

Thermal and mass transport analysis of radiative hybrid nanofluid flow with viscous dissipation and activation energy over a stretching sheet for engineering applications

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Abstract

Hybrid nanofluids (HNFs), due to their superior thermal conductivity and heat transfer performance compared to conventional fluids and simple nanofluids, have attracted increasing attention in diverse engineering and industrial sectors including microelectronics cooling, chemical and manufacturing processes, biomedical devices, and microfluidic systems. The present article discusses thermal convection and mass diffusion in radiative hybrid nanofluid (HNF) flow over a sheet, considering Joule heating, chemical reaction, and activation energy. Water (H_2O) serves as the base fluid, while copper (Cu) and aluminum oxide (Al_2O_3) particles act as HNF. The governing partial differential equations are reduced to nonlinear ordinary differential equations through similarity transformations and solved numerically using MATLAB. The effects of key parameters in this article are illustrated graphically and in tabular form, with results validated against existing literature. The key findings useful for optimizing heat and mass transfer in applications such as chemical reactor enhancement, advanced industrial thermal management and electronic cooling. activation energy. Water (H_2O) serves as the base fluid, while copper (Cu) and aluminum

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Keywords: Joule heating, Hybrid nanofluid, Activation energy, Viscous dissipation.

1. Introduction

Research on stretching sheets has been widely applied to study the heat transfer behavior of fluids, leading to numerous valuable industrial applications. Such studies are significant in processes involving heat preservation, fabrication of complex synthetic materials, crystal growth, and the design of advanced transport systems. Industrial applications of heat transfer and momentum analysis are particularly important, as the quality of a finished product often depends on how efficiently heat exchange occurs and how quickly velocity gradients transform [1, 2]. Hybrid Nano fluid (HNF) mixed in a base fluid, that has the prospective to overcome both ordinary cooling fluids and nanofluids in terms of heat transfer. It is sound predictable that combination of nanoparticles is significant in numerous industrialized and scientific regions and that the combination and reliability of hybrid nanoparticles have been deliberate in abundant literature studies [3, 4]. When it comes to creating a long-lasting hybrid nanofluid, one of the most essential elements to consider is choosing suitable combination of nanoparticles. Additionally, the dimension and solid volume fractions of the nanoparticles are all crucial aspects in optimizing the heat conductivity of the HNF. Waini, Ishak, and Pop [5] considered HNF flow over a permeable surface. The effects of nanoparticles of Cu and Ag past a plate are examined by Rawat, Mishra, and Kumar [6]. During crystals designing melting in the field of industry a fluid is stimulated by electricity strong magnetic field. The vast array of industrial and engineering applications that use the magnetic effect, such as power generators, medical equipment, magnetic resonance graphs make them immensely important [7, 8, 9]. Joule heating provides effective control over the movement of hydromagnetic fluids, which is crucial in various scientific and technological fields. Often referred to as Ohmic heating, Joule heating involves converting electrical energy into heat

through resistive losses in a material. This principle is effectively utilized in most electrical and electronic devices. The primary application of Joule heating lies in transferring electrical potential to manage losses by minimizing current. Numerous studies [10, 11, 12] have explored the movement of various MHD fluids influenced by Joule heating. Thermal radiation plays a vital role in high-thermal processes and has ample of practical applications. Its impact becomes significant when convective heat transfer coefficients are low, due to this researcher to investigate its effects on nanofluid and hybrid nanofluid flows. Ghadikolaei, Hosseinzadeh, and Ganji [13] developed the radiative squeezing flow of an Eyring–Powell fluid for parallel surfaces and found that radiation parameters reduce the fluid temperature. Khan, Malik, and Salahuddin [14] prepared an analytical model of Carreau nanofluid flow over a variable thickness sheet. Hasona et al. [15] comprehensively prepared a model through mathematical on therapy for cancer treatment. Zainal et al. [16] has illustrated radiative influence on Maxwell hybrid nanofluids at the stagnation point flow of two-dimensional system. Heat transfer mechanisms (temperature distribution, radiation, or Joule heating) along with fluid flow influenced by external forces like magnetic fields (MHD) or pressure gradients has been elaborately studied by [17, 18, 19].

