

Nonlinear studies on complex modified trapped Korteweg-de Vries burgers' equation

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Abstract

In this paper, the effect of electrons and positrons trapping on positive and negative ions are analysed in collision-less magnetised system. Relying on the widely used reductive perturbation technique (RPT), a three-dimensional (3D) modified trapped Korteweg-de Vries Burgers' (mTKdVB) equation is governed due to the presence of trapped positrons, trapped electrons, and both kinematic as well as bulk viscosity. This study examines the influence of different parameters, such as negative ion number densities, electron ion densities, and positron ion density on dynamical and structural properties. Also, the impact of the viscosity coefficient and magnetised parameter of the small-amplitude shock waves has been examined. The current theoretical research may be helpful to understand the shock

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wave structure in space environments related to astrophysics such as white dwarfs, neutron stars etc.

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1. Introduction

Research on nonlinear studies has gained attention from physicists and applied mathematicians because to its broad implications in natural phenomena and astronomical space environments. Plasma can generate numerous types of nonlinear waves and structures when different properties are appropriately balanced. Therefore, a range of nonlinear wave structures can help in better understanding of these plasma behaviours. In various plasma the behaviours of nonlinear wave such as solitary waves, shock waves, rough waves, double layers and wave modulations have extensively investigated by many researchers [1-10]. Ongoing research focuses on the characteristics of the nonlinear wave processes in plasmas with ions having opposite charge polarity. Comprehending shock wave structures is crucial for grasping various astrophysical events since shock waves are believed to accelerate the speed of charged particles in the universe, contributing to the formation of cosmic rays.

Mace and Hellberg [11] performed an empirical investigation on development of shock waves. These wave structures depend on nonlinearity and dissipation of the system. This structure is generated through many factors such as wave particles, landau damping and viscosity. The behaviour of a nonlinear wave in a medium with dissipation and dispersion can be accurately described by the Korteweg-de Vries Burger (KdVB) equation. The shock wave is produced due to the Burger term. During problematic conditions, the Burger term in the KdVB equation decreases to a very tiny amount. The KdVB equation can be converted into the Korteweg-de Vries (KdV) equation, which allows for the existence of solitons. Studying the KdVB equation in various plasma settings becomes more interesting. The KdVB equations were used to investigate shock wave production in various plasma settings by a group of researchers [12–15]. Saha and Saeed [16] investigated ion-acoustic shock waves (IASWs) in an electron-positron-ion (EPI) plasma by establishing the KdVB equation. They noticed that steepness (amplitude) of the

IAShWs becomes less with increased positron density. However, it does so in direct correlation with the dissipation coefficient occurs due to kinematic viscosity. The plasma consists of negatively charged Maxwellian ions that have trapped electrons, as well as positively charged and nonthermality of ions. In a multicomponent electron-ion plasma performed a theoretical investigation on IAShWs and ion-acoustic solitary waves (IASWs) [17]. The investigations derived the KdVB equation and observed that the amplitudes of both waves rise with negative ion concentrations as well as the temperature ratios, but dropped with rising ion concentration. In a collisionless dusty plasma, the impact of temperature ratio on shock wave propagation in heavy negative ions and cold positive ions are studied by Dev et al. [18]. Considering the effect of dissipation and dispersion, Masood et al. [19] investigated on IAShWs phenomenon in plasmas having trapped ions and electrons. They reached the KdVB equation, which included terms representing fractional nonlinearity. In a plasma system containing trapped electrons both solitary and shock waves is observed in magnetized plasma by Orani, Akter, and Mamun [20]. In their observation they conclude that the amplitude expanded by raising the strength of magnetic field and trapped electrons. Guo et al. [21] studied the influence of electrons and viscosity on the profile of IAShWs. They investigate that the amplitude of the wave profile increases by rising viscosity of the medium, while the width of the medium remains same. Adhikary et al. [22] has recently examined the attributes of dust acoustic shock waves in a uniform, non-magnetized, collisionless dusty plasma that includes positive and negative ions. The width of the shock profile is influenced by the coefficient of viscosity is observed when phase speeds are constant. Hussain, Rehman, and Mahmood [23] examined the impact of a dense magnetic field and viscosity on shock waves when a trapping effect is present. The increase in viscosity increases the strength of the shock wave. In a study Yeashna et al. [24] shown that the viscosity of the medium does not affect the amplitude of the IAShWs profile. However, the steepness of the shock wave profile decreases. Berbri, Younsi, and Laghbeche [25] investigated the effect of magnetic field on magnetized dusty plasma. They observed that the amplitude of the IAShWs profile grows as the influence of magnetic field increases. Willington and Venugopal [26] studied the development of IAShWs in a magnetised plasma with anisotropy. They found that the magnetic field significantly influenced the breadth and propagation of the IAShWs. Higher magnetic field leads to decreased shock wave diameter. In a magnetised dissipative plasma, a theoretical investigation on periodic, solitary and shock waves are

observed by Ozah and Deka [27]. In their observation the shock profiles' amplitude remains steady, but their width fluctuates due to changes in the viscosity of ions. EL-Moonier et al. [28] investigated the amplitude and width of the oscillatory shock wave structure in magnetized plasma, and they concluded that due to the existence of a restoring force, the amplitude and width of the wave decrease.

