.

Remark 1.5: [5] If $B \in EO(\mathcal{K})$, then B is a Λ_e-set.

Lemma 1.6: [5] *For a topological space $B \in EO(\mathcal{K})$, the following properties satisfied:*

(1) *Every union of Λ_e-sets is a Λ_e-set.*

(2) *The subsets and* $\mathcal{K}$ *are* Λ_e*-sets.*

(3) *Every intersection of* Λ_e*-sets is a* Λ_e*-set.*

2. Generalized (Λ, e)-closed sets

In this section, I introduce and studies the notion of generalized (Λ, e)-closed sets and some properties concerning of generalized (Λ, e)-closed sets have been studded and obtained..

Definition 2.1: A subset B of a topological space $(\mathcal{K}, \Gamma)$ is considered to be (Λ, e)-closed if $B = T \cap C$, where T is a Λ_e-set and C is a e-closed set. The collection of all (Λ, e)-closed sets in a topological space $(\mathcal{K}, \Gamma)$ is represented by $EO(\mathcal{K})$.

Every Λ_e-set (resp. e-closed set) is (Λ, e)-closed

Lemma 2.2:

Theorem 2.3: *If* $(\mathcal{K}, \Gamma)$ *is a topological space and* $B \subseteq \mathcal{K}$, *then the following properties are equivalent:*

(1) B is (Λ, e)-closed.

(2) $B = T \cap eCl(B)$, where T is an Λ_e-set.

(3) $B = B^{\lambda_e} \cap eCl(B)$.

Proof: (1) ⇒ (2) Let $B = T \cap C$, where T is an Λ_e-set and C is an e-closed set. Since $B \subseteq C$, we have $eCl(B) \subseteq C$ and $B = T \cap C \supseteq T \cap ECl(B)$õ$B$. This implies that $B = T \cap eCl(B)$.

(2) ⇒ (3) Let $B = T \cap ECl(B)$, where T is an Λ_e-set. Since $B \subseteq T$, we have $B^{\Lambda_e} \subseteq T^{\Lambda_e} = T$ and hence, $B \subseteq B^{\Lambda_e} \cap eCl(B) \subseteq T \cap eCl(B) = B$ This implies that, we obtain $B = B^{\Lambda_e} \cap eCl(B)$.

(3) ⇒ (1) Since B^{Λ_e} is an Λ_e-set, $eCl(B)$ is e-closed and $B = B^{\Lambda_e} \cap eCl(B)$. Then I get that B is (Λ, e)-closed.

Definition 2.4: A subset B of a topological space $(\mathcal{K}, \Gamma)$ is said to be (Λ, e)-open if the complement of B is (Λ, e)-closed. The collection of all (Λ, e)-open sets in a topological space $(\mathcal{K}, \Gamma)$ is defined by $EO(\mathcal{K}, \Gamma)$.

Theorem 2.5: *Let* $B_\lambda (\lambda \in \Delta)$ *be a subset of a topological space* $(\mathcal{K}, \Gamma)$. *Then the following properties hold:*

(1) *If* B_λ *is* (Λ, e)*-closed for each* $\lambda \in \Delta$, *hence* $\cap \{B_\lambda | \lambda \in \Delta\}$ *is* (Λ, e)*-closed.*

(2) *If* B_λ *is* (Λ, e)*-open for each* $\lambda \in \Delta$, *hence* $\cup \{B_\lambda | \lambda \in \Delta\}$ *is* (Λ, e)*-open.*

Proof:

(1) Let B_λ is (Λ, e)-closed for each $\lambda \in \Delta$. Therefore, for all λ there exist an Λ_e-set T and an e-closed set C_λ such that $A_\lambda = T_\lambda \cap C_\lambda$. I have

$\bigcap_{\lambda\in\Delta} B_\lambda = \bigcap_{\lambda\in\Delta} (T_\lambda \cap C_\lambda) = (\bigcap_{\lambda\in\Delta} T_\lambda) \cap (\bigcap_{\lambda\in\Delta} C_\lambda)$. By Lemma 1.6, $\bigcap_{\lambda\in\Delta} T_\lambda$ is an Λ_e-set and $\bigcap_{\lambda\in\Delta} C_\lambda$ is an e-closed. This shows that $\bigcap_{\lambda\in\Delta} B_\lambda$ is (Λ, e)-closed.

(2) Suppose that B_λ is (Λ, e)-closed for each $\lambda \in \Delta$. Then $\mathcal{K} - B_\lambda$ is (Λ, e) and $\mathcal{K} - \bigcup_{\lambda\in\Delta} B_\lambda = \bigcap_{\lambda\in\Delta} (\mathcal{K} - B_\lambda)$. Then by Theorem 2.5(1) $\mathcal{K} - \bigcup_{\lambda\in\Delta} B_\lambda$ is (Λ, e)-closed and therefore, $\bigcup_{\lambda\in\Delta} B_\lambda$ is (Λ, e)-open.

Definition 2.6: Let B represent a subset of a topological space denoted as $(\mathcal{K}, \Gamma)$. A point k belonging to $\mathcal{K}$ is defined as a (Λ, e)-cluster point of B if for every (Λ, e)-open set V of $\mathcal{K}$ that includes k, the intersection of B and V is empty. The collection of all (Λ, e)-cluster points is referred to as the (Λ, e)-closure of B, symbolized as BpÎŻ,eq.

Lemma 2.7: *For a topological space $(\mathcal{K}, \Gamma)$, let Let B and A be subsets of $\mathcal{K}$. For the (Λ, e)-closure, then the following properties hold:*

(1) $B \subseteq B^{(\Lambda,e)}$ *and* $[B^{(\Lambda,e)}]^{(\Lambda,e)} = B^{(\Lambda,e)}$.

(2) *If* $B \subseteq A$, *then* $B^{(\Lambda,e)} \subseteq A^{(\Lambda,e)}$.

(3) $B^{(\Lambda,e)} = \cap \{G | B \subseteq G\}$ *and* G *is* (Λ, e)*-closed* .

(4) $B^{(\Lambda,e)}$ *is* (Λ, e)*-closed.*

(5) B *is* (Λ, e)*-closed if and only if* $B = B^{(\Lambda,e)}$.

Lemma 2.8: *For a topological space $(\mathcal{K}, \Gamma)$, let B subsets of $\mathcal{K}$. Then the following properties hold:*

(1) *If* B *is* (Λ, e)*-closed, then* $B = B^{(\Lambda,e)} \cap B^{(\Lambda,e)}$.

(2) *If* B *is e-closed, then* B *is* (Λ, e)*-closed*

Proof: Let B be (Λ, e)-closed, then there exist a Λ_e-set T and a e-closed set C such that $B = T \cap C$. By $B \subseteq T$, we have $B \subseteq B^{\Lambda_e} T^{\Lambda_e} = T$, and therefore by $B \subseteq C$, $B \subseteq B^{(\Lambda,e)} \subseteq C^{(\Lambda,e)} = C$. This implies $B \subseteq B^{\Lambda_e} \cap B^{(\Lambda,e)} \subseteq T \cap C = B$. Then, $B \subseteq B^{\Lambda_e} \cap B^{(\Lambda,e)}$.