Building upon the insights from previous literature, it is well established that hybrid nanofluids (HNFs) exhibit significantly higher heat transfer rates compared to nanofluids (NFs) and conventional base fluids. This study motivates the flow of a HNF over an elongating or contracting sheet, where the base fluid is water and the nanoparticles are metal and metallic oxides. The analysis includes the effects of heat source/sink, and activation energy, which are particularly relevant due to energy losses encountered in various industrial processes. The governing nonlinear ODEs, derived via similarity transformations, are simulated using MATLAB. The effects of physical parameters are illustrated through detailed tables and graphical results. Furthermore, a comparative study with previously published works confirms the validity of the mathematical formulation and numerical methodology adopted in this research.

2. Formulation of the problem

A two-dimensional incompressible HNF laminar flow with suction and injection velocity $v_0 = -\sqrt{av_f S}$, over a stretching sheet is considered. In the present analysis, Cu and Al_2O_3 are considered nanoparticles while water as conventional fluid. The reference axes x and y are along horizontal

and vertical axes respectively. A sheet with velocity $u_w(x) = ax$ which is stretching along x direction and is applied perpendicular to it (see Figure 1). Let the temperature and concentration are T_w and C_w at the surface while far away from it which are T_∞ and C_∞ respectively. The flow assumes to be optically thick, where the fluid medium is sufficiently dense.

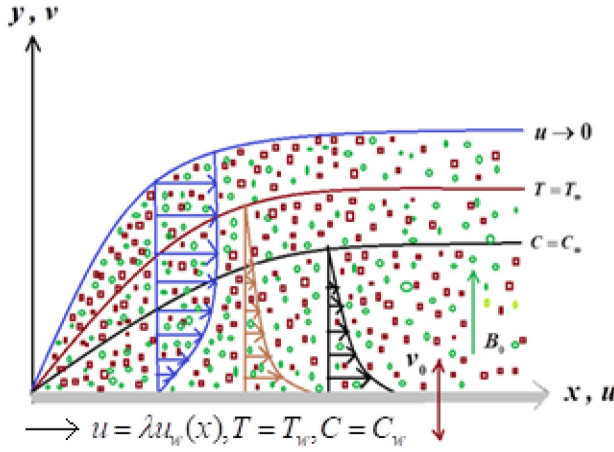


Figure 1
Flow Pattern

In the context of above flow conditions, the controlling equations representing steady flow of HNF are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{B_0^2 \sigma_{hmf}}{\mu_{hmf}} u = \frac{\mu_{hmf}}{\rho_{hmf}} \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$(\rho C_p)_{hmf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{hmf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \sigma_{hmf} B_0^2 u^2 + \mu_{hmf} \left(\frac{\partial u}{\partial y} \right)^2 + Q^* (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) - k_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^n e^{\left(\frac{-E_a}{k_B T} \right)}. \quad (4)$$

$$\left. \begin{aligned} u &= \lambda ax, v = -\sqrt{Sav_f}, C = C_w, T = T_w \text{ at } y = 0 \\ u &\rightarrow 0, C \rightarrow C_\infty, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (5)$$

Using Rosseland approximation, Now

$$q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (6)$$

here k^*, σ^* Stefan-Boltzmann constant and mean absorption coefficient. then by Taylor series approximation $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$ From the above relation (6), we have

$$\frac{\partial q_r}{\partial y} = -\frac{16}{3} \frac{T_\infty^3 \sigma^*}{k^*} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

Hence, Eq. (3) becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho C_p)_{hmf}} \left[\left(\frac{3k^* k_{hmf} + 16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \mu_{hmf} \left(\frac{\partial u}{\partial y} \right)^2 + \sigma_{hmf} B_0^2 u^2 + Q^* (T - T_\infty) \right] \quad (8)$$

Here B_0 and Q^* represent the magnetic field strength, heat source coefficient, and stretching parameter respectively for non-dimensional temperature and concentration of the HNFs.