To our knowledge, there has been no research conducted on the complex mTKdVB equation in pair of ions with trapped electrons and positrons, considering both kinematic and bulk viscosity. This study focuses on magnetized collisionless with kinematic and bulk viscosity, resulting in the development of the complex mTKdVB equation. The presence of dissipation terms in this equation leads to the development of shock waves.

2. Theoretical Model

In the present plasma model, we study the IAShWs in a 3D plasma having positive ion and negative ions along with electron and positron is considered. Without violating the charge neutrality, we have $\mu_m + \mu_e = 1 + \mu_p$, where $\mu_m = n_{-0}/n_{+0}$, $\mu_e = n_{e0}/n_{+0}$ and $\mu_p = n_{p0}/n_{+0}$ are the ratios of negative ion, electron and positron density normalised with their equilibrium values. The fluid dynamics of ions governed by continuity and momentum equations ions are

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \vec{v}_j) = 0 \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \vec{v}_j \cdot \nabla \right) \vec{v}_j = \frac{Z_j e}{m_j} (\alpha \vec{\nabla} \phi + \vec{V}_j \times B) - \frac{1}{m_j n_j} \nabla P_j + \eta_j \vec{\nabla}^2 \vec{v}_j + (\eta_j + \mu_j) \vec{\nabla} (\vec{\nabla} \cdot \vec{v}_j) \quad (2)$$

Overall system is closed by the Poisson's equation as

$$\nabla^2 \phi = 4\pi e (n_e + Z_- n_- - Z_+ n_+ - n_p) \quad (3)$$

In the above equation, n_j ($j = \pm$) stands for concentration of ion species, v_j stands ion fluid velocity, m_j for mass and P_j stands for pressure of ions respectively; e is the electric charge and Z_j is the charged state of ions. The kinematic and bulk viscosities of ions are denoted by η_j and μ_j respectively. The adiabatic state for momentum equation i.e. for Eq. (2) $P_j / P_{j0} = (n_j / n_{j0})^\gamma$ and $P_{j0} = n_{j0} K_B T_e$. γ (adiabatic index), which describes

how pressure changes with density in the plasma and defined as $\Upsilon = (2 + D_f / 3) = 5/3$, where D_f is the degrees of freedom of the system. Here, $\alpha = +1$ (-1) for positive (negative) ions.

The physical quantities electrostatic potential function scaled by Boltzmann constant $\phi = e\phi / K_B T_e$, the density of ions scaled by equilibrium value n_{j0} as $N_j = n_j / n_{j0}$ and the ion fluid velocity with acoustic speed (C_s) along x, y and z direction is scaled by $V_j = v_j / C_s$. The space variable (x) is scaled by Debye length $\lambda_D (= (K_B T_e / 4\pi n_e e^2)^{1/2})$ as X / λ_D , time variable (t) scaled by wave plasma frequency $\omega_{pi} (= \sqrt{4\pi n_{+0} e^2 / m_+})$ as T / ω_{pi} and the induced magnetic field is given by $\Omega_j = Z_j e B \lambda_D / m_j C_s$. So, using Eqs. (1) – (3), we get the normalized equations as follows

$$\frac{\partial N_j}{\partial T} + \frac{\partial(N_j V_{jx})}{\partial X} + \frac{\partial(N_j V_{jy})}{\partial Y} + \frac{\partial(N_j V_{jz})}{\partial Z} = 0 \quad (4)$$

$$\left. \begin{aligned} \frac{\partial V_{jx}}{\partial T} + \left(V_{jx} \frac{\partial}{\partial X} + V_{jy} \frac{\partial}{\partial Y} + V_{jz} \frac{\partial}{\partial Z} \right) V_{jx} &= \alpha \gamma Z_j \frac{\partial \phi}{\partial X} + (\vec{V}_{jx} \times \Omega_-) - \frac{5}{3} \gamma \sigma_j \frac{1}{N_j^{1/3}} \frac{\partial N_j}{\partial X} \\ + \eta'_j \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right) (V_{jx}) &+ (\eta'_j + \mu'_j) \left(\frac{\partial^2 V_{jx}}{\partial X^2} + \frac{\partial^2 V_{jy}}{\partial X \partial Y} + \frac{\partial^2 V_{jz}}{\partial X \partial Z} \right) \end{aligned} \right\} \quad (5)$$