(2) It is enough observability that $B = \mathcal{K} \cap B$, where the whole set $\mathcal{K}$ is an Λ_e-set.

Let $\mathcal{N}$ be a subset of a topological space $(\mathcal{K}, \Gamma)$, then $\mathcal{N}$ is called (Λ, e)-neighbourhood of a point $k \in \mathcal{K}$ if $\exists$ an (Λ, e)-open set V such that $k \in V \subseteq \mathcal{N}$.

Lemma 2.9: *For a subset of a topological space $(\mathcal{K}, \Gamma)$ is (Λ, e)-open in $(\mathcal{K}, \Gamma)$ iff it is an (Λ, e)-neighbourhood of each of its points.*

Definition 2.10: For a subset B of a topological space $(\mathcal{K}, \Gamma)$. The union of all (Λ, e)-open sets contained in B is said to be (Λ, e)-interior of B and is denoted by $B_{(\Lambda,e)}$.

Lemma 2.11: *For a topological space* $(\mathcal{K},\Gamma)$, *let Let* B *and* A *be subsets of* $\mathcal{K}$. *For the* (Λ,e)*-interior, then the subsequent properties are valid:*

(1) $B \subseteq B_{(\Lambda,e)}$ *and* $[B_{(\Lambda,e)}]_{(\Lambda,e)} = B_{(\Lambda,e)}$.

(2) *If* $B \subseteq A$, *then* $B_{(\Lambda,e)} \subseteq A_{(\Lambda,e)}$.

(3) $B_{(\Lambda,e)} = \cup\{F|F \subseteq B\}$ *and* F *is* (Λ,e)*-open.*

(4) $B_{(\Lambda,e)}$ *is* (Λ,e)*-open.*

(5) B is (Λ,e)-closed if and only if $\mathrm{B} = \mathrm{B}_{(\Lambda,\mathrm{e})}$.

Definition 2.12: For a topological space $(\mathcal{K},\Gamma)$, let B and A be subsets of $\mathcal{K}$, then B is called a generalized (Λ,e)-closed (in short $g-(\Lambda,e)$-closed) set if $B^{(\Lambda,e)}$ whenever $B \subseteq V$ and $V \in \Lambda_e O(\mathcal{K})$.

Definition 2.13: For a topological space $(\mathcal{K},\Gamma)$ is called an e-symmetric if for $k,s \in \mathcal{K}$, $k \in \{s\}^{(\Lambda,e)}$ therefore $s \in \{k\}^{(\Lambda,e)}$.

Theorem 2.14: *A topological space* $(\mathcal{K},\Gamma)$ *is* e*-symmetric iff* r *is* $g-(\Lambda,e)$*-closed for every* $k \in \mathcal{K}$.

Proof: Assume that $k \in \{s\}^{(\Lambda,e)}$ but $s \notin \{k\}^{(\Lambda,e)}$. This mean that the complement of $\{k\}^{(\Lambda,e)}$ contains s. Then the set s is a subset of the complement of $\{k\}^{(\Lambda,e)}$. Therefore, $\{s\}^{(\Lambda,e)}$ represents a subset of the complement of $\{k\}^{(\Lambda,e)}$. Furthermore, the complement of the complement of $\{k\}^{(\Lambda,e)}$ contains k which is a contradiction.

Conversely, let $\{k\} \subseteq U \in \Lambda_e O(\mathcal{K})$ but $\{k\}^{(\Lambda,e)}$ is can not be a subset of U. This implies that $\{k\}^{(\Lambda,e)}$ and the complement of U are not disjoint. Let s belongs to their intersection. This implies k is a subset of the complement of U and $k \notin U$. But this is a contradiction.

Theorem 2.15: *For a topological space* $(\mathcal{K},\Gamma)$, *let* B *be* $g-(\Lambda,e)$*-closed iff* $B^{(\Lambda,e)} - B$ *contains no nonempty* (Λ,e)*-closed set.*

Proof: Let G be an (Λ,e)-closed subset of $B^{(\Lambda,e)} - B$. Now $B \subseteq \mathcal{K} - G$ and since B is $g-(\Lambda,e)$-closed, we have $B^{(\Lambda,e)} \subseteq \mathcal{K} - G$ or $G \subseteq \mathcal{K} - B^{(\Lambda,e)}$. Thus $G \subseteq B^{(\Lambda,e)}[\mathcal{K} - B^{(\Lambda,e)}] = \phi$ and G is empty.

Conversely, let $B \subseteq V$ and that V is (Λ,e)-open. If $B^{(\Lambda,e)} \subsetneq V$, then $B^{(\Lambda,e)} \cap (\mathcal{K} - V)$ is a nonempty (Λ,e)-closed subset of $B^{(\Lambda,e)} - B$.

Corollary 2.16: *Let* B *be a* $g-(\Lambda,e)$*-closed subset of a topological space* $(\mathcal{K},\Gamma)$. *Then* B *is* (Λ,e)*-closed if and only if* $B^{(\Lambda,e)} - B$ *is* (Λ,e)*-closed.*

Proof: If B is (Λ,e)-closed, then $B^{(\Lambda,e)} - B = \phi$

Conversely, suppose that $B^{(\Lambda,e)} - B$ is (Λ,e)-closed. But B is $g-(\Lambda,e)$-closed and $B^{(\Lambda,e)} - B$ is a (Λ,e)-closed subset of itself. By Theorem 2.15, $B^{(\Lambda,e)} - B = \phi$ so that $B^{(\Lambda,e)} = B$.

Proposition 2.17: For a subset B and A of a topological space $(\mathcal{K}, \Gamma)$. Consequently, the subsequent properties are valid:

(1) If B is (Λ, e)-closed, then B is $g-(\Lambda, e)$-closed.

(2) If B is $g-(\Lambda, e)$-closed and (Λ, e)-open, then B is (Λ, e)-closed.

(3) If B is $g-(\Lambda, e)$-closed and $B \subseteq A \subseteq B^{(\Lambda,e)}$, then A is $g-(\Lambda, e)$-closed.

Proof:

(1) Suppose that B is (Λ, e)-closed and $B \subseteq V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Then, by Lemma 2.7 $B = B^{(\Lambda,e)} \subseteq V$therfore and hence, B is $g-(\Lambda, e)$-closed.

(2) Let B be $g-(\Lambda, e)$-closed and (Λ, e)-open. Hence $B^{(\Lambda,e)} \subseteq B$ so that, B is (Λ, e)-closed.