The non-dimensional variables used in this problem are,

$$\eta = \sqrt{\frac{a y^2}{v_f}}, \theta = \frac{T_\infty - T}{T_\infty - T_w}, \psi = \sqrt{ax^2 v_f} f(\eta), \phi = \frac{C_\infty - C}{C_\infty - C_w} \quad (9)$$

Where v_f is the kinematic viscosity and velocity components are

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \quad (10)$$

$$f''' = \frac{\rho_{hmf} \mu_f}{\rho_f \mu_{hmf}} \left(\frac{\rho_f \sigma_{hmf}}{\rho_{hmf} \sigma_f} M f' + f'^2 - f f'' \right) \quad (11)$$

$$\begin{aligned} & \frac{1}{Pr} \frac{(\rho C_p)_f}{(\rho C_p)_{hmf}} \left(\frac{k_{hmf}}{k_f} + \frac{4}{3} R \right) \theta'' + f \theta' + \\ & \frac{(\rho C_p)_f \sigma_{hmf}}{(\rho C_p)_{hmf} \sigma_f} MEcf'^2 + \frac{(\rho C_p)_f \mu_{hmf}}{(\rho C_p)_{hmf} \mu_f} Ec f''^2 + \frac{(\rho C_p)_f}{(\rho C_p)_{hmf}} Q \theta = 0 \end{aligned} \quad (12)$$

$$\phi'' + Sc f \phi' - \left(Kr Sc (1 + \delta_1 \theta)^n e^{\left(\frac{-E}{1 + \delta_1 \theta} \right)} \right) \phi = 0 \quad (13)$$

$$\begin{aligned} \theta(0) = 1, \phi(0) = 1, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \\ f(0) = S, f'(0) = \lambda, f'(\infty) \rightarrow 0 \end{aligned} \quad (14)$$

$$\begin{aligned} \text{Where } Pr = \frac{(\mu C_p)_f}{k_f}, M = \frac{\sigma_f B_0^2}{\rho_f U_e}, R = \frac{4\sigma^* T_\infty^3}{3k k^*}, Ec = \frac{u_w^2}{(C_p)_f (T_w - T_\infty)}, \\ Sc = \frac{\nu_f}{D_B}, \delta_1 = \frac{T_w - T_\infty}{T_\infty}, Kr = \frac{kr^2}{a}, E = \frac{E_a}{k_B T_\infty}, Q = \frac{Q^*}{a(\rho C_p)_f} \end{aligned}$$

are Prandtl number, magnetic parameter, radiation parameter, Eckert number, Schmidt Number, temperature difference parameter, chemical reaction parameter, activation energy parameter, and heat source/sink parameter respectively and if $\lambda > 0$ for stretching, $\lambda < 0$ for shrinking and $\lambda = 0$ for rigid cases. Besides $\phi_1 = \phi_2 = 0$ characterizes the case pertaining to base fluid. Tables 1 and 2 exemplify the thermal and physical properties of NF and HNF.

The physical interest quantities like, C_f , Nu_x and Sh_x are given below.

$$C_f = \frac{\tau_w}{\rho_f u_e^2} = \frac{\mu_{hmf}}{\rho_f u_e^2} \left(\frac{\partial u}{\partial y} \right)_{y=0} \Rightarrow Re_x^{1/2} C_f = \frac{\mu_{hmf}}{\mu_f} f''(0), \quad (15)$$

$$Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)} \Rightarrow Re_x^{(-1/2)} Nu_x = - \left(\frac{k_{hmf}}{k_f} + \frac{4R}{3} \right) \theta'(0) \quad (16)$$

$$Sh_x = \frac{x q_m}{D_B (C_w - C_\infty)} \Rightarrow Re_x^{(-1/2)} Sh_x = -\phi'(0) \quad (17)$$

