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On R_i spaces with $\alpha_{(\gamma_{os}, \gamma'_{os})}$-open sets

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Abstract

An innovative collection of *open*$_R$ *sets*$_R$, specifically $\alpha_{\gamma_{os}}$-*open*$_R$ *sets*$_R$ are offered by this paper authors and its rudimentary properties utilizing the respective $\tau_{\gamma_{os}}$-interior^R and $\tau_{\gamma_{os}}$- closure^R operators^R are premeditated in their previous papers. As well, it has been substantiated that the idea of γ_{os}-*open*$_R$ *sets*$_R$ besides $\alpha_{\gamma_{os}}$ - *open*$_R$ *sets*$_R$ are autonomous and each γ_{os}-*open*$_R$ *set*$_R$ is an α_{os} *open*$_R$ *set*$_R$, but the contrary need not be exact. The perceptions of $\alpha_{\gamma_{os}}$ T_i *spaces*$_R$ also their properties are familiarized and recognized the connection among these *spaces*$_R$ by establishing several Theorems, undistinguishable statements, consequences besides corollaries.

Another new collection of *open*$_R$ sets$_R$ namely $\alpha_{(\gamma_{os}, \gamma'_{os})}$-open$_R$ *sets*$_R$was introduced and their properties were presented with the corresponding interior^R and closure^R operators. Further the idea of $\alpha_{(\gamma_{os}, \gamma'_{os})}$ T_i *spaces*$_R$ were adapted and documented the connection among these spaces by providing several theorems.

The theory of $\alpha_{(\gamma_{os}, \gamma'_{os})}$ R_i *spaces*$_R$ have been familiarized and their descriptions have been premeditated by illustrating $\tau_{\alpha(\gamma_{os}, \gamma'_{os})}$ -kernel and several statements comprehend to that idea. Also the association amongst these $\alpha_{(\gamma_{os}, \gamma'_{os})}$ R_i *spaces*$_R$ have been formed and evaluated by means of several axioms and opposite affirmations.

Subject Classification: *54A05, 54A10.*

Keywords: α_{γ_o}-*open*$_R$ *sets*$_R$, τ_{γ_o}- *interior*^{R}, τ_{γ_o}- *closure*^{R} , α_{γ_o} T_i *spaces*$_R$ *(i = 0, 1/2), 1, 2)*, $\alpha_{(\gamma_{os}, \gamma'_{os})}$-*open set*, $\tau_{(\gamma_{os}, \gamma'_{os})}$- *interior*^{R}, $\tau_{(\gamma_{os}, \gamma'_{os})}$-*cl* ,$\alpha_{(\gamma_{os}, \gamma'_{os})}$ R_i *spaces*$_R$ *(i = 0, 1, 2).*

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1. Introduction

The present paper is dedicated to familiarize the synergy between R_0 and R_1 spaces$_{\mathrm{R}}$,which build on $\alpha_{\gamma_{os}}$-open$_{\mathrm{R}}$ sets$_{\mathrm{R}}$, $\alpha_{(\gamma_{os},\gamma'_{os})}$-open$_{\mathrm{R}}$ sets$_{\mathrm{R}}$, and discover their vital properties, association and characterizations.

The numerous themes which are mentioned in the present paper are o-open, c-closed ops-open set, opss-open sets, cls-closed set, clss- closed sets, TO -topology, ope-operation, ma-mapping, pos-power set, st-such that, subs-subset, TOS-Topological space$_{\mathrm{R}}$, TOSS-topological spaces$_{\mathrm{R}}$, γ_O-γ, X_{TO} represents X.

Njastad [1] familiarized α-oss in a TOS. The semi- opss and pre-opss were examined individually by Levine [2], Corson and Michael [3], Andrijevic [4] originated a novel type of TO created by pre-opss. Kasahara [5] illustrated the knowledge of ope on TOSS besides invented α-closed graphs of an ops.

Ogata [6] registered the ope α as γ_{os} ope and introduced the understanding of $\tau_{\gamma_{os}}$ that is the stack of all γ_{os}-opss in a TOS (X_{TO}, τ_O). Krishnan [7] and Krishnan and Balachandran [8] declared γ_{os}-semi-opss and γ_{os}-pre-opss. Subsequently, Kalaivani and Krishnan [9] and Kalaivani and Saravanakumar [10, 11] projected $\alpha_{\gamma_{os}}$-opss, $\alpha_{(\gamma_{os},\gamma'_{os})}$-opss that are created through α-opss and studied some of its rudimentary properties. The concept of $\tau_{\alpha_{\gamma_{os}}}$- which is the assortment of $\alpha_{\gamma_{os}}$-opss in a TOS was familiarized. Also, the perceptions of $\tau_{\alpha_{\gamma_{os}}}$- interior$^{\mathrm{R}}$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- interior$^{\mathrm{R}}$ besides $\tau_{\alpha_{\gamma_{os}}}$- closure$^{\mathrm{R}}$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- closure$^{\mathrm{R}}$ operators were familiarized then their properties were premeditated.The thoughts of α-ops in addition $\alpha_{\gamma_{os}}$-opss was independent was demonstrated. Also, each γ_{os}-ops was an $\alpha_{\gamma_{os}}$-opss, but the contrary need not be exact was demonstrated. Correspondingly, some properties of the TO engendered by $\alpha_{\gamma_{os}}$-opss was considered.