(3) Let $A \subseteq V$ and $V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Hence $B \subseteq V$ and B is $g-(\Lambda, e)$-closed. Therefore, $B^{(\Lambda,e)} \subseteq V$. By Lemma 2.7, $B^{(\Lambda,e)} = A^{(\Lambda,e)}$ this implies, $A^{(\Lambda,e)} \subseteq V$. Then, A is $g-(\Lambda, e)$-closed.

Definition 2.18: Suppose that B is a subset of a topological space $(\mathcal{K}, \Gamma)$. Then the (Λ, e)-frontier of B, (i.e $eFr(B)$), is denoted by,

$$\Lambda_e Fr(B) = B^{(\Lambda,e)} \cap [\mathcal{K} - B]^{(\Lambda,e)}$$

Proposition 2.19: Suppose that B is a subset of a topological space $(\mathcal{K}, \Gamma)$. Then if B is $g-(\Lambda, e)$-closed and $B \subseteq U \in \Lambda_e O(\mathcal{K}, \Gamma)$, then $\Lambda_e Fr(B) \subseteq [\mathcal{K} - B]_{(\Lambda,e)}$.

Proof: Let B be $g-(\Lambda, e)$-closed and $B \subseteq U \in \Lambda_e O(\mathcal{K}, \Gamma)$. Then $B^{(\Lambda,e)} \subseteq U$. Suppose that $k \in \Lambda_e Fr(U)$. Hence $U \in \Lambda_e O(\mathcal{K}, \Gamma)$, $\Lambda_e Fr(U) = U^{(\Lambda,e)} - U$. This implies, $\notin U$ and $k \notin B^{(\Lambda,e)}$. Therefore $k \in [\mathcal{K} - B]_{(\Lambda,e)}$ and this show, $\Lambda_e Fr(B) \subseteq [\mathcal{K} - B]_{(\Lambda,e)}$.

Theorem 2.20: *A subset B of a topological space $(\mathcal{K}, \Gamma)$ is $g-(\Lambda, e)$-open iff $G \subseteq B_{(\Lambda,e)}$, for every $G \subseteq B$ and G is (Λ, e)-closed.*

Proof: Suppose that B is $g-(\Lambda, e)$-open. Let $G \subseteq B$ and G is (Λ, e)-closed. Then $\mathcal{K} - B \subseteq \mathcal{K} - G \in \Lambda_e O(\mathcal{K}, \Gamma)$ and $\mathcal{K} - B$ is $g-(\Lambda, e)$-closed. Thus, we obtain $\mathcal{K} - B_{(\Lambda,e)} = [\mathcal{K} - B]^{(\Lambda,e)} \subseteq \mathcal{K} - G$ and therefore $G \subseteq B_{(\Lambda,e)}$.

Conversely, suppose $\mathcal{K} - B \subseteq V$ and $V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Then $\mathcal{K} - V \subseteq B_{(\Lambda,e)}$ and $\mathcal{K} - V$ is (Λ, e)-closed. By using the hypothesis, I have $\mathcal{K} - V \subseteq B$ and thus, $[\mathcal{K} - B]^{(\Lambda,e)} = \mathcal{K} - B_{(\Lambda,e)} \subseteq V$. Hence, $\mathcal{K} - B$ is $g-(\Lambda, e)$-closed and B is $g-(\Lambda, e)$-open.

Proposition 2.21: Let $(\mathcal{K}, \Gamma)$ be a topological space. For every $k \in \mathcal{K}$, either $\{k\}$ is (Λ, e)-closed or k is $g-(\Lambda, e)$-open.

Proof: Suppose that $\{k\}$ is not (Λ, e)-closed. Then is not and the only (Λ, e)-open set containing $\mathcal{K} - k$ is $\mathcal{K}$ itself. This implies, $[\mathcal{K} - \{k\}]^{(\Lambda,e)} \subseteq \mathcal{K}$ and therefore, $\mathcal{K} - k$ is $g - (\Lambda, e)$-closed. Hence, $\{k\}$ is $g - (\Lambda, e)$-open.

Lemma 2.22: *Suppose B be a subset of a topological space (Λ, e). If $\cap F = \phi$, thus $B^{(\Lambda,e)} \cap F = \phi$ for each $F \in \Lambda_e O(\mathcal{K}, \Gamma)$.*

Corollary 2.23: *Let B, A be subsets of a topological space $(\mathcal{K}, \Gamma)$, the subsequent properties are equivalent:*

(1) *If B is (Λ, e)-open, then B is $g - (\Lambda, e)$-open.*

(2) *If B is $g - (\Lambda, e)$-open and (Λ, e)-closed, then B is (Λ, e)-open.*

(3) *If B is $g - (\Lambda, e)$-open and $B_{(\Lambda,e)} \subseteq A \subseteq A$, then A is $g - (\Lambda, e)$-open.*

Proof: The proof follows from Proposition 2.17.

Theorem 2.24: *Suppose B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then the subsequent properties are equivalent:*

(1) *B is $g - (\Lambda, e)$-closed.*

(2) *$B^{(\Lambda,e)} - B$ not contains a nonempty (Λ, e)-closed set.*

(3) *$B_{(\Lambda,e)} - B$ is $g - (\Lambda, e)$-open*

Proof: (1) $\Rightarrow$ (2): This follows from Theorem 2.15.

(2) $\Rightarrow$ (3): Let $G \subseteq B^{(\Lambda,e)} - B$ and G be (Λ, e)-closed. By (2), we have $G = \phi$ and $G \subseteq [B^{(\Lambda,e)} - B]_{(\Lambda,e)}$. Then by Theorem 2.20 that $B_{(\Lambda,e)} - B$ is $g - (\Lambda, e)$-open.

(3) $\Rightarrow$ (1): For a subsets $B \subseteq V$, and $V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Thus, $B^{(\Lambda,e)} - V \subseteq B^{(\Lambda,e)} - B$. Then by using (3), $B^{(\Lambda,e)} - B$ is $g - (\Lambda, e)$-open. Hence $B^{(\Lambda,e)} - V$ is (Λ, e)-closed and by Theorem 2.20, this implies $B^{(\Lambda,e)} - V \subseteq [B^{(\Lambda,e)} - B]_{(\Lambda,e)} = \phi$. Hence, I have $B^{(\Lambda,e)} \subseteq V$ and therefore, B is $g - (\Lambda, e)$-closed. For the proof of this part $[B^{(\Lambda,e)} - B]_{(\Lambda,e)} = \phi$ is follows as following. Let $[B^{(\Lambda,e)} - B]_{(\Lambda,e)} = \phi$. There exists $k \in [B^{(\Lambda,e)} - B]_{(\Lambda,e)}$. Thus, there exists an $F \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $k \in G \subseteq B^{(\Lambda,e)} - B$. Since $F \subseteq \mathcal{K} - B$, I obtain $F \cap B = \phi$ and $F \in \Lambda_e O(\mathcal{K}, \Gamma)$. By Lemma 2.22, $F \cap B^{(\Lambda,e)} = \phi$ and therefore, $G \subseteq \mathcal{K} - B^{(\Lambda,e)}$. Hence, this implies $F \subseteq [\mathcal{K} - B^{(\Lambda,e)}]^{(\Lambda,e)} = \phi$. Which this is a contradiction.