Here, $\tau_w = \mu_{hmf} \left(\frac{\partial u}{\partial y} \right)_{y=0}$, $q_w = -k_{hmf} \left(\frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_{y=0}$, $q_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}$ and

$$Re_x = \frac{u_e x}{\nu_f}$$

The thermal and physical properties like density, dynamic viscosity, thermal conductivity, heat capacity and electrical conductivity of NF and HNF are taken from existing literatures and Thermal and physical properties of base fluid and hybrid Nano particles. density, specific heat, thermal conductivity and electrical conductivity are 997, 4180, 0.613, 5.5×10^{-6} (for water), 8933, 385, 400, 5.96×10^{-7} (for copper), 3970, 765, 40, 37×10^{-6} (for aluminum oxide) respectively.

3. Numerical treatment

To simulate the system of differential equations (11-13) with conditions (14), the shooting method is applied as follows:

$$\left. \begin{aligned} f' &= p, p' = q, q' = \frac{a_1}{a_2} \left[\frac{a_4}{a_1} M p + p^2 - f q \right], \theta' = z, \\ z' &= -\frac{3 \text{Pr}}{3a_5 + 4R} \left[a_3 f z + a_4 M E c p^2 + a_2 E f q^2 + Q \theta \right] \\ \phi' &= t, t' = - \left[S c f t - \left(K r S c (1 + \delta_1 \theta)^n e^{-\frac{E}{1 + \delta_1 \theta}} \right) \phi \right] \end{aligned} \right\} \quad (18)$$

$$\begin{aligned} \theta(0) &= 1, \phi(0) = 1, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0, \\ f(0) &= S, p(0) = \lambda, p(\infty) \rightarrow 0 \end{aligned} \quad (19)$$

Select an appropriate value for the limit η_∞ .

1. Choose suitable initial guesses for $q(0), z(0), t(0)$.
2. Convert the system of differential equations into an initial value problem and solve it using the Runge-Kutta method.
3. If the absolute difference between the computed $p(\infty), \theta(\infty), \phi(\infty)$ and given boundary values is less than the error tolerance 10^{-5} , the solution is accepted.
4. Otherwise, refine the guessed values using Newton's method and repeat the process until convergence, as illustrated in Figure 2.

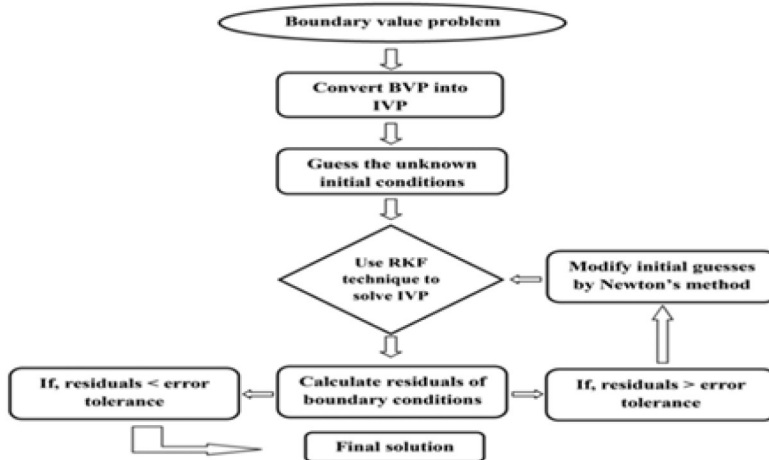


Figure 2

Flow chart of Numerical technique

4. Validation of the problem

Table 1 shows comparison of heat transfer rate $-\theta'(0)$ for Prandtl number Pr in the absence of Magnetic parameter, heat source/sink parameter, activation energy parameter Eckert number, Schmidt number and chemical reaction parameter with existing published results. It is noticed that the outcome is consistent with previous works. This validates the implementation of mathematical model and numerical scheme used to find solution.

