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A fuzzy approach using modified adomian decomposition method for solving fuzzy Duffing differential equation

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Abstract

This study presents a novel application of the fuzzy Adomian decomposition method (FADM) to address the fuzzy Duffing differential equation, a significant non-linear problem frequently encountered in mechanical systems. The traditional methods of solving such equations often struggle to accommodate the inherent uncertainties present within the system parameters. By leveraging fuzzy logic, the FADM enables a more accurate representation of these uncertainties, resulting in robust approximate solutions. The method is systematically outlined, highlighting its effectiveness in producing convergent series solutions. Numerical examples are provided to illustrate the method's application and efficacy in approximating

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solutions under varying fuzzy conditions. Results indicate that the fuzzy approach significantly improves solution accuracy and stability compared to traditional deterministic methods, making it a valuable tool in nonlinear dynamics.

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1. Introduction

Fuzzy differential equations (FDEs) represent a more generalized concept of classical differential equations, specifically addressing the uncertainty and vagueness often present in real-word scenarios. This approach is beneficial in situations where exact numerical parameters may not be available or applicable [1]. Recent techniques for solving such equations, as highlighted in [2], also emphasize methods for obtaining approximate solutions to higher-order nonlinear differential equations.

Fuzzy Duffing differential equation represents a blend of fuzzy logic and the classical Duffing equation, which describes non-linear oscillatory systems. The incorporation of fuzziness allows for handling uncertainties and imprecision in real-world systems more effectively.

The Duffing equation is a non-linear second-order differential equation typically formulated as [3]:

$$\frac{d^2y}{dt^2} + \delta \frac{dy}{dt} + \alpha y + \beta y^3 = F(t), \tag{1}$$

along with the corresponding fuzzy initial values

$$y(0) = \sigma, y'(0) = \varsigma, \tag{2}$$

where:

$y: [0,1] \to \mathbb{R}_F$ is a continuous fuzzy-value function, (δ) is the damping coefficient, (α) represents the linear stiffness, (β) captures the non-linear stiffness, $(F(t))$ is the external forcing function, and $\sigma, \varsigma \in \mathbb{R}_F$ (the set of all fuzzy numbers define on the real number). If $\beta = 0$ this model reduces to simple damped harmonic oscillator, and if $\delta = \beta = 0$ the model can show chaotic dynamical behaviors.

Recent research has focused on numerical methods for solving fuzzy Duffing equation. Techniques such as the extended Runge-Kutta method [4] Laplace transform [5], and Variational iteration [6] have been examined to capture the oscillatory behavior efficiently, allowing for more accurate predictions in practical scenarios. Furthermore, the article [7] explore methods for approximate solutions to initial value problems, which provied insights applicable in this context. These methods enhance the usability of fuzzy Duffing models in real-world applications, offering insights into the behavior of complex dynamic systems under uncertainty.

Fuzzy differential equations incorporate uncertainly and vagueness, making traditional methods less applicable. ADM proves useful in this context by allowing the development of solutions that accommodate fuzzy parameters, enhancing the modeling of systems where inputs are not precisely defined [8-10].

Fuzzy Duffing differential equation represents a significant advancement in the mathematical modeling of complex physical problems. These equations extend the traditional Duffing equation, which describes non-linear oscillations in mechanical systems, by incorporating fuzzy logic to account for uncertainties and imprecisions inherent in real-world situations, fuzzy Duffing differential equation has powerful applications across various fields, particularly where uncertain and dynamic systems are analyzed. These include: Engineering Dynamics, Economics, Environmental Modeling, Modeling Uncertainty, Robotics, Complex Behavior Analysis, Enhanced Solutions, Vibration Analysis, Control Systems, Engineering Mechanics, and Material Science [3-6, 11-13]. Fuzzy Duffing differential equation plays a vital role in addressing complex problems in physics and engineering by integrating fuzzy logic into models, thereby providing effective solutions that accommodate uncertainty in physical systems. This broad applicability underscores their importance in contemporary research and practical applications [14]. The article aims to introduce and develop a novel approach using FADM to solve fuzzy Duffing differential equation. The primary objective of this research is to provide accurate and effective solutions to this differential equation, which are often complex due to it nonlinear characteristics and the presence of fuzzy variables. By employing a modified version of the Adomian decomposition method, the authors seek to enhance the robustness and applicability of the solution technique in handling uncertainties inherent in the Duffing differential equation,leading to improved modeling and analysis of dynamic systems where fuzziness plays a significant role.