$$\left. \begin{aligned} \frac{\partial V_{jy}}{\partial T} + \left(V_{jx} \frac{\partial}{\partial X} + V_{jy} \frac{\partial}{\partial Y} + V_{jz} \frac{\partial}{\partial Z} \right) V_{jy} &= \alpha \gamma Z_j \frac{\partial \phi}{\partial Y} + (\vec{V}_{jy} \times \Omega_-) - \frac{5}{3} \gamma \sigma_j \frac{1}{N_j^{1/3}} \frac{\partial N_j}{\partial Y} + \\ \eta'_j \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right) (V_{jy}) &+ (\eta'_j + \mu'_j) \left(\frac{\partial^2 V_{jx}}{\partial Y \partial X} + \frac{\partial^2 V_{jy}}{\partial Y^2} + \frac{\partial^2 V_{jz}}{\partial Y \partial Z} \right) \end{aligned} \right\} \quad (6)$$

$$\left. \begin{aligned} \frac{\partial V_{jz}}{\partial T} + \left(V_{jx} \frac{\partial}{\partial X} + V_{jy} \frac{\partial}{\partial Y} + V_{jz} \frac{\partial}{\partial Z} \right) V_{jz} &= \alpha \gamma Z_j \frac{\partial \phi}{\partial Z} + (\vec{V}_{jz} \times \Omega_-) - \frac{5}{3} \gamma \sigma_j \frac{1}{N_j^{1/3}} \frac{\partial N_j}{\partial Z} + \\ \eta'_j \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right) (V_{jz}) &+ (\eta'_j + \mu'_j) \left(\frac{\partial^2 V_{jx}}{\partial Z \partial X} + \frac{\partial^2 V_{jy}}{\partial Z \partial Y} + \frac{\partial^2 V_{jz}}{\partial Z^2} \right) \end{aligned} \right\} \quad (7)$$

$$\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} + \frac{\partial^2 \phi}{\partial Z^2} = (\mu_e N_e + \mu_m Z_- N_- - Z_+ N_+ - \mu_p N_p) \quad (8)$$

We have defined mass ratio $\gamma = m_j / m_-$.

The expression for trapped electron and positron number densities is provided after normalization as:

$$N_e = 1 + \phi - v_e \phi^{3/2} + \frac{1}{2} \phi^2 - \dots \quad (9)$$

$$N_p = 1 + \sigma(-\phi) - v_p \sigma^{3/2}(-\phi)^{3/2} + \frac{1}{2} \sigma^2(-\phi)^2 - \dots \quad (10)$$

Where $v_e = 4(1 - \beta_e) / 3\sqrt{\pi}$, $\sigma = T_p / T_e$ and $v_p = 4(1 - \beta_p) / 3\sqrt{\pi}$.

With trapped electron efficiency $\beta_e = T_{ef} / T_{et}$ and trapped positron efficiency $\beta_p = T_{pf} / T_{pt}$, $T_{ef(et)}$ is the free (trapped) electrons temperature & $T_{pf(pt)}$ is the free (trapped) positrons temperature.

3. Derivation of Complex mTKdVB Equation

This section explains how the complex mTKdVB equation is derived by introducing the well-established reductive perturbation approach from Eqs. (4)-(8) to illustrate the dissipative as well as dispersion effects in the presence of both trapped electron and positrons. Hence the independent variables in terms of space and time coordinates ξ and τ such that

$$\xi = \varepsilon^{1/2} (L_x X + L_y Y + L_z Z - \lambda T), \quad \tau = \varepsilon^{3/2} T, \quad \eta'_j = \varepsilon^{1/2} \eta_j \quad \text{and} \quad \mu'_j = \varepsilon^{1/2} \mu_j \quad (11)$$

In which the direction cosines of the vector along the parallel (x -axis) and perpendicular (y (z)- axes) are L_x and L_y (L_z) respectively which must satisfy the relation $L_x^2 + L_y^2 + L_z^2 = 1$, ε is the small dimensionless parameter ($0 < \varepsilon < 1$) which determines the strength of the perturbed amplitude and λ is the velocity of the wave (known as phase velocity) normalized to ion sound speed.