Table 1

Comparison of $-\theta'(0)$ for Pr for $M = Q = E = Ec = Sc = Kr = 0$ and $\lambda = 1$

Pr	Khan and Pop [1]	Ahmad et al. [4]	Current study
70	6.4621	6.47814	6.4619282
20	3.3539	3.35271	3.3535493
7	1.8954	1.89083	1.8949271
2	0.9113	0.91045	0.9106823

5. Findings

The simulation is achieved for system (7) – (9) using $Pr = 1.9, Q = 0.5, R = 0.1, \lambda = 1.0, \phi_1 = 0.01, \phi_2 = 0.01, Sc = 1.5, Kr = 0.5, E = 0.5, M = 0.5, Ec = 0.3,$

$\delta_1 = 0.5$, $S = 0.5$ by MATLAB software. The solution's convergence, stability, and accuracy were tested using a step size of 0.01 and an error tolerance of 0.00001. Figure 3 demonstrated the impact of magnetic parameter and nano fluid volume fraction on velocity of nano fluid above extending surface. Magnetic field offers resistance to the flow regime indicating thinning of momentum boundary layer. For a particular value of magnetic parameter increasing volume fraction further reduces the flow pattern. Figure 4 correlates the importance of suction or blowing parameter. It is obvious that blowing parameter reduces the flow compared to suction one for a particular value of stretching rate. Even higher stretching rate accelerates the flow (solid line compared to dashed line). Figure 5 specifies the thermal transport characteristics with respect to

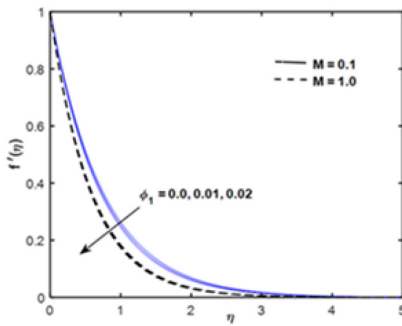


Figure 3

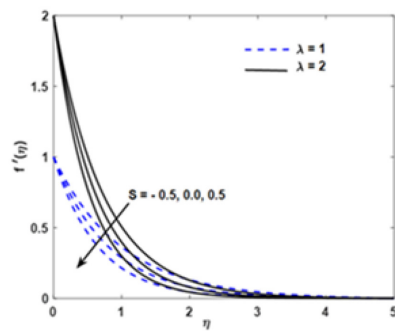
Impact of $f'(\eta)$ for f_1 

Figure 4

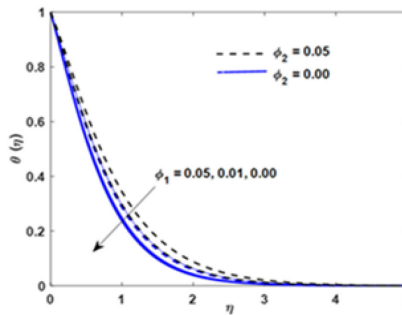
Impact of $f'(\eta)$ for S 

Figure 5

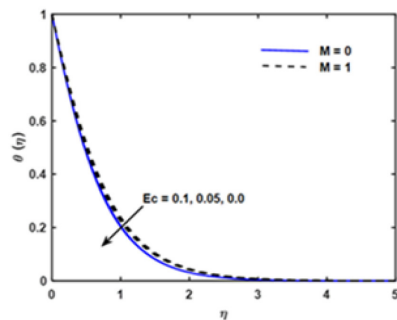
Impact of $\theta(\eta)$ for f_1 

Figure 6

Impact of $\theta(\eta)$ for Ec

$\text{Cu}(\phi_1)$ and $\text{Al}_2\text{O}_3(\phi_2)$ nanoparticles. For simple Nano fluid temperature distribution is low. Increasing volume fraction of nanoparticles further reduces the temperature distribution. However, presence of both nanoparticles i.e Cu and Al_2O_3 enhance the temperature distribution. That strengthened the importance of hybrid Nano fluid compared to pure Nano fluid.