2. Preliminaries

Definition 1: A function $\vartheta: \mathbb{R} \to [0,1]$ is defined as a fuzzy number if it satisfies the following conditions [15]:

- **Normality:** There exists at least one $t_0 \in \mathbb{R}$ such that $\vartheta(t_0) = 1$;
- **Convexity:** For any $t, s \in \mathbb{R}$ and $\Lambda \in [0,1]$, we have
$$\vartheta(\Lambda t + (1-\Lambda)s) \geq \min\{\vartheta(t), \vartheta(s)\};$$
- **Upper semi-continuity:** The function is upper semi-continuous over $\mathbb{R}$;
- **Compact support:** The support set $\{t \in \mathbb{R} | \vartheta(t) > 0\}$ is compact.

Definition 2: An ordered pair of functions $(\underline{\vartheta}(r), \overline{\vartheta}(r)), r \in [0,1]$ that satisfies the following requirements, represents a fuzzy number in the parametric form [16]

1. $(\underline{\vartheta}(r)$ is a bounded left continuous non-decreasing function over [0,1].
2. $\overline{\vartheta}(r)$ is a bounded left continuous non-increasing function over [0,1].
3. $\underline{\vartheta}(r) \leq \overline{\vartheta}(r), r \in [0,1]$.

Definition 3: Let $\mathbb{R}_F$ represent the set of all fuzzy numbers. A function $f(t)$ is referred to as a fuzzy function if it is defined as a mapping $f: \mathbb{R} \rightarrow \mathbb{R}_F$ [6].

Definition 4: A function $F: \Xi \rightarrow \mathbb{R}_F$, where $\Xi \subset \mathbb{R}$ is an interval, is termed a fuzzy functional process [6]. The corresponding r-level set of f is given by:

$$f(t,r) = [\underline{f}(t,r), \overline{f}(t,r)], t \in \Xi, r \in [0,1],$$

where $\underline{f}$ and $\overline{f}$ denote the lower and upper bounds, respectively.

Definition 5: $\int_a^b f(t,r)dt$, for $a, b \in \Xi$, and $r \in [0,1]$, the fuzzy integral of fuzzy process $f(t,r)$ [6] is defined by: $\int_a^b f(t,r)dt = [\int_a^b \underline{f}(t,r)dt, \int_a^b \overline{f}(t,r)dt]$.

Definition 6: If there is a fuzzy number ι such that $\rho! \sigma = \iota$, then $\rho = \sigma + \iota$, where ρ, σ are two fuzzy numbers in the $\mathbb{R}_F$ set. The fuzzy number is known as the Hukuhara difference (H-difference) [17].

Definition 7: Let $x_0 \in (c,d)$. A function $f: (c,d) \rightarrow \mathbb{R}_F$ is said to be H-differentiable at x_0 if there exists an element $f'(x_0) \in \mathbb{R}_F$ such that, for a sufficiently small increment $h \geq 0$, both forward and backward difference quotients converge to $f'(x_0)$ with respect to the metric D. Specifically [18],

$$\lim_{h\to 0} D\left(\frac{f(x_0+h)-f(x_0)}{h}, f'(x_0)\right) = \lim_{h\to 0} D\left(\frac{f(x_0)-f(x_0-h)}{h}, f'(x_0)\right) = 0.$$

3. Fuzzy Modified Adomiam Decomposition Method

In this section, we analyze fuzzy Duffing differential equation by fuzzy modified Adomiam Decomposition Method as follows.