Further, the different dependent quantities such as N_j, V_j and ϕ are expanded in terms of ε as :

$$\left. \begin{aligned} N_j &= 1 + \varepsilon N_j^{(1)} + \varepsilon^2 N_j^{(2)} + \varepsilon^3 N_j^{(3)} + \dots \\ V_{jx} &= \varepsilon V_{jx}^{(1)} + \varepsilon^2 V_{jx}^{(2)} + \varepsilon^3 V_{jx}^{(3)} + \dots \\ V_{jy} &= \varepsilon^{3/2} V_{jy}^{(1)} + \varepsilon^2 V_{jy}^{(2)} + \varepsilon^{5/2} V_{jy}^{(3)} + \dots \\ V_{jz} &= \varepsilon^{3/2} V_{jz}^{(1)} + \varepsilon^2 V_{jz}^{(2)} + \varepsilon^{5/2} V_{jz}^{(3)} + \dots \\ \phi &= \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \dots \end{aligned} \right\} \quad (12)$$

Substitute Eqs. (11) & (12) into the normalized Eqs. (4)-(8), write the terms of lowest powers of ε and after simplification, we obtain the relation between the first order expression as:

$$\left. \begin{aligned} V_{+x}^{(1)} &= \frac{\lambda}{L_x} N_+^{(1)}, N_+^{(1)} = \frac{-Z_+ L_x^2}{\left(\lambda^2 - \frac{5}{3} \sigma_+ L_x^2\right)} \phi^{(1)}, V_{+x}^{(1)} = \frac{-\lambda Z_+ L_x}{\left(\lambda^2 - \frac{5}{3} \sigma_+ L_x^2\right)} \phi^{(1)} \\ V_{-x}^{(1)} &= \frac{\lambda}{L_x} N_-^{(1)}, N_-^{(1)} = \frac{\gamma Z_- L_x^2}{\left(\lambda^2 - \frac{5}{3} \sigma_- \gamma L_x^2\right)} \phi^{(1)}, V_{-x}^{(1)} = \frac{\lambda \gamma Z_- L_x}{\left(\lambda^2 - \frac{5}{3} \sigma_- \gamma L_x^2\right)} \phi^{(1)} \end{aligned} \right\} \quad (13)$$

Similarly, from the lowest order ε of Eq. (8) i.e. from Poisson's equation, we get

$$\mu_e \phi^{(1)} + \mu_m Z_- N_-^{(1)} - Z_+ N_+^{(1)} - \mu_p \sigma (-\phi^{(1)}) = 0 \quad (14)$$

Using Eq. (13) in Eq. (14), we found the following expression for the wave phase velocity:

$$\lambda = \left[p \pm \sqrt{p^2 - 4qr} / 2q \right]^{1/2} \quad (15)$$

$$\text{where, } p = \left(\left(Z_+^2 L_x^2 + \gamma \mu_m Z_-^2 L_x^2 \right) + \left(\mu_e + \mu_p \sigma \right) \left(\frac{5}{3} \sigma_+ L_x^2 + \frac{5}{3} \sigma_- \gamma L_x^2 \right) \right),$$

$$q = (\mu_e + \mu_p \sigma),$$

$$r = \left(\left(\frac{5}{3} Z_+^2 \sigma_- \gamma L_x^4 + \frac{5}{3} Z_-^2 \gamma \mu_m \sigma_+ L_x^4 \right) + \frac{25}{9} \sigma_+ \sigma_- \gamma L_x^4 (\mu_e + \mu_p \sigma) \right).$$

On the other hand, next higher order terms of ε , are collected from the normalized Eqs. (4)-(10) and accordingly we obtained the expression for

the perturbation terms $N_j^{(2)}$, $V_j^{(2)}$ and $\phi^{(2)}$. Finally removing the higher order perturbations terms, with the help of first order expression one can obtain the differential equation of the form of the complex mTKdVB [29] equation as:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)1/2} \frac{\partial \phi^{(1)}}{\partial \xi} + B \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + F \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} - G \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = 0 \quad (16)$$

Where, $A_1 = b_1/a$, $A_2 = b_2/a$, $B = c/a$, $G = d/a$, $F = f/a$ and

$$A = \sqrt{A_1^2 + A_2^2},$$

$$a = \left\{ -\frac{2\lambda Z_+^2 L_x^2}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right)^2} - \frac{2\lambda\gamma\mu_- Z_-^2 L_x^2}{\left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)^2} \right\},$$

$$b_1 = -\frac{3}{2}\mu_e \alpha_e, b_2 = -\frac{3}{2}\mu_p \alpha_p \sigma^{3/2},$$