The relation between kinetic energy and potential energy is shown in Figure 6. Due to viscous force the thermal behaviour of the nano fluid increases i.e increase in Eckert number. Addition of magnetic field further makes the temperature to rise. Figure 7 elaborates the effect of R . The temperature change in the boundary layer is due to R . Enhancement of temperature in the boundary layer raise the kinetic energy. This signifies increase in velocity. The Arrhenius condition signifies influence of thermal process on chemical reaction and effect on concentration distribution. The minimum energy needed to activate molecules in reaction is known as activation energy. Figure 8 exemplifies the increase in species concentration profile due to increase in activation energy. Higher the activation energy, the slower the chemical reaction rate and most of the molecules are not participating in this process, resulting an increase in concentration profile away from the surface. Again, higher the temperature difference parameter (Figure 9) lowers the concentration profiles. However, the temperature difference parameter has no control over concentration profiles away from the surface.

Figure 10 elaborates the distribution of temperature with respect to heat source/sink parameter. The profiles attain maximum value unity and these rapidly drop to zero away from the surface. Increasing the heat source parameter heat transfer rate enhances signifying net drop in

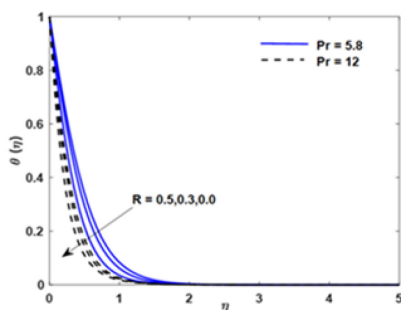


Figure 7
Impact of $\theta(\eta)$ for R

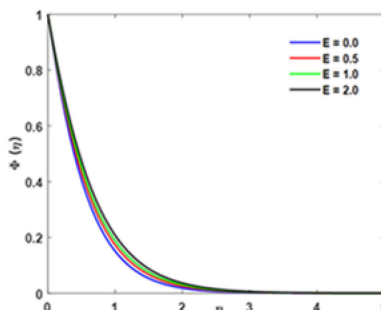


Figure 8
Impact of $\theta(\eta)$ for E .

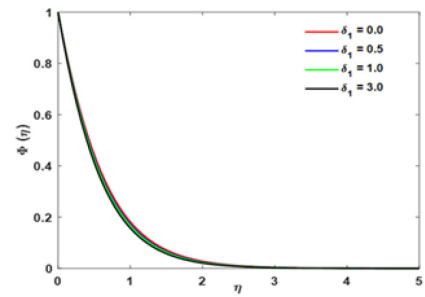


Figure 9
Impact of $\theta(\eta)$ for d_1

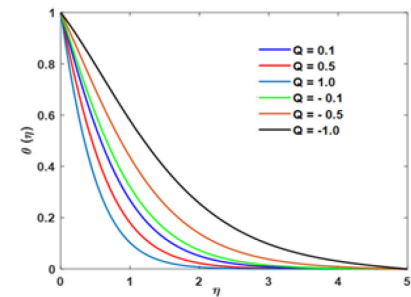


Figure 10
Impact of $\theta(\eta)$ for Q

Table 2
Computational values of C_f, Nu_x, Sh_x when $M = 0.5, Ec = 0.3, Pr = 1.9, Q = 0.5, R = 0.1, \lambda = 1.0, \phi_1 = 0.01, \phi_2 = 0.01, Sc = 1.5, Kr = 0.5, E = 0.5, \delta_1 = 0.5, S = 0.5$