We can write Eq.(1) in an operator form

$$Ly = F(t) - \beta y^3, \tag{3}$$

where

$$Ly = e^{(i\sqrt{v}-\delta)t} \frac{d}{dt} e^{(-2i\sqrt{v}+\delta)t} \frac{d}{dt} e^{i\sqrt{vt}}(y), \tag{4}$$

and

$$L^{-1}y = e^{-i\sqrt{vt}} \int_0^t e^{(2i\sqrt{v}-\delta)t} \int_0^t e^{(-i\sqrt{v}+\delta)t}(y), \tag{5}$$

here $\alpha = (v + i\sqrt{v}\delta)$. If we applying Eq.(5) on Eq.(3) we get

$$\underline{y} = \underline{\zeta} - L^{-1}F(t) - L^{-1}\beta\underline{y^3},$$

$$\bar{y} = \overline{\zeta} - L^{-1}F(t) - L^{-1}\beta\overline{y^3},$$

the series solution in the ADM [19-22] defines as follows

$$\underline{y} = \sum_{j=0}^{\infty} \underline{y}_j, \tag{6}$$

$$\overline{y} = \sum_{j=0}^{\infty} \overline{y}_j, \tag{7}$$

and the nonlinear part y^3 is decomposed as

$$\underline{y^3} = \sum_{j=0}^{\infty} \underline{A}_j, \tag{8}$$

$$\overline{y^3} = \sum_{j=0}^{\infty} \overline{A}_j, \tag{9}$$

where $\underline{A}_j, \overline{A}_j$ are Adomian polynomials

$$\underline{A}_j = \frac{1}{j!}\frac{d^j}{d\gamma^n}\left[N\left(\sum_{i=0}^{\infty} \gamma^i \underline{y}_i\right)\right], \text{where } \gamma = 0, j = 0,1,2,3, \dots . \tag{10}$$

$$\overline{A}_j = \frac{1}{j!}\frac{d^j}{d\gamma^n}\left[N\left(\sum_{i=0}^{\infty} \gamma^i \overline{y}_i\right)\right], \text{where } \gamma = 0, j = 0,1,2,3, \dots . \tag{11}$$

The all terms can get by following

$$\underline{y}_0 = \underline{\zeta} - L^{-1}F(t), \tag{12}$$

$$\overline{y}_0 = \overline{\zeta} - L^{-1}F(t), \tag{13}$$

and

$$\underline{y}_{j+1} = -L^{-1}\underline{A}_j, \tag{14}$$

$$\overline{y}_{j+1} = -L^{-1}\overline{A}_j. \tag{15}$$

4. Numerical Application

The examples presented in this section will demonstrate the effectiveness of the modified Adomian Method for solving fuzzy Duffing equation.

Example 1: Consider the following fuzzy Duffing equation $(\delta = 0, \alpha = 3, \beta = -2)$ [3] [6]

$$y''(t) + 3y(t) - 2y^3(t) = \cos t \sin 2t, \quad t \geq 0 \tag{16}$$

subject to the following initial conditions represented as fuzzy set

$$y(0) = [r-1, 1-r], y'(0) = [r, 2-r],$$
$$\forall r \in [0,1].$$

We can write Eq.(16) in the form:

$$y''(t) + 3y(t) = \cos t \sin 2t + 2y^3(t),$$
$$Ly = \cos t \sin 2t + 2y^3(t), \tag{17}$$