Recent works in the Journal of Interdisciplinary Mathematics have significantly contributed to this area. Kalaivani et al. [12] introduced α-γ-compact, regular, and normal spaces, establishing important relationships between generalized open sets and classical topological properties. Nasef, El-Maghrabi, and El-Monsef [13]–[15] further extended these ideas by incorporating grill and nano topological structures, introducing concepts such as grill bi-open sets, generalized α-sets, and semi-open sets in grill nano topology. These studies emphasize the growing importance of combining multiple operators and structures to develop more flexible and comprehensive topological frameworks.

However, despite these advancements, the study of hybrid open sets involving multiple operators such as $\alpha_{(\gamma_{os},\gamma'_{os})}$-open sets and their applications in defining $\alpha_{(\gamma_{os},\gamma'_{os})}T_i$ spaces$_{\mathrm{R}}$ remains largely unexplored. This motivates the present work to investigate such hybrid structures and their implications in generalized topology.

In section 4 the notion of $\alpha_{(\gamma_{os},\gamma'_{os})}R_i$ spaces_R (i = 0,1,2) are familiarized. Also, their properties as well the association amongst them are examined.

2 Prologue

The essential expressions and maxims linked to the present paper are explored from the reference papers [1]-[10].

A subs D of X_{TO} is thought to be an $\alpha_{\gamma_{OS}}$-ops [9] with the condition that D $\subseteq \tau_{\gamma_{OS}}$- int^R ($\tau_{\gamma_{OS}}$ -cl(τ_{γ_o}- int^R (D))). The $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- interior^R of D [9] is the union of the entire $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -opss enclosed in D and it is expressed by $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- int^R (D). Specifically $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- int^R (D) = {W: W is an $\alpha_{\gamma_{OS}}$ops and W $\subseteq$ D}. The $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- closure^R of D [9] is the intersection of the complete $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -clss subsisting of D and it is presaged by $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl^R (D). Specifically $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl^R(D) = $\left\{\text{Q: Q is a } \tau_{\alpha(\gamma_{os},\gamma'_{os})}\text{cls and D} \subseteq \text{Q}\right\}$.

3. Preliminaries Discussion about the novel $\alpha_{(\gamma_{os},\gamma'_{os})}T_i$ $spaces_R$

We introduce the general operation approaches on T_i spaces_R where i = 0, $\frac{1}{2}$, 1, 2 . At this time $\gamma_{os}, \gamma'_{os}$ are operations on τ_O.

Definition 3.1: A TOS (X_{TO} , τ_O) is called an $\alpha_{(\gamma_{os},\gamma'_{os})}T_0$ space_R if for respective discrete points f, g $\in X_{TO}$ there occurs an $\alpha_{(\gamma_{os},\gamma'_{os})}$- ops M such that f $\in$ M besides g $\notin$ M or else g $\in$ M and f $\notin$ M.

Definition 3.2: A TOS (X_{TO},τ_O) is called an $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ space_R if for every dissimilar points f, g $\in X_{To}$ there occurs $\alpha_{(\gamma_{os},\gamma'_{os})}$- opss M, N accommodating f and g respectively such that g $\notin$ M and f $\notin$ N.

Definition 3.3: A TOS (X_{TO},τ_O) is called an $\alpha_{(\gamma_{os},\gamma'_{os})}T_2$ space_R if for each distinct points f, g $\in X_{TO}$ there exists $\alpha_{(\gamma_{os},\gamma'_{os})}$-opss M, N alike that f $\in$ M, g $\in$ N besides M $\cap$ N $= \phi$.

Definition 3.4: A subs P of X_{TO} is named as $\alpha_{(\gamma_{os},\gamma'_{os})}$ g-cls if $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl(P) $\subseteq$ Q whensoever P $\subseteq$ Q besides Q is an $\alpha_{(\gamma_{os},\gamma'_{os})}$- ops in ($X_{TO}$,$\tau_O$).

Remark 3.1: From the above 3.3 Definition, every $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls in ($X_{TO}$,$\tau_O$) is an $\alpha_{(\gamma_{os},\gamma'_{os})}$ g- cls. However, the contrary need not be factual.

Definition 3.5: A TOS (X_{TO} , τ_O) is named an $\alpha_{(\gamma_{os},\gamma'_{os})}T_{\frac{1}{2}}$ space_R if every $\alpha_{(\gamma_{os},\gamma'_{os})}$ g-cls in (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls.