$$c = \left\{ \mu_e - \mu_p \sigma_p^2 + \frac{3\lambda^2 Z_+^3 L_x^4}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right)^3} - \frac{\frac{5}{9}\sigma_+ Z_+^3 L_x^6}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right)^3} + \frac{3\mu_- \lambda^2 \gamma^2 Z_-^3 L_x^4}{\left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)^3} - \frac{\frac{5}{9}\mu_- \sigma_- \gamma^3 Z_-^3 L_x^6}{\left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)^3} \right\},$$

$$d = \left\{ \frac{\lambda Z_+^2 \eta_+ L_x^2}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right)^2} + \frac{\lambda Z_+^2 (\eta_+ + \mu_+) L_x^4}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right)^2} + \frac{\lambda\mu_- \gamma Z_-^2 L_x^2 \eta_-}{\left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)^2} + \frac{\lambda\mu_- \gamma Z_-^2 (\eta_- + \mu_-) L_x^4}{\left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)^2} \right\},$$

$$f = \left\{ \frac{Z_+^2 \lambda^2 (1 - L_x^2)}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right) \Omega_+^2} + \frac{\frac{5}{3}\lambda^2 \sigma_+ Z_+^2 L_x^2 (1 - L_x^2)}{\left(\lambda^2 - \frac{5}{3}\sigma_+ L_x^2\right)^2 \Omega_+^2} + \frac{\mu_- \lambda^2 \gamma Z_-^2 (1 - L_x^2)}{\Omega_-^2 \left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)} + \frac{\frac{5}{3}\lambda^2 \mu_- \sigma_- \gamma^2 Z_-^2 L_x^2 (1 - L_x^2)}{\Omega_-^2 \left(\lambda^2 - \frac{5}{3}\sigma_- \gamma L_x^2\right)^2} - 1 \right\}.$$

In Eq. (16), A and B stands for nonlinear coefficients, $F(G)$ stands for the dispersion (dissipation) coefficients.

4. Results and Discussion

For the numerical investigation based on Eqs. (15) & (16), we analyse the dynamic effect of distinct plasma parameters on shock wave propagation specially in astrophysical environments. For this purpose, we choose the density of charge particle as $10^{26} - 10^{30} \text{ cm}^{-3}$, the strength of magnetic field as $10^9 - 10^{12} \text{ Gauss}$ and we find the viscosity for those parameters in the range $\eta_+ = 0.01 - 0.1$, $\eta_- = 0.01 - 0.1$ [8, 23, 30, 31]. The fixed values of all other parameters are $Z_j = 1$, $m_+ = 40$, $m_- = 146$, $\beta_e = 0 - 0.9$, $\beta_p = 0 - 0.9$ [1, 3].

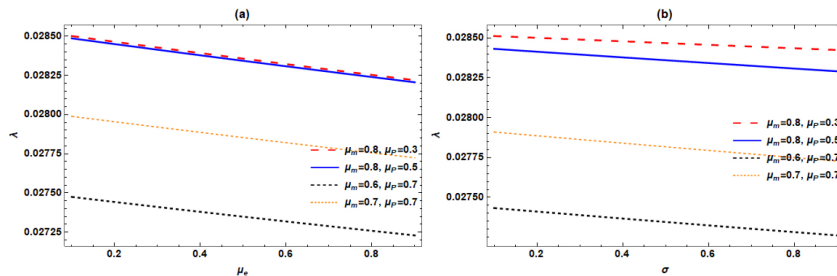


Figure 1

Variation of lower phase velocity λ with (a) μ_e and (b) σ for different combinations of μ_m and μ_p . The specific values of others parameter are $L_x = 0.11$, $\sigma_+ = 0.1$ and $\sigma_- = 0.1$

Figure 1 (a) (Figure 1 (a)) explore that the variation of lower phase velocity λ (small positive roots of Eq. (15)) of the wave for different values of μ_m and μ_p w.r.t μ_e (the symbols have usual meanings). Further, the variation of lower λ for certain combinations of μ_m and μ_p w.r.t temperature ratio (σ) is shown in Figure 1 (b). It is noticed from Figure 1 (a) and (b) that the lower phase velocity varies up to $\lambda \cong 0.02850$ for the values of the parameters. Figure 1 (a) shows that as μ_e increases the λ of the wave decreases. However, for a certain value of μ_e in plasma, increasing μ_m increases the λ of the wave for a chosen set of plasma parameters. This is due to the increased restoring force of the plasma environment due to the increase of negative ions in plasma. Also, this figure shows that for a certain value of μ_e in plasma increase in positron density reduces λ of the wave. This is may be due to increased efficacy of energetic trapped electrons, positrons in plasma. However, the variation in the λ causes by the modify in the value of μ_m is found to be more effective than that of change in the value of μ_p in plasma. From Figure 1 (b), it is seen that as the

increases the phase velocity of the wave decreases. Also, we noticed that as the μ_m increases the λ increases which is decrease with increase in μ_p [18, 30].