Theorem 2.25: *Let B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then B is $g - (\Lambda, e)$-closed iff $G \cap B^{(\Lambda,e)} = \phi$ whenever $B \cap G = \phi$ and G is (Λ, e)-closed.*

Proof: Let B is (Λ, e)-closed. Suppose that$B \cap G = \phi$ and G be (Λ, e)-closed. Hence $B \subseteq \mathcal{K} - G \in \Lambda_e O(\mathcal{K}, \Gamma)$ and $B^{(\Lambda,e)} \subseteq \mathcal{K} - G$. Then, we got $G \cap B^{(\Lambda,e)} = \phi$.

Conversely, suppose that $B \subseteq V$ and $V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Hence $B \cap (\mathcal{K} - V) = \phi$ and $\mathcal{K} - V$ is (Λ, e)-closed. Then by the hypothesis, $(\mathcal{K} - V) \cap B^{(\Lambda,e)} = \phi$ and therefore $B^{(\Lambda,e)} \subseteq V$. Hence, B is (Λ, e)-closed.

Theorem 2.26: *Let B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then B is $g - (\Lambda, e)$-closed iff $B \cap \{k\}^{(\Lambda,e)} \neq \phi$ for each $k \in B^{(\Lambda,e)}$.*

Proof. Let B is $g - (\Lambda, e)$-closed. Suppose that $B \cap \{k\}^{(\Lambda,e)} = \phi$ for each $k \in B^{(\Lambda,e)}$. By Lemma 2.7, $\{k\}^{(\Lambda,e)}$ is (Λ, e)-closed, then $B \subseteq \mathcal{K} - \{k\}^{(\Lambda,e)} \in \Lambda_e O(\mathcal{K}, \Gamma)$. Hence B is $g - (\Lambda, e)$-closed, $B^{(\Lambda,e)} \subseteq \mathcal{K} - \{k\}^{(\Lambda,e)} \subseteq \mathcal{K} - \{k\}$. This is contradicts $k \in B^{(\Lambda,e)}$.

Conversely, let a subset B is not $g - (\Lambda, e)$-closed. Hence $\phi \neq B^{(\Lambda,e)} - V$ for some $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ containing B. Then there exists $k \in B^{(\Lambda,e)} - V$. Then $k \notin V$, by Lemma 2.8 $V \cap \{k\}^{(\Lambda,e)} = \phi$, therefore $B \cap \{k\}^{(\Lambda,e)} \subseteq V \cap \{k\}^{(\Lambda,e)} = \phi$ This implies that $B \cap \{k\}^{(\Lambda,e)} = \phi$ for some $k \in B^{(\Lambda,e)}$.

Corollary 2.27: *Let B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then the following properties are hold:*

(1) *B is $g - (\Lambda, e)$-open.*

(2) *$B - B_{(\Lambda,e)}$ contains no nonempty (Λ, e)-closed set.*

(3) *$B - B_{(\Lambda,e)}$ is $g - (\Lambda, e)$-open.*

(4) *$(\mathcal{K} - B) \cap \{k\}^{(\Lambda,e)} = \phi$ for each $k \in B - B_{(\Lambda,e)}$.*

Proof: The proof is follows from Theorems 2.20, 2.24 and 2.26

Theorem 2.28: *Let $(\mathcal{K}, \Gamma)$ be a topological space, then the following properties are equivalent:*

(1) *Every (Λ, e)-open set V, $V^{(\Lambda,e)} \subseteq V$.*

(2) *For each subset of $\mathcal{K}$ is $g - (\Lambda, e)$-closed.*

Proof: (1) $\Rightarrow$ (2): For any subset B of $\mathcal{K}$ and $B \subseteq V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Then by (1), $V^{(\Lambda,e)} \subseteq V$ also by $B^{(\Lambda,e)} \subseteq V^{(\Lambda,e)} \subseteq V$. Then, B is $g - (\Lambda, e)$-closed.

(2) $\Rightarrow$ (1) Let$B \subseteq V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Then by (2), V is $g - (\Lambda, e)$-closed this implies $V^{(\Lambda,e)} \subseteq V$.

Theorem 2.29: *Let $(\mathcal{K}, \Gamma)$ be a topological space and let B be a subset of $\mathcal{K}$, then B is $g - (\Lambda, e)$-open iff $V = \mathcal{K}$ whenever V is (Λ, e)-open and $(\mathcal{K} - B) \cup B_{(\Lambda,e)} \subseteq V$.*

Proof: Suppose that B is $g - (\Lambda, e)$-open and $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $(\mathcal{K} - B) \cup B_{(\Lambda,e)} \subseteq V$. Hence $\mathcal{K} - V \subseteq [\mathcal{K} - B]^{(\Lambda,e)} - (\mathcal{K} - B)$. Hence $\mathcal{K} - B$ is $g - (\Lambda, e)$-closed and $\mathcal{K} - V$ is (Λ, e)-closed, by Theorem 2.15 $\mathcal{K} - V = \phi$ and then, $\mathcal{K} = B$.

Conversely, suppose that $G \subseteq B$ and G is (Λ, e)-closed. By Lemma 2.11, since $(\mathcal{K} - B) \cup B_{(\Lambda,e)} \subseteq (\mathcal{K} - G) \cup B_{(\Lambda,e)} \in \Lambda_e O(\mathcal{K}, \Gamma)$. Now by the hypothesis, $\mathcal{K} = (\mathcal{K} - G) \cup B_{(\Lambda,e)}$. Then, $G = G \cap [(\mathcal{K} - G) \cup B_{(\Lambda,e)}] = G \cap B_{(\Lambda,e)} \subseteq B_{(\Lambda,e)}$. This follows from Theorem 2.20 that B is $g - (\Lambda, e)$-open

Proposition 2.30; Let $(\mathcal{K}, \Gamma)$ be a topological space and let B, A be a subset of $\mathcal{K}$, then if B is $g - (\Lambda, e)$-open and $B_{(\Lambda,e)} \subseteq A \subseteq B$, then A is $g - (\Lambda, e)$-open.

Proof: Since $\mathcal{K} - B \subseteq \mathcal{K} - A \subseteq \mathcal{K} - B_{(\Lambda,e)} = [\mathcal{K} - B]^{(\Lambda,e)}$. I have $\mathcal{K} - B$ is $g - (\Lambda, e)$-closed, now from the Proposition 2.17(3) we have $\mathcal{K} - A$ is $g - (\Lambda, e)$-closed and then, A is $g - (\Lambda, e)$-open.