Theorem 3.1: *A subs S^R of X_{TO} is aforesaid to be an $\alpha_{(\gamma_{os},\gamma'_{os})}$ g-cls if and only if $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-$cl(\{a\}) \cap S^R \neq \phi$ holds for every $a \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-$cl(S^R)$.*

Theorem 3.2: *A subs Q of X_{TO} is forenamed to be an $\alpha_{(\gamma_{os},\gamma'_{os})}$ g- cls, then $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- cl^R (Q)$-$Q does not encompass a non-empty $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls.*

Proof: Proof can be referred from [10]

Theorem 3.3: *For each $d \in X_{TO}$, $\{d\}$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls or $X_{TO} - \{d\}$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$ g-cls in (X_{TO},τ_O).*

Theorem 3.4: *A TOS (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_{\frac{1}{2}}$ $space_R$ if and only if is $\{a\}$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls or an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops in ($X_{TO}$,$\tau_O$).*

Theorem 3.5: *A TOS (X_{TO}, τ_O) an $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ $space_R$ if and only if any singleton of X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls.*

Proof: The Proof of the above Theorems 3.1, 3.2, 3.3 ,3.4 and 3.5 can be referred from the reference paper [10].

4. Characterizations of $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ and $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ Spaces_R

Definition 4.1: Let (X_{TO},τ_O) be a TOS. B be a subs of X_{TO}. The $\alpha_{(\gamma_{os},\gamma'_{os})}$-kernel of B is defined as the intersection of the entire $\alpha_{(\gamma_{os},\gamma'_{os})}$-opss that contains B and it is expressed as $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(B). That is $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(B)= $\cap$ {F: F $\in$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$: B $\subseteq$ F}.

Theorem 4.1: *Let (X_{TO},τ_O) be a TOS, C^R and ð^R be subsets of X_{TO}, then*

(i) c$\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R ($C^R$) if and only if C^R $\cap$ F $\neq \phi$ for an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls F that contains c.

(ii) C^R $\subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R ($C^R$) and C^R = $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R ($C^R$) if C^R is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops.

(iii) If $C^R \subseteq D^R$, formerly $\tau_{\alpha_{\gamma_{OS}}}$-ker^R($C^R$) $\subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R ($D^R$).

Proof: (i) Let F_0 be the set of all c $\in$ X_{TO} such that F $\cap$ $C^R \neq \phi$ for every $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls F and c $\in$ F. To establish the statement it is sufficient to explain that $F_0 = \tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R ($C^R$). Let c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R ($C^R$). Let us take up that c $\notin F_0$ then there occurs a $\alpha_{(\gamma_{os},\gamma'_{os})}$- cls U_c of c with the condition that $U_c \cap C^R = \phi$. This implies that $C^R \subseteq X_{TO} - U_c$ and hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R

$(C^R) \subseteq X_{TO} - U_c$. Therefore, c $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ - ker^R (C^R) which is an inconsistent statement henceforward $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R $(C^R) \subseteq F_0$.

Conversely, let F_1 be a set such that $C^R \subseteq F_1$ and $(X_{TO} - F_1)$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls. Let c $\notin F_1$ then c $\in (X_{TO} - F_1)$ and $(X_{TO} - F_1) \cap C^R = \phi$. This implies c $\notin F_0$. Therefore $F_0 \subseteq F_1$. Hence $F_0 \subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$- ker^R (C^R). Hence the proof.

(ii) as well (iii) The validation trails after the 4.1. Definition.

Theorem 4.2: *Let (X_{TO},τ_O)be a TOS. Assume that e,f as dual different points in X_{TO}. Formerly y $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{e\}$) if and only if e $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{f\}$).*

Proof: Let f $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker ({e}). If e $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({f}), then e $\notin \cap \left\{F_1: X_{TO} - F_1 \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}\right\}$ and $\{f\} \subseteq F_1$ implies that e does not belong to at least one F_1, say F_0 such that e $\in X_{TO} - F_0$ and f $\notin X_{TO} - F_0$. Therefore f $\notin \cap \{X_{TO} - F_1: X_{TO} - F_1 \in \tau_{\alpha_{\gamma_O}}\}$ and $\{e\} \subseteq X_{TO} - F_1$ and hence f $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({e}), which is a contradiction. Thus e $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl ({f}).

Conversely, Let e $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-*cl* ({f}). If f $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{e\}$), then f $\notin \cap \left\{U_c: U_c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}\right\}$ and $\{e\} \subseteq U_c$ implies that f $\in X_{TO} - U_c$ and e $\notin X_{TO} - U_c$. Therefore e $\notin \cap \left\{X_{TO} - U_c: U_c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}\}\right\}$ and $\{f\} \subseteq X_{TO} - U_c$ and hence $\{e\} \notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl ({f}), a contradiction. Thus, f $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({e}).

Theorem 4.3: *Let H be a sub of X_{TO}. Then $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(H) $= \{e \in X_{TO}: \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{e\}) \cap H \neq \phi\}$.*

Proof: Let e $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(H). If $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({e}) $\cap$ H $= \phi$, then *e* $\notin X_{TO} - \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-*cl(*$\{e\}$), which is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-os containing H. Thus e $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(H), a contradiction. Hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({e}) $\cap$ H $\neq \phi$.

Contrarywise, let e $\in X_{TO}$ with the condition that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({e}) $\cap$ H $\neq \phi$. If probable, let e $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(H). Formerly there occurs a $U_c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$alike that e $\notin U_c$ and H $\subseteq U_c$. Let f $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({e}) $\cap$ H. Then f $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({e}) besides f $\in U_c$, which offers e $\in U_c$, an inconsistent statement. Hence forth e $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker(H).