Where

$$Ly = e^{i\sqrt{3}t}\frac{d}{dt}e^{-2i\sqrt{3}t}\frac{d}{dt}e^{i\sqrt{3}t}(y), \tag{18}$$

and

$$L^{-1}y = e^{-i\sqrt{3}t}\int_0^t e^{2i\sqrt{3}t}\int_0^t e^{-i\sqrt{3}t}(y)dtdt, \tag{19}$$

by applying Eq.(19) to Eq.(17) we get

$$\underline{y}_0 = -1 + r + rt - \frac{3}{2}(-1+r)t^2 - \frac{1}{6}(-2+3r)t^3 + \frac{3}{8}(-1+r)t^4 + \frac{1}{120}(-20+9r)t^5 + \dots,$$

$$\overline{y}_0 = 1 - r - (-2+r)t + \frac{3}{2}(-1+r)t^2 + \frac{1}{6}(-4+3r)t^3 - \frac{3}{8}(-1+r)t^4 + \dots,$$

$$\underline{y}_{j+1} = -2e^{-i\sqrt{3}t}\int_0^t e^{2i\sqrt{3}t}\int_0^t e^{-i\sqrt{3}t}(\underline{A}_j)dtdt, \quad (20)$$

$$\overline{y}_{j+1} = -2e^{-i\sqrt{3}t}\int_0^t e^{2i\sqrt{3}t}\int_0^t e^{-i\sqrt{3}t}(\overline{A}_j)dtdt. \quad (21)$$

When $j = 0$ in Eq.(20) and Eq.(20) we get

$$\underline{y}_1 = (-1+r)^3t^2 + (-1+r)^2rt^3 - \frac{1}{2}(-1+r)(2-4r+r^2)t^4 + \ldots,$$

$$\overline{y}_1 = -(-1+r)^3t^2 - (-2+r)(-1+r)^2t^3 + \frac{1}{2}(2-2r-r^2+r^3)t^4 + \ldots,$$

Similarly, the other terms can be found, and we get the solution of Eq.(16) by MADM by collecting these terms.

Table 1

Compare solutionn of Eq.(16) at $t = 0.1$ of 5-terms

	Lower			
r	FMADM	VIM [6]	RPSM [3]	Exact Solution
0	-0.9946548739	-0.9946548745	-0.9946548333	-0.9946548739
0.25	-0.7176219070	-0.7176219076	-0.7176218148	-0.7176219070
0.5	-0.4435578790	-0.4435578806	-0.4435578646	-0.4435578799
0.75	-0.1714170382	-0.1714170389	-0.1714170902	-0.1714170382
1	0.09983341664	0.09983341601	0.09983341667	0.09983341664

	Upper			
r	FMADM	VIM [6]	RPSM [3]	Exact Solution
0	1.19652475343	1.19652475384	1.19652366667	1.19652475400
0.25	0.91852280395	0.91852280319	0.91852284342	0.91852280370
0.5	0.6437715610	0.64377156027	0.64377182292	0.64377156090
0.75	0.37122034072	0.37122034006	0.37122049382	0.37122034070
1	0.09983341664	0.09983341601	0.09983341667	0.09983341664

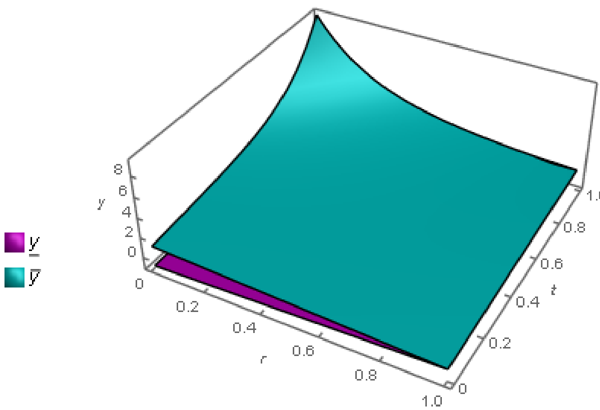

Figure 1

3D approximate lower and upper bound solutions of Eq. (16) at $t = 0.1$ by MADM.