Definition 2.31: Let B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then B is said to be locally (Λ, e)-closed if $B = V \cap G$, where $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and G is (Λ, e)-closed.

Theorem 2.32: *Let $(\mathcal{K}, \Gamma)$ be a topological space and let B be a subset of $\mathcal{K}$, then the following properties are equivalent:*

(1) *B is locally (Λ, e)-closed;*

(2) *$B = V \cap B^{(\Lambda,e)}$ for some V;*

(3) *$B^{(\Lambda,e)} - B$ is (Λ, e)-closed;*

(4) $B \cup [\mathcal{K} - B^{(\Lambda,e)}] \in \Lambda_e O(\mathcal{K}, \Gamma)$;

(5) $B \subseteq [A \cup [\mathcal{K} - B^{(\Lambda,e)}]]^{(\Lambda,e)}$.

Proof: (1) ⇒ (2): Let $B = V \cap G$, where $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and G is (Λ, e)-closed. Since $B \subseteq G$, we have $B^{(\Lambda,e)} \subseteq G^{(\Lambda,e)} = G$. Since $B \subseteq V$, $B \subseteq V \cap B^{(\Lambda,e)} \subseteq V \cap G = B$. Hence, I have $B = V \cap B^{(\Lambda,e)}$ for some $V \in \Lambda_e O(\mathcal{K}, \Gamma)$.

(2) ⇒ (3): Suppose that $B = V \cap B^{(\Lambda,e)}$ for some $V \in \Lambda_e O(\mathcal{K}, \Gamma)$. Therefore, $B^{(\Lambda,e)} - B = (\mathcal{K} - [V \cap B^{(\Lambda,e)}]) \cap B^{(\Lambda,e)} = (\mathcal{K} - V) \cap B^{(\Lambda,e)}$. Since $(\mathcal{K} - V) \cap B^{(\Lambda,e)}$ is (Λ, e)-closed and hence, $B^{(\Lambda,e)} - B$ is (Λ, e)-closed.

(3) ⇒ (4): Suppose $\mathcal{K} - [B^{(\Lambda,e)} - B] = [\mathcal{K} - B^{(\Lambda,e)}] \cup B$ and therefore, by (3) we have $B \cup [\mathcal{K} - B^{(\Lambda,e)}] \in \Lambda_e O(\mathcal{K}, \Gamma)$.

(4) ⇒ (5): I have by (4) $B \subseteq B \cup [\mathcal{K} - B] = [B \cup [\mathcal{K} - B^{(\Lambda,e)}]_{(\Lambda,e)}$.
(5) ⇒ (1): Suppose that $V = [B \cup \left[\mathcal{K} - B^{(\Lambda,e)}\right]]_{(\Lambda,e)}$. Therefore $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and $B = B \cap V \subseteq V \cap B^{(\Lambda,e)} \subseteq [A \cup [\mathcal{K} - B^{(\Lambda,e)}]] \cap B^{(\Lambda,e)} = B \cap B^{(\Lambda,e)} = B$. Then, this implies $B = V \cap B^{(\Lambda,e)}$, where $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and $B^{(\Lambda,e)}$ is (Λ, e)-closed. Then B is locally (Λ, e)-closed.

Theorem 2.33: *A subset B of a topological space $(\mathcal{K}, \Gamma)$ is called (Λ, e)-closed iff B is locally (Λ, e)-closed and $g - (\Lambda, e)$-closed.*

Proof: Suppose B is (Λ, e)-closed. Then by Proposition 2.17(1), B is $g-(\Lambda, e)$-closed. Since $\mathcal{K} \in \Lambda_e O(\mathcal{K}, \Gamma)$ and $B = \mathcal{K} \cap B$, B is locally (Λ, e)-closed.

Conversely, let B is locally (Λ, e)-closed and $g-(\Lambda, e)$-closed. Since B is locally (Λ, e)-closed, by Theorem 2.32 this shows

$$B \subseteq [B \cup [\mathcal{K} - B^{(\Lambda,e)}]]_{(\Lambda,e)}$$

By Lemma 2.11, $[B \cup [\mathcal{K} - B^{(\Lambda,e)}]]_{(\Lambda,e)} \in \Lambda_e O(\mathcal{K}, \Gamma)$ and B is $g-(\Lambda, e)$-closed. Then we have

$$B^{(\Lambda,e)} \subseteq [B \cup [\mathcal{K} - B^{(\Lambda,e)}]]_{(\Lambda,e)} \subseteq B \cup [\mathcal{K} - B^{(\Lambda,e)}]$$

and hence, $B^{(\Lambda,e)} = B$ and by Lemma 2.7 B is (Λ, e)-closed.

Definition 2.34: A subset B of a topological space $(\mathcal{K}, \Gamma)$ is called a generalized $\Lambda_{(\Lambda,e)}$-set (in short $g-\Lambda_{(\Lambda,e)}$-set) if $\Lambda_{(\Lambda,e)}(B)$ $B \subseteq G$ and G is (Λ, e)-closed.

Definition 2.35: Suppose B be a subset of a topological space $(\mathcal{K}, \Gamma)$. The subset (Λ, e) is denoted as follows:

$$\Lambda_{(\Lambda,e)} = \cap \{V \in \Lambda_e O(\mathcal{K}, \Gamma) | B \subseteq V\}$$

Lemma 2.36: *For subsets B, A of a topological space $(\mathcal{K}, \Gamma)$, the following properties hold:*

(1) $B \subseteq \Lambda_{(\Lambda,e)}(B)$.

(2) *If $B \subseteq A$, then $\Lambda_{(\Lambda,e)}(B) \subseteq \Lambda_{(\Lambda,e)}(A)$.*

(3) $\Lambda_{(\Lambda,e)}[\Lambda_{(\Lambda,e)}(B)] = \Lambda_{(\Lambda,e)}(B)$.

(4) *If B is (Λ, e)-open, $\Lambda_{(\Lambda,e)}(B) = B$.*

Definition 2.37: A topological space $(\mathcal{K}, \Gamma)$ is said to be a $\Lambda_e - T_{\frac{1}{2}}$-space if every $g-\Lambda_{(\Lambda,e)}$-closed set of $(\mathcal{K}, \Gamma)$ is (Λ, e)-closed.

Definition 2.38: A subset B of a topological space $(\mathcal{K}, \Gamma)$ is called a $(\mathcal{K}, \Gamma)$-set if $B = \Lambda_{(\Lambda,e)}(B)$. The family of all $\Lambda_{(\Lambda,e)}$-sets of $(\mathcal{K}, \Gamma)$ is denoted by $\Lambda_{(\Lambda,e)}(\mathcal{K}, \Gamma)$ (or simply $\Lambda_{(\Lambda,e)}$).