Definition 4.2: A TOS (X_{TO},τ_O) is said to be an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space if for each $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops U, c $\in$ H implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) $\subseteq$ H.

Example 4.1: Let $X_{TO} = \{l, m, n\}$, $\tau = P(X)$, illustrate an ope γ_{OS} on τ_O

$$D^{\gamma_o} = \begin{cases} D \cup \{c\} \text{if } D = \{l\} \text{or} \{m\} \\ D \cup \{a\} \text{if } D = \{n\} \\ D \text{ if } D \neq \{l\}, \{m\} \text{ and } \{n\} \end{cases} \quad \text{besides } D^{\gamma_o'} = \begin{cases} D \cup \{a\} \text{ if } D = \{l\} \text{or} \{m\} \\ D \cup \{c\} \text{ if } D = \{n\} \\ D \text{ if } D \neq \{l\}, \{m\} \text{ and } \{n\} \end{cases}$$

Then $\tau_{(\gamma_{os},\gamma'_{os})} = \{\phi, X_T, \{l,m\}, \{l,n\}, \{m,n\}\}$, and $\tau_{\alpha(\gamma_{os},\gamma'_{os})} = \{\phi, X_T, \{l,m\}, \{l,n\}, \{m,n\}\}$. Then (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space.

Theorem 4.4: *Let (X_{TO},τ_O)be a TOS. Formerly the proclamations specified underneath are comparable:*

(i) *(X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ $space^R$;*

(ii) *Let M^R be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls besides a point $m \notin M^R$. There exists an $N^R \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ such that $m \notin N^R$ and $M^R \subseteq N^R$;*

(iii) *If M^R is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls and a point $c \notin M^R$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-$cl\{c\} \cap M^R = \phi$.*

Proof: (i) ⇒ (ii) Let M^R be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls and $m \notin M^R$. By (i), $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) \subseteq X_{TO} - M^R$. Let $N^R = X_{TO} - \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\})$. Then $N^R \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ and also $M^R \subseteq N^R$ and $m \notin N^R$.

(ii) ⇒ (iii) Let M^R be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cs besides a point $m \notin$ F. Presume the specified condition holds. Formerly by (ii), there occurs $N^R \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ with a condition that $m \notin N^R$ and $M^R \subseteq N^R$. Thus $m \in X_{TO} - N^R \subseteq X_{TO} - M^R$. Henceforth $X_{TO} - N^R$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls containing m, we have $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) \cap (X_{TO} - N^R) \neq \phi$ implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) \subseteq X_{TO} - N^R$. Therefore $N^R \cap \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) = \phi$ and hence $M^R \cap \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) = \phi$.

(iii) ⇒ (i) Let $P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ and $m \in P$. Then $X_{TO} - P$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls and $m \notin X_{TO} - P$. By (iii), $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) \cap (X_{TO} - P) = \phi$ implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{m\}) \subseteq P$. Hence (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space.

Theorem 4.5: *Let (X_{TO},τ_O) be a TOS and c, d be dual points in X_{TO}. Formerly the declarations stated below are comparable:*

{i} (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space

{ii} $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker$(\{c\})$

{iii} $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker$(\{c\})$ if and only if $c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker $(\{d\})$

{iv} $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\})$ if and only if $c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$

{v} Let E^R be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls. Consider a point $c \notin E^R$. At that time there occurs a $P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ and $E^R \subseteq P$.

{vi} Each $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls E^R can be stated as
$E^R = \cap \left\{P: P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})} \text{ and } E^R \subseteq P\right\}$.

{vii} Each $\alpha_{(\gamma_{os},\gamma'_{os})}$- ops P, P= $\cup \left\{E^R: X_{TO} - E^R \in \tau_{\alpha(\gamma_{os},\gamma'_{os})} \text{ and } E^R \subseteq P\right\}$

{viii} Let E be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls and c $\notin$ E implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) ∩ E = ϕ.

Proof: {i} ⇒ {ii} By Definition 4.1, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker({c}) =$\cap \left\{P: P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})} \text{and } \{x\} \subseteq U\right\}$. Then by (i), $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({c}) ⊆ P. Hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) ⊆ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({c}).

{ii} ⇒ {iii} If d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker({c}) , then by Theorem 4.2, c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({d}). By (ii) c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker({d}). Similarly, if c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({d}), then d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({c}).

{iii} ⇒ {iv} If d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({c}), then by Theorem 4.2, c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}). By (iii) d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({c}). Again by Theorem 4.2, d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}). Similarly, if c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}), then d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}).

$\{iv\} \Rightarrow \{v\}$ Let E be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls besides a point c $\notin$ E. At that moment for any d ∈ E,$\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}) ⊆ E and so c $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}). By (iv), c $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({d}) implies d $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({c}) , that is there occurs a $P_d \in \tau_{\alpha_{\gamma_o}}$ alike that d $\in P_d$ and c $\notin P_d$. Let P = $\cup_{d\in E} \left\{P_d: P_d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}, d \in P_d \text{and } c \notin P_d\right\}$. Then by Theorem 2.1 [10], P is $\alpha_{(\gamma_{os},\gamma'_{os})}$-os with c $\notin$ P besides E ⊆ P.