As noted in Table 1, the results obtained by the MADM closely approximate the exact solution compared to VIM and RPSM. Additionally, Figure 1 illustrates the upper and lower fuzzy solutions obtained by MADM, demonstrating the convergence behavior and the narrowness of the fuzzy band.

Example 2: Consider the following fuzzy Duffing equation $(\delta = 1, \alpha = 0, \beta = 1)$ [3] [6] [23]

$$y''(t) + y'(t) = -y^3(t), \quad t \geq 0, \tag{22}$$

with the initial conditions defined in the fuzzy context

$$y(0) = [0.9 + 0.1r, 1.1 - 0.1r], y'(0) = [0.9 + 0.1r, 1.1 - 0.1r],$$
$$r \in [0,1].$$

This equation does not have an exact solution .

We can write Eq.(22) as following

$$Ly = -y^3(t), \tag{23}$$

where

$$Ly = e^{-t}\frac{d}{dt}e^{t}\frac{d}{dt}(y), \tag{24}$$

and

$$L^{-1}(y) = \int_0^t e^{-t}\int_0^t e^{t}(y), \tag{25}$$

By applying Eq.(25) to Eq.(23) we get

$$\underline{y}_0 = 0.1\,(9.+1.r) + 0.1\,(9.+1.r)\,t - 0.05\,(9.+1.r)\,t^2$$
$$+0.0166667\,(9.+1.r)\,t^3 - 0.00416667\,(9.+1.r)\,t^4 + \ldots,$$
$$\overline{y}_0 = -0.1\,(-11.+1.r) - 0.1\,(-11.+1.r)\,t + 0.05\,(-11.+1.r)\,t^2$$
$$-0.0166667\,(-11.+1.r)\,t^3 + 0.00416667$$
$$(-11.+1.r)\,t^4 + \ldots,$$

$$\underline{y}_{j+1} = -\int_0^t e^{-t}\int_0^t e^{t}(\underline{A}_j)dtdt, \tag{26}$$

$$\overline{y}_{j+1} = \int_0^t e^{-t}\int_0^t e^{t}(\overline{A}_j)dtdt. \tag{27}$$

When $j = 0$ we get

$$\underline{y}_1 = 1.77636\;10^{-15}\,r\,(12.+r)$$
$$+2.77556\;10^{-17}\,(-38.3303 + r)\,(-1.6697 + r)\,(8.+r)\,t$$
$$-0.0005\,(9.00054 + r)\,(80.9951 + 17.9995\,r + r^2)\,t^2$$
$$-0.000333333\,(8.9994 + r)\,(81.0054 + 18.0006\,r + r^2)\,t^3$$
$$-0.0000416667\,(9.00089 + r)\,(80.992 + 17.9991\,r + r^2)\,t^4 + \ldots,$$
$$\overline{y}_1 = 1.42109\;10^{-14}\,(10.+1.r)\,t$$
$$+0.0005\,(-11.0005 + r)\,(120.995 - 21.9995\,r + r^2)\,t^2$$
$$+0.000333333\,(-10.9993 + r)\,(121.008 - 22.0007\,r + r^2)\,t^3$$

$$+0.0000416667\,(-11.0013+r)\,(120.985-21.9987\,r+r^2)\,t^4+\ldots,$$

Similarly, the other terms can be found, and we get the solution of Eq.(22) by MADM by collecting these terms. As previously noted, Eq. (22) lacks an exact analytical solution. To assess the reliability of the obtained 5-term MADM approximation, we compute the residual error using the following expression [24]: $E(t,r) = |y''(t,r) + y'(t,r) + y^3(t,r)|$

Table 2

Upper and lower fuzzy approximations solution by MADM for Eq. (22) when $t = 0.1$.