ϕ_1	ϕ_2	M		R	E	Q	δ_1	C_f	Nu_x	Sh_x
0.01								-1.61913	1.555686	1.541447
0.1								-2.33838	1.458696	1.5768828
	0.01							-1.61913	1.5556862	1.54144707
	0.1							-2.03427	1.5403438	1.5931796
		0.5						-1.61913	1.5556862	1.5414470
		5.0						-2.88938	0.7027356	1.4773094
			1					-1.61913	1.555686	1.541447
			2					-3.99902	0.1786249	1.852010
				0.2				-1.61913	1.6159744	1.559334
				2				-1.61913	2.4074232	1.7951689
					0.1			-1.61913	1.555686	1.6277018
					1.0			-1.61913	1.5556862	1.4614401
						0.5		-1.61913	1.5556862	1.54144707
						3.0		-1.61913	2.9615870	1.4422646
							0.5	-1.61913	1.5556862	1.54144707
							3.0	-1.61913	1.5556864	1.6317967

temperature distribution but reverse effect is observed in case of sink. The computational values of C_f, Nu_x, Sh_x are presented in Table 2. The surface drag for hybrid nano fluid is less than simple nano fluid and skin friction tend to decrease in step with volume fraction. The enhancement in heat transfer rate as depicted above shows a promising aspect in case of hybrid nano fluid compared to simple nano fluid. Magnetic field produces magnetic resistance, which increases skin friction. This resulted decrease in heat transfer rate. As we increase stretching rate, the velocity of flow increases tending to subside skin friction. As evident from calculation radiation parameter has no influence on C_f , however heat transfer rate rises with radiation parameter. Similarly surface drag is not affected by Eckert number. Nu_x is enhanced with respect to Ec . Heat source or sink has no effect on skin friction. However, heat source parameter tends to increase heat transfer rate, while heat sink parameter acts in the opposite manner. The mass transfer coefficient is more in case of aluminium than copper. It shows increasing tendency with volume fraction. Higher magnetic parameter reduces mass transfer coefficient with respect to Ec and Q .

5. Conclusion

In above discussion, we highlighted the impact of hybrid nano fluid over an extending surface. The summary of discussion is outlined below.

- Higher volume fraction accelerates the flow.
- Temperature as well as thermal distribution is more in case of multi-phase fluid than compared to single phase fluid.
- Heat transfer rate achieved by HNF is more than single phase fluid
- Molecules having high activation energy slowed down chemical reaction.
- Temperature difference parameter also lowers chemical diffusion.

Nomenclature

a : constant, B_0 : magnetic field strength, v_0 : suction velocity, S : suction/injection parameter, k_f : thermal conductivity of base fluid, k_{hmf} : thermal conductivity of hybrid nanofluid, k_{nf} : thermal conductivity of nanofluid, f : dimensionless velocity, C_∞ : ambient concentration, D_B : Brownian motion coefficient, C_w : concentration at the sheet, Re_x : local Reynolds number, C : concentration, Sc : Schmidt number, Sh_x : local

Sherwood number, T : temperature, T_w : temperature at the sheet, E : activation energy parameter, λ : stretching rate, μ_{mf} : viscosity of hybrid nanofluid, μ_{nf} : viscosity of nanofluid, v_f : kinetic velocity of base fluid, v_{mf} : kinetic velocity of hybrid nanofluid, ϕ_1 : dimensionless copper volume fraction, ϕ : dimensionless concentration, σ_{nf} : electrical conductivity of nanofluid, M : magnetic parameter, Nu_x : local Nusselt number, Pr : Prandtl number, q_r : radiative heat flux, R : Parameter due to radiation, u, v : velocity components, Ec : Eckert number, T_∞ : ambient temperature, Kr : Chemical reaction parameter, C_f : local skin friction coefficient, Q : heat source/sink parameter, δ_1 : temperature difference parameter, ρ_f : density of base fluid, ρ_{mf} : density of hybrid nanofluid, ρ_{nf} : density of nanofluid, $(\rho C_p)_{mf}$: heat capacitance of hybrid nanofluid, $(\rho C_p)_{nf}$: heat capacitance of nanofluid, ϕ_2 : dimensionless aluminum oxide volume fraction, σ_{mf} : electrical conductivity of hybrid nanofluid, θ : Non-dimensional temperature.

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