{v} ⇒ {vi} Let E^R be an $\alpha_{(\gamma_{os},\gamma'_{os})}$ -cls and H^R = $\cap \left\{P: P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})} \text{and } E^R \subseteq P\right\}$. $E^R \subseteq H^R$ in addition it remains to illustrate that $H^R \subseteq E^R$. Let $c \notin E^R$. Formerly through (v), there occurs an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops alike that $c \notin$ P and $E^R \subseteq$ P then henceforth $c \notin H^R$. Consequently, every individul $\alpha_{(\gamma_{os},\gamma'_{os})}$ -cls E^R can be expressed as E^R = $\cap \left\{P: P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})} \textit{and } E^R \subseteq P\right\}$.

$\{vi\} \Rightarrow \{vii\}$ Trails after the Definitions 3.16 and 3.18 [10].

$\{vii\} \Rightarrow \{viii\}$ Let E^R be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls and $c \notin E^R$. Then $X_{TO} - E^R$ = P is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops containing c. Then by $\{vii\}$, P can be written as the union of $\alpha_{(\gamma_{os},\gamma'_{os})}$-clss and so there is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls H^R such that c $\in E^R \subseteq$ P and $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) ⊆ P. Thus $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) ∩ $E^R = \phi$.

$\{viii\} \Rightarrow \{i\}$ Let P be an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops and c ∈ P. Take $E^R = X_{TO} - P$. Formerly by means of (viii), there occurs an $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls E^R alike that c $\in E^R \subseteq$ P in addition $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl ($\{c\}$) ∩ $E^R \neq$ ϕ. Therefore

$\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\subseteq E^R$ and hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\subseteq P$. Thus X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space^R.

Theorem 4.6 For a TOS the specified declarations are alike:

(i) (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ $space^R$.

(ii) For any two distinct points c, d of X_{TO}, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $=$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) or $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) $= \phi$.

Proof: (i) $\Rightarrow$ (ii) Let X_{TO} be an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) and $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) $\neq \phi$. Let s $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) and c $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$). Then c $\in X_{TO} - \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$. But $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\notin X_{TO} -$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$), since s $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$), which is a incongruity to the hypothesis that X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space. Hence either $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $=$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) or $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl ($\{d\}$) $= \phi$.

(ii) $\Rightarrow$ (i) Let W $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ with c $\in$ W. To show that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\subseteq$ W. Let d $\notin$ W.Then d $\neq$ c and c $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$).Thus $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) .Hence by (ii), $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) $= \phi$ for each d $\notin$ W and hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $[\bigcup_{d \in X_{TO}-W} \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$)$] = \phi$. Again, since W $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ and d $\in X_{TO} -$ W, d $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) $\subseteq X_{TO} -$ W. Thus $X_{TO} - W =$ $\bigcup_{d \in X_{TO}-W} \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$). Therefore ($X_{TO} -$ W) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $= \phi$ and hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\subseteq$ W.

Corollary 4.1: *A TOS (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ $space^R$ if and only if for any points $c, d \in X_{TO}$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$) $= \phi$.*

Proof: Follows from the Theorem 4.6.

Theorem 4.7: *Let c, d be any two distinct points in a TOS X_{TO}. Formerly these announcements are corresponding:*

{i} $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{c\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{d\}$);

{ii} $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$).

Proof: {i} $\Rightarrow$ {ii} Let $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{c\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{d\}$). Then there prevails z $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{d\}$)(say). By Theorem 4.2, c $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$)

and d $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({z}).As $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) $\subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({z}), we have d $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}). Hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}).

{ii} ⇒ {i} Let $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}). Then there occurs z $\in X_{TO}$,and z $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d})(say), which indicates that there ensues a M^R $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ like that z $\in$ M^R , d $\notin$ M and c $\in$ M^R infers that d $\notin$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker({c}). Hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}).

Theorem 4.8: *A TOS (X_{TO}, τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space in case that for respective p,q $\in X_{TO}$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$) implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$) $= \phi$.*

Proof: Let X_{TO} be an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space and $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$) where p,q $\in X_{TO}$. Suppose that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{d\}$) $\neq \phi$. Let s $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$). Then s $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) and s $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$). By Theorem 4.5(iii), p $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{s\}$) and q $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{s\}$). Hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\subseteq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{s\}$) $\subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{q\}$) and $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{q\}$) $\subseteq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{s\}$) $\subseteq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{p\}$) implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{p\}$) $=$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{q\}$), a contradiction. Hence $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$) $= \phi$.

Contrariwise, suppose that for individual p,q $\in X_{TO}$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\neq$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$) implies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{q\}$) $= \phi$. If $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl($\{p\}$) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl($\{q\}$) then by Theorem 4.7, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{p\}$) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{q\}$) . Then by the given condition $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{q\}$) $= \phi$ which implies $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{p\}$) $\neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{q\}$) $= \phi$. Because if z $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl($\{q\}$) by Theorem 4.2, p $\in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{z\}$), implies $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{z\}$) $\neq \phi$ and similarly $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ker($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{z\}$) $\neq \phi$. So by hypothesis $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{p\}$) $= \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{z\}$) $= \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -ker($\{q\}$), which is a contradiction. Thus $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{p\}$) $\cap$ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{q\}$) $= \phi$ and by 4.1. Corollary, the TOS X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ $space^R$.