Lemma 2.39: *Suppose $(\mathcal{K}, \Gamma)$ be a topological space. Consequently, the subsequent properties are equivalent:*

(1) *For every $k \in \mathcal{K}$, the singleton $\{k\}$ is (Λ, e)-closed or $\mathcal{K} - k$ is $g-\Lambda_{(\Lambda,e)}$- closed.*

(2) *For every $k \in \mathcal{K}$, the singleton $\{k\}$ is (Λ, e)-open or $\mathcal{K} - k$ is $g-\Lambda_{(\Lambda,e)}$-set.*

Proof:

(1) Suppose $k \in \mathcal{K}$ and the singleton $\{k\}$ be not (Λ, e)-closed. Hence $\mathcal{K} \in k$ is not (Λ, e)-open and $\mathcal{K}$ is the only (Λ, e)-open set containing $\mathcal{K} - k$ and then, $\mathcal{K} - k$ is $g - \Lambda_{(\Lambda,e)}$-closed.

(2) Now, let $k - \mathcal{K}$ and the singleton k is not (Λ, e)-open. Therefore $\mathcal{K} - k$ can not be (Λ, e)-closed and there is one set only (Λ, e)-closed set which contains $\mathcal{K} - k$ is $\mathcal{K}$ and then, $\mathcal{K} - k$ is an $g - \Lambda_{(\Lambda,e)}$-set.

Theorem 2.40: *For a topological space $(\mathcal{K}, \Gamma)$, the following properties are equivalent:*

(1) $(\mathcal{K}, \Gamma)$ *is an* $\Lambda_e - T_{\frac{1}{2}}$*-space.*

(2) *For every* $k \in \mathcal{K}$*, the singleton* $\{k\}$ *is* (Λ, e)*-open or* (Λ, e)*-closed.*

(3) *Every* $g - \Lambda_{(\Lambda,e)}$*-set is an* $\Lambda_{(\Lambda,e)}$*-set.*

Proof: (1) ⇒ (2): By using Lemma 2.39, for every $k \in \mathcal{K}$, the singleton $\{k\}$ is (Λ, e)-closed or $\mathcal{K} - k$ is $g - (\Lambda, e)$-closed. Hence $(\mathcal{K}, \Gamma)$ is a $\Lambda_e - T_{\frac{1}{2}}$-space, and $\mathcal{K} - k$ is (Λ, e)-closed and therefore, $\{k\}$ is (Λ, e)-open in the latter case. This implies, the singleton $\{k\}$ is (Λ, e)-open or (Λ, e)-closed.

(2) ⇒ (3): Let there exists a $g - \Lambda_{(\Lambda,e)}$-set B which is not a $\Lambda_{(\Lambda,e)}$-set. Then there exists $k \in \Lambda_{(\Lambda,e)}$ such that $k \notin B$. In case the singleton $\{k\}$ is (Λ, e)-open, $B \subseteq \mathcal{K} - k$ and $\mathcal{K} - k$ is (Λ, e)-closed. Since B is a $g - \Lambda_{(\Lambda,e)}$-set, $\Lambda_{(\Lambda,e)}(B) \subseteq \mathcal{K} - k$. Which mean this represents a contradiction. In case the singleton $\{k\}$ is (Λ, e)-closed, $B \subseteq \mathcal{K} - k$ and $\mathcal{K} - k$ is (Λ, e)-open. By Lemma 2.36, $\Lambda_{(\Lambda,e)}(B) \subseteq \Lambda_{(\Lambda,e)}(\mathcal{K} - k)$. This represents a contradiction. Hence, every $g - \Lambda_{(\Lambda,e)}$-set is a $\Lambda_{(\Lambda,e)}$-set.

(3) ⇒ (1): Suppose that $(\mathcal{K}, \Gamma)$ is not a $\Lambda_e - T_{\frac{1}{2}}$-space. Hence, there exists a $g - (\Lambda, e)$-closed set B which is not (Λ, e)-closed. Since B is not (Λ, e)-closed, there exists a point $k \in B^{(\Lambda,e)}$ such that $k \in B$. By Lemma 2.39, the singleton $\{k\}$ is (Λ, e)-open or $\mathcal{K} - k$ is a $\Lambda_{(\Lambda,e)}$-set. (I) In case $\{k\}$ is (Λ, e)-open, since $k \in B^{(\Lambda,e)}$, $\{k\} \cap B \neq \phi$ and $k \in B$. This represents a contradiction with (II) In case $\mathcal{K} - k$ is a $\Lambda_{(\Lambda,e)}$-set, if k is not (Λ, e)-closed, $\mathcal{K} - k$ is not (Λ, e)-open and $\Lambda_{(\Lambda,e)}(\mathcal{K} - k) = \mathcal{K}$. Then, $\mathcal{K} - k$ is not a $\Lambda_{(\Lambda,e)}$-set. This represents a contradiction with (III). If $\{k\}$ is (Λ, e)-closed, $B \subseteq \mathcal{K} - k \in \Lambda_e O(\mathcal{K}, \Gamma)$ and B is $g - (\Lambda, e)$-closed. Therefore we have $B^{(\Lambda,e)} \subseteq \mathcal{K} - k$.This is not true since $k \in B^{(\Lambda,e)}$. Then, $(\mathcal{K}, \Gamma)$ is a $\Lambda_e - T_{\frac{1}{2}}$-space.

3. Λ_e-normal spaces and Λ_e-regular spaces

Here, I define and present the concepts of Λ_e-normal spaces and Λ_e-regular spaces. Some characterizations of Λ_e-normal spaces and Λ_e-regular spaces are obtained.

Definition 3.1: If, for each pair of disjoint (Λ, e)-closed sets G and Q, there exist disjoint (Λ, e)-open sets V and U such that $G \subseteq V$ and $Q \subseteq U$, then a topological space $(\mathcal{K}, \Gamma)$ is Λ_e-normal.

Theorem 3.2: *For a subset B of a topological space $(\mathcal{K}, \Gamma)$, then the following properties are equivalent:*

(1) *$(\mathcal{K}, \Gamma)$ is Λ_e-normal.*

(2) *For every pair of (Λ, e)-open sets V and U whose union is $\mathcal{K}$, there exist (Λ, e)-closed sets G and Q such that $G \subseteq V$, $Q \subseteq U$ and $G \cup Q = \mathcal{K}$.*

(3) *For each (Λ, e)-closed set G and each (Λ, e)-open set F containing G, there exists a (Λ, e)-open set V such that $G \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F$.*

Proof: (1) ⇒ (2): Suppose that V and U be a pair of (Λ, e)-open sets in a Λ_e-normal space $(\mathcal{K}, \Gamma)$ such that $\mathcal{K} = V \cup U$. Therefore $\mathcal{K} - V$ and $\mathcal{K} - U$ are disjoint (Λ, e)-closed sets. Hence $(\mathcal{K}, \Gamma)$ is Λ_e-normal, there exist disjoint (Λ, e)-open sets F and N such that $\mathcal{K} - V \subseteq F$ and $\mathcal{K} - U \subseteq N$. Let $G = \mathcal{K} - F$ and $Q = \mathcal{K} - N$. Then G and Q are (Λ, e)-closed sets such that $G \subseteq V$, $Q \subseteq U$ and $G \cup Q = \mathcal{K}$.