r	$\underline{y}$	$\underline{E}$	$\overline{y}$	$\overline{E}$
0	0.9817642968	6.29891×10^{-7}	1.197598637	1.03747×10^{-6}
0.25	1.0088113025	7.59263×10^{-7}	1.1706904676	9.05757×10^{-7}
0.5	1.035839938	9.05317×10^{-7}	1.14376100450	7.80308×10^{-7}
0.75	1.06284971323	1.06885×10^{-6}	1.11681073354	6.62773×10^{-7}
1	1.0898401411	1.250455×10^{-6}	1.08984014077	5.54262×10^{-7}

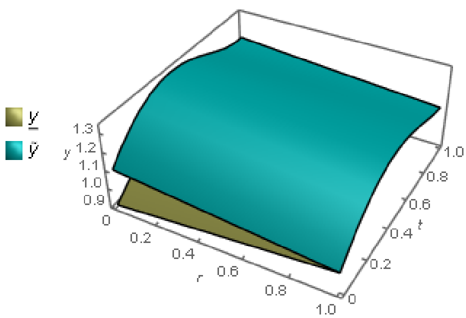

Figure 2
Three-dimensional fuzzy approximation of the solution by MADM bounds for Eq. (22) when $t = 0.1$.

Example 3: Consider the following fuzzy Duffing equation $(\delta = 1, \alpha = -2, \beta = 1)$

$$y''(t) + y'(t) - 2y(t) = -y^3(t),\ \ t \geq 0, \tag{28}$$

with the specified fuzzy initial conditions

$$y(0) = [0.5 + r, 1 - 0.1r], y'(0) = [0.5 + r, 1 - 0.1r],$$
$$\forall r \in [0,1].$$

This equation does not have an exact solution .

We can write Eq.(28) as following

$$Ly = -y^3(t), \tag{29}$$

where

$$Ly = e^{-2t}\frac{d}{dt}e^{3t}\frac{d}{dt}e^{-t}(y), \tag{30}$$

and

$$L^{-1}(y) = e^{t}\int_0^t e^{-3t}\int_0^t e^{2t}(y)dtdt, \tag{31}$$

By applying Eq.(31) to Eq.(29) we get

$$\underline{y}_0 = 0.5 + r + (0.5 + r)\,t + 0.5\,(0.5 + 1.r)\,t^2 + 0.166667\,(0.5 + 1.r)\,t^3 + 0.0416667\,(0.5 + r)\,t^4 + \ldots,$$

$$\overline{y}_0 = 1 - 0.1\,r + (1 - 0.1\,r)\,t - 0.05\,(-10. + 1.r)\,t^2 - 0.0166667\,(-10. + 1.r)\,t^3 - 0.00416667\,(-10. + 1.r)\,t^4 + \ldots,$$

$$\underline{y}_{j+1} = -e^{t}\int_0^t e^{-3t}\int_0^t e^{2t}(\underline{A}_j)dtdt, \tag{32}$$

$$\overline{y}_{j+1} = -e^{t}\int_0^t e^{-3t}\int_0^t e^{2t}(y)(\overline{A}_j)dtdt. \tag{33}$$

When $j = 0$ we get

$$\underline{y}_1 = -1.11022\,10^{-17}\,(0.5 + r)^3\,t - 0.5\,(0.5 + r)^3\,t^2 - 0.333333\,(0.5 + r)^3\,t^3 - 0.375\,(0.5 + r)^3\,t^4 + \ldots,$$

$$\overline{y}_1 = -2.08167\,10^{-16} + 3.46945\,10^{-17}\,r - 1.30104\,10^{-18}\,r^2 + (-2.08167\,10^{-16} + 3.46945\,10^{-17}\,r - 2.1684\,10^{-18}\,r^2 + 2.71051\,10^{-20}\,r^3)\,t + 0.0005\,(-1000. + 300.r - 30.r^2 + 1.r^3)\,t^2 + 0.000333333\,(-1000. + 300.r - 30.r^2 + 1.r^3)\,t^3 + 0.000375\,(-1000. + 300.r - 30.r^2 + 1.r^3)\,t^4 + \ldots,$$

Similarly, the other terms can be found, and we get the solution of Eq.(22) by MADM by collecting these terms.