Theorem 4.9: *In a TOS (X_{TO}, τ_O), the announcements specified below are corresponding:*

(i) *(X_{TO}, τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ space;*

(ii) *Let c be any point in (X_{TO}, τ_O). Then $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $= \{c\}$;*

(iii) (X_{TO}, τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ and $\alpha_{(\gamma_{os},\gamma'_{os})}T_0$ space.

Proof: (i) ⇒ (ii) If $d \notin \{c\}$, then there occurs $c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ like that $d \in P$, $c \notin P$. Consequently $P \cap \{c\} = \phi$ and henceforth $d \notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$).

(ii) ⇒ (iii) Let c, $d \in X_{TO}$ with $c \neq d$. Then $\{c\}$ and $\{d\}$ are $\alpha_{(\gamma_{os},\gamma'_{os})}$-cls and hence $X_{TO} - \{c\}$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops encompassing d but not c viewing X_{TO} to be $\alpha_{(\gamma_{os},\gamma'_{os})}T_0$.Suppose that P is an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops and $c \in$ P. Then by (ii), $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $= \{c\} \subseteq$ P. Hence (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space.

(iii) ⇒ (i) Let c, $d \in X_{TO}$ with $c \neq d$. Then there occurs $P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ alike that $c \in P$ and $d \notin P$ (say), which infers that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\subseteq$ P and so $d \notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$). Hence $c \in P \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$, $d \notin P$ and $d \in X_{TO} - \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $\notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$, $c \notin X_{TO} - \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$). So (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ space.

Definition 4.3: *Let (X_{TO},τ_O) be a TOS and $c \in X_{TO}$. A net $\{c_\alpha : \alpha \in D\}$ in $(X_{TO},\tau_O)$is said to be $\alpha_{(\gamma_{os},\gamma'_{os})}$-convergent to c in X_{TO} if for every $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-ops P encompassing c there occurs $\alpha_0 \in D$ such that $c_\alpha \in P$, for individual $\alpha \geq \alpha_0$.*

Lemma 4.1: *Let p, q be points of a TOS (X_{TO},τ_O) such that every point in (X_{TO},τ_O) converging to q also $\alpha_{(\gamma_{os},\gamma'_{os})}$-converging to p. Then $p \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{q\}$).*

Proof: Supposing that $p_n = q$ for each $n \in N$ (N is the set of natural numbers). Then $\{p_n\}$ is a net in $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{y\}$). By the fact that $\{x_n\}$, $\alpha_{(\gamma_{os},\gamma'_{os})}$-converges to y, $\{x_n\}$ is $\alpha_{(\gamma_{os},\gamma'_{os})}$ converging to p. Thus $p \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{q\}$).

Theorem 4.10: *Aimed at a TOS the statements quantified beneath are equal:*

(i) *(X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ Space*

(ii) *If $c, d \in (X_{TO},\tau_O)$, formerly $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) if and only if every net in (X_{TO},τ_O) $\alpha_{(\gamma_{os},\gamma'_{os})}$-converging to d also $\alpha_{(\gamma_{os},\gamma'_{os})}$-converging to c.*

Proof: (i)⇒(ii) Let $c, d \in X_{TO}$ alike that $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$). Let $\{c_\alpha\}_{\alpha\in D}$ be a net in (X_{TO},τ_O) such that $\{c_\alpha\}_{\alpha\in D}\alpha_{(\gamma_{os},\gamma'_{os})}$-converges to d. Since $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) by Theorem 4.6., $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{c\}$) $= \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$). Therefore $c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$). Thus $\{c_\alpha\}_{\alpha\in D}\alpha_{(\gamma_{os},\gamma'_{os})}$-converging towards c. Contrariwise, let $c, d \in (X_{TO},\tau_O)$ such that every net in (X_{TO},τ_O) $\alpha_{(\gamma_{os},\gamma'_{os})}$-converging to d also $\alpha_{(\gamma_{os},\gamma'_{os})}$-convergence in the direction of c.

Then by Lemma 4.1, $c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl($\{d\}$). Thus by Theorem 4.6, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) = $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}).Thus $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}).

(ii)⇒(i) Let c, d be points of (X_{TO}, τ_O) like that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) ∩ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}) ≠ ϕ. Specified Theorem 4.6, it is appropriate to show that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl($\{c\}$) = $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({d}). Let z ∈ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({c}) ∩ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}). So there occurs a net $\{c_\alpha\}_{\alpha\in D}$ in $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) such that $\{c_\alpha\}_{\alpha\in D}$ converges to z. Subsequently z ∈ $\tau_{\alpha_{\gamma_o}}$-cl({d}), $\{c_\alpha\}_{\alpha\in D}$ congregates to d.Hence by (ii) z ∈ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}) ⇒ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({z}) ⊆ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}) (1). By Lemma 4.1, d ∈ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({z}) implies $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({d}) ⊆ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({z}) (2). Hence by (1) and (2), $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({d}) = $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({z}). Similarly, it can be shown that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({c}) = $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({z}) by taking the net in $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({d}). So $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -cl({c}) = $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}).