(2) ⇒ (3): Suppose that G be a (Λ, e)-closed set and F a (Λ, e)-open set containing G. So that $\mathcal{K} - G$ and F are (Λ, e)-open sets whose union is $\mathcal{K}$. Hence by (2), there exist (Λ, e)-closed sets I and J such that $I \subseteq \mathcal{K} - G$, $J \subseteq F$ and $I \cup J = \mathcal{K}$. Therefore $G \subseteq \mathcal{K} - I$, $\mathcal{K} - F \subseteq \mathcal{K} - J$ and $(\mathcal{K}I) \cap (\mathcal{K} - J) = \phi$. Let $V = \mathcal{K} - I$ and $U = \mathcal{K} - J$. So that V and U are disjoint (Λ, e)-open sets such that $G \subseteq V\mathcal{K} - U \subseteq F$. As $\mathcal{K} - U$ is (Λ, e)-closed set, this implies $V^{(\Lambda,e)} \subseteq \mathcal{K} - U$ and $G \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F$.

(3) ⇒ (1): Let G and Q be two disjoint (Λ, e)-closed sets. Then $G \subseteq \mathcal{K} - Q$ nd $\mathcal{K} - Q$ is (Λ, e)-open and since there exists a (Λ, e)-open set V such that $G \subseteq V \subseteq V^{(\Lambda,e)}\mathcal{K} - Q$. Let $U = \mathcal{K} - V^{(\Lambda,e)}$. Then V and U are disjoint (Λ, e)-open sets such that $G \subseteq V$ and $Q \subseteq U$. This implies $(\mathcal{K}, \Gamma)$ is Λ_e-normal.

Definition 3.3: A topological space $(\mathcal{K}, \Gamma)$ is said to be Λ_e-regular if for every (Λ, e)-closed sets G not containing k, there exist disjoint (Λ, e)-open sets V and U such that inV and $G \subseteq U$.

Theorem 3.4: *Let B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then the subsequent properties are equivalent:*

(1) *$(\mathcal{K}, \Gamma)$ is Λ_e-regular.*

(2) *For every $k \in \mathcal{K}$ and every $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $k \in V$, there exists $U \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $k \in U \subseteq U^{(\Lambda,e)} \subseteq V$.*

(3) *For every (Λ, e)-closed set G, $\cap \{U^{(\Lambda,e)} | G \subseteq U \in \Lambda_e O(\mathcal{K}, \Gamma)\} = G$.*

(4) *For every $\mathcal{K}$ and every $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $B \cap V \neq \phi$ there exists $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $B \cap U = \phi$ and $U^{(\Lambda,e)} \subseteq V$.*

(5) *For every nonempty subset B of $\mathcal{K}$ and each (Λ, e)-closed set G of $\mathcal{K}$ such that $B \cap G = \phi$, there exist $U, W \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $B \cap U = \phi$, $G \subseteq W$ and $U \cap W = \phi$.*

(6) *For every (Λ, e)-closed set G and $k \in G$, there exist $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and a $g-(\Lambda, e)$-open set U such that $k \in V$, $G \subseteq U$ and $V \cap U = \phi$.*

(7) *For every subset $B \in \mathcal{K}$ and every (Λ, e)-closed set G such that $B \cap G = \phi$, there exist $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and a $g-(\Lambda, e)$-open set U such that $B \cap V = \phi$, $G \subseteq U$ and $V \cap U = \phi$.*

Proof: (1) ⇒ (2): Let $F \in \Lambda_e O(\mathcal{K}, \Gamma)$ and $k \in F$. Hence $k \notin \mathcal{K} - F$, and there exist disjoint $V, U \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $\mathcal{K} - F \subseteq V$ and $k \in U$. Therefore $U \subseteq \mathcal{K} - V$ and so $k \in U \subseteq U^{(\Lambda,e)} \subseteq \mathcal{K} - V \subseteq V$.

(2) ⇒ (3): For all $G \in \Lambda_e C(\mathcal{K}, \Gamma)$, then $G \subseteq \cap \{U^{(\Lambda,e)} | G \subseteq U \in \Lambda_e O(\mathcal{K}, \Gamma)\}$ always satisfies. On the other hand, suppose$G \in \Lambda_e C(\mathcal{K}, \Gamma)$ such that $k \in \mathcal{K} - G$. Hence by (2) there exist $U \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $kinV \subseteq V^{(\Lambda,e)} \subseteq \mathcal{K} - G$. Therefore $G \subseteq \mathcal{K} - V^{(\Lambda,e)} = U \in \Lambda_e O(\mathcal{K}, \Gamma)\}$ and $V \cap U = \phi$. Hence $k \in U^{(\Lambda,e)}$. This implies $G \supseteq G \subseteq \cap \{U^{(\Lambda,e)} | G \subseteq U \in \Lambda_e O(\mathcal{K}, \Gamma)\}$.

(3) ⇒ (4): Suppose B be a subset of $\mathcal{K}$ and $U \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $B \cap V \neq \phi$. For $k \in B \cap V$. Hence $k\mathcal{K} - V$. Then by (3), there exists $N \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $\mathcal{K} - V \subseteq N$ and $k \in N^{(\Lambda,e)}$. Now said $U = \mathcal{K} - N^{(\Lambda,e)}$ which is a (Λ, e)-open set containing k and then $B \cap U \neq \phi$. Now $U \subseteq \mathcal{K} - N$ and hence $V^{(\Lambda,e)} \subseteq \mathcal{K} - N \subseteq V$.

(4) ⇒ (5): Suppose B be a nonempty subset of $\mathcal{K}$ and G be a (Λ, e)-closed set such that $B \cap G = \phi$. Hence $\mathcal{K} - G \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $B \cap (\mathcal{K} - G) \neq \phi$ and so by (4), there exists $U \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $B \cap U \neq \phi$ and $U^{(\Lambda,e)} \subseteq \mathcal{K} - G$. For $N = \mathcal{K} - U^{(\Lambda,e)}$, therefore N is a (Λ, e)-open set such that $G \subseteq N$ and $N \cap U = \phi$.

(5) ⇒ (1): Suppose B be a (Λ, e)-closed set not containing k. Hence $G \cap \{k\} = \phi$. So that by (5), there exist $U, N \in \Lambda_e O(\mathcal{K}, \Gamma)$ such that $k \in U$ $G \subseteq N$ and $U \cap N = \phi$.

(1) ⇒ (6): Straightforward.

(6) ⇒ (7): Suppose B be a subset of $\mathcal{K}$ and G be a (Λ, e)-closed set such that $B \cap G = \phi$. Hence for $k \in B$ and $k \notin G$, and then by (6), there exist $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and a $g-(\Lambda, e)$-open set U such that $k \in V$, $G \subseteq U$ and $V \cap U = \phi$. So $B \cap V = \phi$, $G \subseteq U$ and $V \cap V = \phi$.