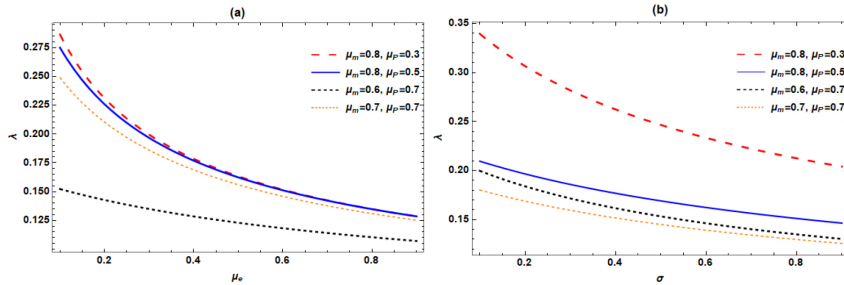


Figure 2

Variation of higher phase velocity λ with (a) μ_e and (b) σ for different combinations of μ_m and μ_p . The specific values of others parameter are $L_x = 0.11$, $\sigma_+ = 0.1$ and $\sigma_- = 0.1$.

Figure 2 (a) & (b) is plotted for the variation of higher λ (large positive root of Eq. (15)) for different values of μ_m , μ_p , μ_e and temperature ratio σ respectively. From Figure 2 (a), it is shown that raising the values of μ_m rises the λ , while raise the values of μ_p and μ_e decreases the λ . As the μ_m grows, the λ decreases, while increasing the ratio of μ_p and μ_e , also reduces the λ . On the other hand, increasing the values of μ_m , μ_p and σ decreases the phase velocity. It may be due to the same physical scenario like the case of Figure 1.

Figure 3 (a) & (b) illustrates that the efficacy of trapped electron (β_e) and positron (β_p) on the profile of IAShWs for (a) lower phase velocity and (b) for higher phase velocity. From both the figure, the magnitude of amplitude of the rarefactive shock wave potential profile ϕ is minimum for flat-topped distribution of electron and positron. However, it is maximum when electrons and positrons move towards thermal equilibrium. From both the figures we clearly see that the influence of electrons trapping is stronger than positrons trapping. Furthermore, the graph clearly demonstrates that when the trapping parameter increases, there is a noticeable rise in potential and width of the rarefactive shock. Greater amount of inertia is observed in the wave propagation once the flat-trapped distribution of electrons and positrons are increased and thus the system becomes less nonlinear, resulting in a significant increase in IAShWs potential [30, 31].

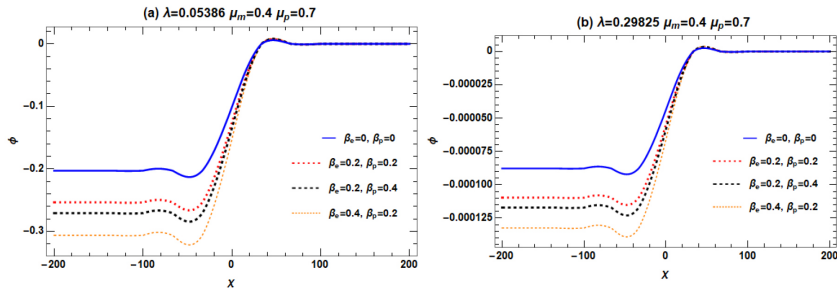


Figure 3

The plot of the potential function ϕ as a function of χ of complex mTKdVB equation for different combinations of β_e and β_p for (a) lower λ and (b) higher λ . The specific values of other parameter are $L_x = 0.11$, $\Omega_+ = 0.5$, $\Omega_- = 0.13$, $\sigma_+ = 0.5$, $\sigma_- = 0.3$, $\eta_+ = 0.07$, $\eta_- = 0.05$ and $\sigma = 0.2$.