Definition 4.4: A TOS (X_{TO},τ_O) is said to be an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ space if for c,d ∈ X_T,$\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) ≠ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}), there exists P, Q ∈ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ such that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({c}) ⊆ P and $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({d}) ⊆ Q and P ∩ Q = ϕ.

Example 4.2: Let $X_{TO} = \{f, g, h\}$, $\tau_O = \{\phi, X_{TO}, \{f\}, \{g\}, \{f, g\}\}$, define γ_o on τ_O alike that γ_o(D)= cl(D) for every D ∈ τ_O. Formerly (X_{TO} , τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ space^R.

Theorem 4.11: *If (X_{TO},τ_O) is an $\alpha_{\gamma_o}R_1$ space, then it is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0 space^R$.*

Proof: Let f, g be discrete points in X_{TO}.Formerly there occurs an $\alpha_{(\gamma_{os},\gamma'_{os})}$-ops M^R with the condition that f ∈ M^R besides g ∉ M^R. This indicates that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl ({f}) ≠ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ - cl^R({g}). Since X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ space signifies that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl({f}) ∩ $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-$cl^R$({g}) = Null set. Thereafter by 4.6 Theorem policies, (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ space^R.

Remark 4.1: The transposal of the directly above doctrine 4.10 need not be precise commonly.

Theorem 4.12: *In a TOS (X_{TO},τ_O), these declarations are equal:*

(i) (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_2$ space^R

(ii) (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ and $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ space^R

(iii) (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ and $\alpha_{(\gamma_{os},\gamma'_{os})}T_0$ space^R

Proof: (i) ⇒ (ii) Let (X_{TO},τ_O) be an $\alpha_{(\gamma_{os},\gamma'_{os})}T_2$ space. Then (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ space. If $c,d \in (X_{TO},\tau_O)$ with $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$, then $c \neq d$ and so there exists $P, Q \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ such that $c \in P$, $d \in Q$ and $P \cap Q = \phi$. Hence by Theorem 4.8, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{x\}) = \{x\} \subseteq U$ and $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\}) = \{d\} \subseteq Q$ and $P \cap Q = \phi$. Then $((X_{TO},\tau_O)$ is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ space^R.

(ii) ⇒ (iii) The validation foollows after the Definitions 3.1 and 3.2.

(iii) ⇒ (i) Let (X_{TO},τ_O)be an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ and $\alpha_{(\gamma_{os},\gamma'_{os})}T_0$ space. Then by Theorem 4.9, (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_0$ and $\alpha_{(\gamma_{os},\gamma'_{os})}T_0$ space. By Theorem 4.9, (X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_1$ space. Let c, $d \in X_{TO}$ through $c \neq d$. Formerly $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) = \{c\} \neq \{d\} = \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$. As X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ space, there exists P, $Q \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ such that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) = \{c\} \subseteq P$, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\}) = \{d\} \subseteq Q$ and $P \cap Q = \phi$. Thus X_{TO} is an $\alpha_{(\gamma_{os},\gamma'_{os})}T_2$ space.

Theorem 4.13: *In a TOS (X_{TO},τ_O), the statements quantified below are comparable:*

(i) *(X_{TO},τ_O) is an $\alpha_{(\gamma_{os},\gamma'_{os})}R_1$ Space;*

(ii) *If c,d are dissimilar points $\in (X_{TO},\tau_O)$, at that time, one of the ensuing holds;*

  (a) *For $B^R \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}, c, \in B^R$ if and only if $d \in B^R$;*

  (b) *there exists $B^R, Q \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$ such that $c \in B^R$ and $d \in Q$ and $B^R \cap Q = \phi$;*

(iii) *If $c,d \in$ such that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$, then there exists $\alpha_{(\gamma_{os},\gamma'_{os})}$-clss E_1 and E_2 such that $c \in E_1$, $d \notin E_1$, $d \in E_2$, $c \notin E_2$ and $X_{TO} = E_1 \cup E_2$.*

Proof: (i) ⇒ (ii) Let $c,d \in X_{TO}$. Formerly $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) = \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$ or $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$. Suppose $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) = \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$ and $c \in B^R \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$. This implies that $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\}) = \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \subseteq B^R$. Henceforth $d \in B^R$. Likewise, it can be demonstrated that if $d \in B^R$, at that time $c \in B^R$. Presume $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$. Formerly there occurs B^R, $Q \in \tau_{\alpha_{\gamma_o}}$ alike that $c \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \subseteq B^R$ and $d \in \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\}) \subseteq Q$ and $B^R \cap Q = \phi$.