(7) ⇒ (1): Let G be a (Λ, e)-closed set such that $k \notin G$. Put $\{k\} \cap G = \phi$, now by(7) there exist $V \in \Lambda_e O(\mathcal{K}, \Gamma)$ and a $g-(\Lambda, e)$-open set N such that $k \in V$, $G \subseteq N$ and $V \cap N = \phi$. Hence N is $g-(\Lambda, e)$-open, by Theorem 2.20 we got that $G \subseteq N_{(\Lambda,e)} = U \in \Lambda_e O(\mathcal{K}, \Gamma)$ and therefore $V \cap U = \phi$.

Theorem 3.5: *Let B be a subset of a topological space $(\mathcal{K}, \Gamma)$, then the subsequent properties are equivalent:*

(1) *$(\mathcal{K}, \Gamma)$ is Λ_e-normal.*

(2) *For each pair of disjoint (Λ, e)-closed sets G and Q, there exist disjoint $g-(\Lambda, e)$-open sets V and U such that $G \subseteq V$ and $Q \subseteq U$.*

(3) *For every (Λ, e)-closed set G and each (Λ, e)-open set W containing G, there exists a $g-(\Lambda, e)$-open set V such that $G \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F$.*

(4) *For every (Λ, e)-closed set G and each $g-(\Lambda, e)$-open set W containing G, there exists a (Λ, e)-open set V such that $G \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F_{(\Lambda,e)}$.*

(5) *For every (Λ, e)-closed set G and each $g-(\Lambda, e)$-open set W containing G, there exists a $g-(\Lambda, e)$-open set V such that $G \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F_{(\Lambda,e)}$.*

(6) *For every $g-(\Lambda, e)$-closed set G and each (Λ, e)-open set W containing G, there exists a (Λ, e)-open set V such that $G^{(\Lambda,e)} \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F$.*

(7) *For every $g-(\Lambda, e)$-closed set G and each (Λ, e)-open set W containing G, there exists a $g-(\Lambda, e)$-open set V such that $G^{(\Lambda,e)} \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F$.*

Proof: (1) $\Rightarrow$ (2): The proof is obvious. (2) $\Rightarrow$ (3): Suppose that G be a (Λ, e)-closed set and W be a (Λ, e)-open set containing G. Therefore G and $\mathcal{K} - F$ are two disjoint (Λ, e)-closed sets. So that by (2) there exist disjoint $g-(\Lambda, e)$-open sets V and U such that $G \subseteq V$ and $\mathcal{K} - F \subseteq U$. Hence U is $g-(\Lambda, e)$-open and $\mathcal{K} - F$ is (Λ, e)-closed, by Theorem 2.20, $\mathcal{K} - F \subseteq U_{(\Lambda,e)}$. Since, $[\mathcal{K} - U]^{(\Lambda,e)} = U_{(\Lambda,e)} \subseteq F$. Then, $G - V \subseteq U^{(\Lambda,e)} \subseteq F$.

(3) $\Rightarrow$ (1): Suppose G and Q be two disjoint (Λ, e)-closed sets. Therefore G is a (Λ, e)-closed set and $\mathcal{K} - Q$ is a (Λ, e)-open set containing G. Since by (3), there exists a $g-(\Lambda, e)$-open set V such that $G \subseteq V \subseteq U^{(\Lambda,e)} \subseteq \mathcal{K} - Q$ Thus by Theorem 2.20, $G \subseteq V^{(\Lambda,e)}$, $Q \subseteq \mathcal{K} - V^{(\Lambda,e)}$, where $V_{(\Lambda,e)}$ and $\mathcal{K} - V^{(\Lambda,e)}$ are two disjoint (Λ, e)-open sets. This shows that (Λ, e) is e-normal.

(4) $\Rightarrow$ (5): Straightforward.

(6) $\Rightarrow$ (7): Straightforward.

(3) $\Rightarrow$ (5): Let G be a (Λ, e)-closed set and W be a $g-(\Lambda, e)$-open set containing G. Since W is $g-(\Lambda, e)$-open and G is (Λ, e)-closed, by Theorem 2.20 $G \subseteq F_{(\Lambda,e)}$. Thus by (3), there exists a $g-(\Lambda, e)$-open set V such that $G \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F^{(\Lambda,e)}$.

(6) $\Rightarrow$ (4): Suppose that G be a (Λ, e)-closed set and W be a $g-(\Lambda, e)$-open set containing G. Hence by Theorem 2.20, we got $G \subseteq F_{(\Lambda,e)}$. Since G is $g-(\Lambda, e)$-closed and $F_{(\Lambda,e)}$ is (Λ, e)-open, by (6) there exists a (Λ, e)-open set V such that $G = G^{(\Lambda,e)} \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F_{(\Lambda,e)}$.

(5) $\Rightarrow$ (6): Suppose that G be a $g-(\Lambda, e)$-closed set and W be a (Λ, e)-open set containing G. Therefore $G^{(\Lambda,e)}$ W. Hence W is $g-(\Lambda, e)$-open, there exists a $g-(\Lambda, e)$-open set V such that $G^{(\Lambda,e)} \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F$. Since V is $g-(\Lambda, e)$-open and $G^{(\Lambda,e)}$ is (Λ, e)-closed, by using Theorem 2.20 we got $G^{(\Lambda,e)} \subseteq V_{(\Lambda,e)}$. Now let $U = V_{(\Lambda,e)}$. This implies U is (Λ, e)-open and then, $G^{(\Lambda,e)} \subseteq V \subseteq V^{(\Lambda,e)} \subseteq F = [V_{(\Lambda,e)}]^{(\Lambda,e)} \subseteq V^{(\Lambda,e)} \subseteq F$.

4. Conclusion

Numerous writers are responsible for the ideas of e-open sets, e-continuity, e-compactness, and related research in topological spaces. The following advances in intuitionistic fuzzy topological spaces are examined in this paper: intuitionistic fuzzy e-continuity, intuitionistic fuzzy e-compactness, intuitionistic fuzzy e^*-open sets, intuitionistic fuzzy α-δ-open sets, and intuitionistic fuzzy e-open sets. We introduce neutrosophic e-compactness sets, neutrosophic e-irresolute, and other topological concepts related to the results after introducing the basic ideas of neutrosophic sets and neutrosophic topological spaces. Numerous features and preservation characterizations pertaining to neutrosophic locally e-compactness have been studied and acquired.

5. Author's contributions

This article was written in collaboration with all of the contributors. The final manuscript was read and approved by all writers.

6. Data availability

On request, the data used to support the findings of this study can be obtained from the corresponding author.

7. Conflict of Interests

The authors declare that there is no conflict of interests regarding this manuscript.

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