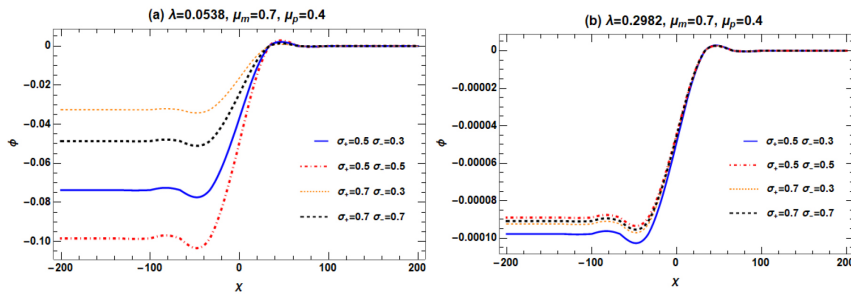


Figure 4

The plot of the potential function ϕ as a function of χ of the complex mTKdVB equation for different combinations of temperature ratios σ_+ and σ_- for (a) lower λ and (b) higher λ . The specific values of other parameters are $L_x = 0.11$, $\mu_m = 0.7$, $\mu_p = 0.4$, $\beta_e = 0.5$, $\beta_p = 0.5$, $\Omega_+ = 0.5$, $\Omega_- = 0.13$, $\eta_+ = 0.07$, $\eta_- = 0.05$ and $\sigma = 0.2$.

The variation of potential function ϕ w.r.t χ using the complex mTKdVB equation for observing the effect of temperature ratios of σ_+ and σ_- is depicted in Figure 4. Figure 4 (a) and Figure 4 (b) represents the cases of low and high phase velocity respectively. In Figure 4 (a), it is found that enhancing the values of σ_- increases the magnitude of amplitude as well as steepness of the IAShWs. However, increasing σ_+ declines the amplitude and steepness of the wave. On the otherhand, Figure 4 (b)

shows that as the temperature ratio of both ion increases the magnitude of amplitude declines. From both the figures we observed that the influence of temperature is stronger for slow acoustic mode and negligible in fast acoustic mode. Both images depict the development of rarefactive IAShWs profiles [30].

Figure 5 represents the role of magnetic field affects the propagation of rarefactive IAShWs profile ϕ w.r.t x . From Figure 5 (a) & (b) and we see that as magnetic field acting on the positive and negative ion increases respectively i.e. increasing Ω_+ and Ω_- , both potential as well as steepness of the IAShWs profile increases. The impact is assessed using the coefficient of dispersion, which is measured w.r.t cyclotron frequency.

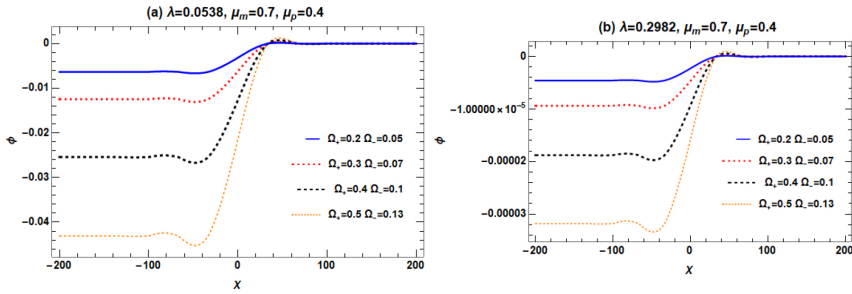


Figure 5

The variations of potential function ϕ against of complex mTKdVB for different combinations external Ω_+ and Ω_- for (a) lower λ and (b) higher λ . The specific values of other parameter are $\mu_m = 0.7$, $\mu_p = 0.4$, $\beta_e = 0.2$, $\beta_p = 0.2$, $\eta_+ = 0.07$, $\eta_- = 0.05$, $\sigma_+ = 0.5$, $\sigma_- = 0.3$ and $\sigma = 0.2$

In Figure 6 (a) & (b), the influence of both the viscosity parameter η_+ and η_- on the wave solution of complex mTKdVB equation is depicted by choosing the value as $\beta_e = 0.2$, $\beta_p = 0.2$, $\sigma_+ = 0.5$, $\sigma_- = 0.1$, $\Omega_+ = 0.5$, and $\Omega_- = 0.13$. Figure 6 (a) and Figure 6 (b) represents the cases of lower and higher phase velocity. From Figure 6 (a) & (b), It can be inferred that the magnitude of amplitude and steepness of shock structure grows with growing the effect of viscosity for both ions. Further from Figure 6 (a), it displays that a rapid change in shock wave potential occurs due to the change in viscosity of positive ions and by increasing the negative ion density declines the shock wave potential. But from Figure 6 (b), it displays that the rapid change in the potential function ϕ occurs due to negative ion viscosity instead of positive ion viscosity. This rapid change in shock wave potential in Figure 6 is due to viscosity because viscosity of ion is

source of dissipation in the plasma system and it dominate over dissipative terms of the medium [30, 31].