(ii) ⇒ (iii) Let c, $d \in X_{TO}$ with the condition that $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{c\}) \neq \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$. Then $c \notin \tau_{\alpha(\gamma_{os},\gamma'_{os})}$-cl$(\{d\})$, so that there exists $M^R \in$

$\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$ such that $c \in M^R$ and $d \notin M^R$. Accordingly through (ii) there be present B^R, $Q \in \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$alike that $c \in B^R$ and $d \in Q$ and $B^R \cap Q = \phi$. Let $E_1 = X_{TO} - Q$ besides $E_2 = X_{TO} - B^R$. At that time E_1 and E_2 are $\alpha_{(\gamma_{OS},\gamma'_{OS})}$-css in addition $c \in E_1$, $d \notin E_1$, $d \in E_2$, $c \notin E_2$ besides $X_{TO} = E_1 \cup E_2$.

(iii) ⇒ (i) Let $B^R \in \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$ besides $c \in B^R$. Then $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl ({c}) $\subseteq B^R$. Otherwise, there exists $d \in \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl({c}) $\cap (X_{TO} - B^R)$. Then $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl({c}) $\neq \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl({d}) and by (iii), there exists $\alpha_{(\gamma_{OS},\gamma'_{OS})}$-css E_1 and E_2 such that $c \in E_1$, $d \notin E_1$, $d \in E_2$, $c \notin E_2$ and $X_{TO}=E_1 \cup E_2$. Then $d \in E_2 - E_1 = X_{TO} - E_1$ and $c \notin X_{TO} - E_1$, where $X_{TO} - E_1 \in \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$, which is an incongruity to the datum that $d \in \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$- cl^R ({c}). Hence $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$- cl^R ({c}) $\subseteq B^R$. Thus X_{TO} is an $\alpha_{(\gamma_{OS},\gamma'_{OS})}$ R_0. To show X_{TO} to be an $\alpha_{(\gamma_{OS},\gamma'_{OS})}R_1$ space assume that $a_1, b_1 \in X_{TO}$ with $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl($\{a_1\}$) $\neq \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$- cl^R ($\{b_1\}$). Then,there exists $\alpha_{(\gamma_{OS},\gamma'_{OS})}$-clss A_1 and A_2 such that $c \in A_1$, $d \notin A_1$, $d \in A_2$, $c \notin A_2$and $X_{TO} = A_1 \cup A_2$. Thus $a_1 \in A_1 - A_2 \in \tau_{\alpha_{\gamma_O}}$, $b_1 \in A_2 - A_1 \in \tau_{\alpha_{\gamma_O}}$. So $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$ - cl^R ($\{a_1\}$) $\subseteq A_1 - A_2$, $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$ - cl^R($\{b_1\}$) $\subseteq A_2 - A_1$. Thus (X_{TO},τ_O)is an $\alpha_{(\gamma_{OS},\gamma'_{OS})}R_1$ $space^R$.

Theorem 4.14

(i) *$Space^R$ (X_{TO},τ_O) is an $\alpha_{(\gamma_{OS},\gamma'_{OS})}T_2$ $Space^R$ if and only if for c, $d \in X_{TO}$ with $c \neq d$, there occurs an $\alpha_{(\gamma_{OS},\gamma'_{OS})}$-clss E_1 and E_2 such that $c \in E_1$, $d \notin E_1$, $d \in E_2$, $c \notin E_2$ and $X_{TO} = E_1 \cup E_2$.*

(ii) *Space X_{TO} is an $\alpha_{(\gamma_{OS},\gamma'_{OS})}R_1$ space contigent upon c, $d \in X_{TO}$ with $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-ker($\{c\}$) $\neq \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-ker($\{d\}$), there exists B^R, $Q \in \tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$ such that $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl($\{c\}$) $\subseteq B^R$ and $\tau_{\alpha(\gamma_{OS},\gamma'_{OS})}$-cl($\{d\}$) $\subseteq Q$ and $B^R \cap Q = \phi$.*

Proof:

(i) The proof is evident from the doctrine 4.12 and 4.13.

(ii) The proof is prompt from doctrine 4.7 and Definition 4.4.

5. Conclusion

In this article the idea of $\alpha_{(\gamma_{OS},\gamma'_{OS})}T_i$ spaces that are generated through $\alpha_{(\gamma_{OS},\gamma'_{OS})}$, (γ_O,γ'_{OS}) and α-open sets besides some of their essential properties are used by the authors to propose the insight and credentials of $\alpha_{(\gamma_{OS},\gamma'_{OS})}R_i$ spaces. The $\alpha_{(\gamma_{OS},\gamma'_{OS})}$generalized -closed set($\alpha_{(\gamma_{OS},\gamma'_{OS})}$-gcls) has been demarcated and statements connected are recognized.The idea of $\alpha_{(\gamma_{OS},\gamma'_{OS})}$the convergence of net has been elaborated. The concepts of $\alpha_{(\gamma_{OS},\gamma'_{OS})}R_0$, $\alpha_{(\gamma_{OS},\gamma'_{OS})}R_1$are elaborated using $\alpha_{(\gamma_{os},\gamma'_{os})}$ -closed sets, $\alpha_{(\gamma_{os},\gamma'_{os})}$ -open sets and their corresponding $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -closure, $\tau_{\alpha(\gamma_{os},\gamma'_{os})}$ -interior operators. Besides the relationship among them have been built and their nature have been learnt efficiently.

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