Similarly, since Eq. (28) does not admit an exact solution, the accuracy of the 5-term MADM approximate result is evaluated by defining a residual error function as follows [24]: $E(t,r) = |y''(t,r) + y'(t,r) + y^3(t,r)|$

Table 3

Numerical estimation of the upper and lower bounds of Eq. (28) evaluated at $t = 0.1$.

r	$\underline{y}$	$\underline{E}$	$\overline{y}$	$\overline{E}$
0	0.5519143120	1.97742×10^{-12}	1.0998125439	4.03838×10^{-9}
0.25	0.8266149706	1.70864×10^{-10}	1.0725745286	3.05735×10^{-9}
0.5	1.0998125439	4.03838×10^{-9}	1.0453170354	2.29794×10^{-9}
0.75	1.3710138207	4.69059×10^{-8}	1.0180405557	1.71404×10^{-9}
1	1.6397322063	4.69059×10^{-7}	0.9907455815	1.26826×10^{-9}

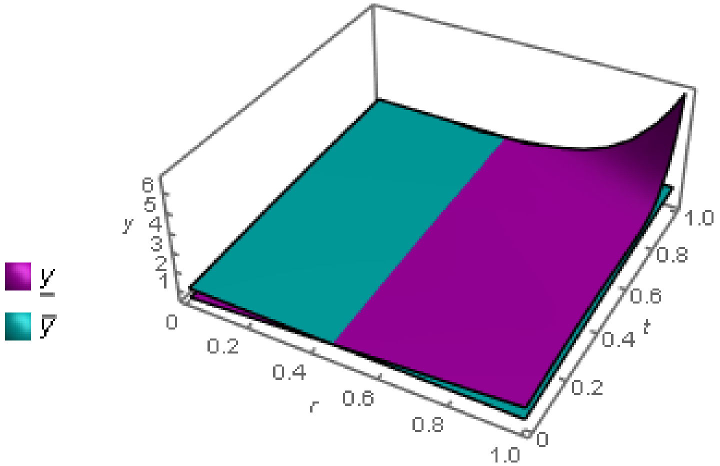

Figure 3
Visualization of the upper and lower 3D solutions by MADM of fifth-order of Eq. (28) evaluated at $t = 0.1$.

As presented in Tables 2 and 3, the computed upper and lower fuzzy solutions using the MADM are listed for different values of and . The corresponding graphical representations in Figures 2 and 3 illustrate the behavior of these solutions, showing how the fuzzy band evolves and confirming the stability and reliability of MADM in capturing the solution bounds.

Conclusion

The FMADM has demonstrated significant effectiveness in addressing the Fuzzy Duffing Differential Equation. Based on the results presented in examples 1, 2, and 3, in Example 1, the results obtained using the FMADM were in close proximity to the exact solution when compared to those derived from the Variational Iteration Method (VIM) and the Residual Power Series Method (RPSM). This indicates the robustness of the FMADM in providing accurate approximations for the Fuzzy Duffing Differential Equation. For Examples 2 and 3, where exact solutions are not available, we defined the residual error, $(E(t,r) = |y_{,,}(t,r) + y_{,}(t,r) + y^3(t,r)|)$, to assess the performance of the MADM. The analysis of residual errors suggests that the FMADM continues to perform well, further supporting its utility in scenarios where traditional methods may fall short. The figures and tables presented substantiate the effectiveness of the FMADM. The method not only yields results that are close to exact solutions when applicable but also offers valuable insights in cases lacking closed-form solutions. In summary, the Fuzzy Modified Adomian Decomposition Method stands out as a reliable computational technique for solving the Fuzzy Duffing Differential Equation, thereby contributing positively to the field of fuzzy differential equations and their applications.

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