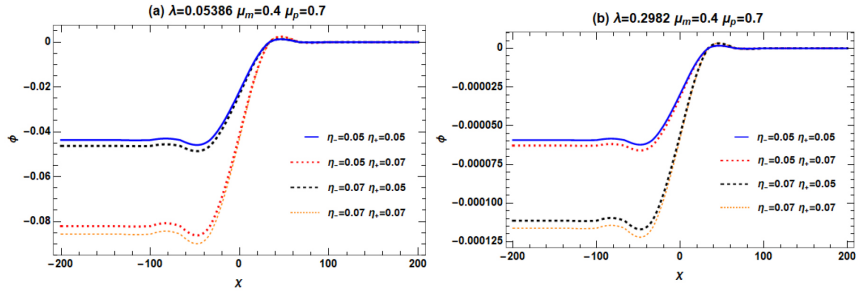


Figure 6

The plot of the potential function ϕ as a function of χ of the complex mTKdVB equation for different combinations of η_- and η_+ for (a) lower λ and (b) higher λ . The specific values of other parameter are $L_x = 0.11$, $\beta_e = 0.2$, $\beta_p = 0.2$, $\Omega_+ = 0.5$, $\Omega_- = 0.13$, $\sigma_+ = 0.5$, $\sigma_- = 0.1$ and $\sigma = 0.2$.

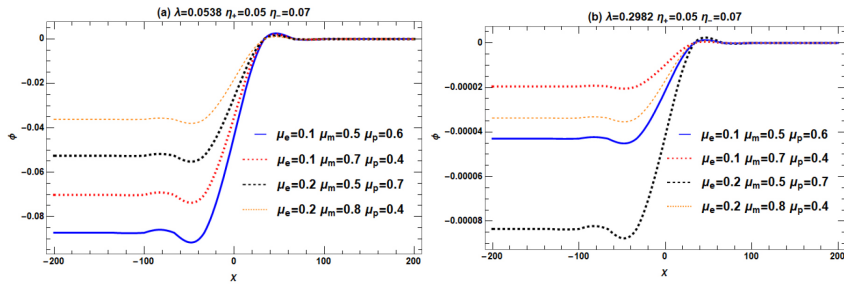


Figure 7

The plot of the potential function ϕ as a function of χ for various combinations of μ_e , μ_m and μ_p for the complex mTKdVB equation. The specific values of other plasma parameter are $L_x = 0.11$, $\beta_e = 0.2$, $\beta_p = 0.2$, $\eta_+ = 0.05$ and $\eta_- = 0.07$, $\sigma_+ = 0.5$, $\sigma_- = 0.3$, $\Omega_+ = 0.5$ and $\Omega_- = 0.13$ for (a) lower λ and (b) higher λ .

Figure 7 demonstrate that the potential profile ϕ w.r.t χ for observing the influence of μ_m , μ_p and μ_e with other parameters remaining constant. Figure 7 (a) & (b) displays for two situations i.e. for lower/higher phase velocity respectively. In Figure 7 (a), we can see that the magnitude of amplitude and steepness of the rarefactive IAShWs goes down with enhancement(diminution) of μ_m (μ_p). As we increase μ_e and μ_p by

keeping μ_m as constant value (blue solid-black dashed curve) the dimension of amplitude and steepness declines, this result remain same if we increase μ_e and μ_m by keeping μ_p as constant (red dotted-orange dashing curve). From Figure 7 (a), we conclude that increasing the density ratios decrease the amplitude and steepness of the IAShWs. On the otherhand, from Figure 7 (b) i.e. for higher phase velocity, we can see that as we increase values of μ_e and μ_p by keeping μ_m constant the magnitude of amplitude increases, while in other cases as we discussed in Figure 7 (a) the amplitude of magnitude decreases. This is because as the number density increases in slow acoustic waves the nonlinearity of the plasma increases, which will result in decrease in magnitude of amplitude and steepness of the rarefactive IAShWs profile. However, increased phase velocity with increasing electron and negative ion density enhances the system's nonlinearity, which will result increases the magnitude of amplitude decreases. Additionally, increasing positron ion density ratio reduces the system's nonlinearity, which will result in increasing the magnitude of amplitude and steepness [30, 31].

5. Conclusion

In the present plasma system, a theoretical investigation on the wave dynamics of electrostatic shock wave have been carried out in magnetized containing pair of ions, dissipation and trapped electrons as well as trapped positrons. The kinematic and bulk viscosities term. The dissipation term in our basic model is accountable for the development of shock wave structure, due to the presence of both types of viscosities. The traditional methodology of reductive perturbation is used for derivation of evolution equation. From the numerical solution of complex mTKdVB equation the several characteristics of shock wave is investigated for various parameters such as ion density ratios, trapping effect, external magnetic field and viscosity.

The main finding from our analysis can be concisely described as follows. The λ i.e. phase velocity is a function of multiple variables, including μ_m , the μ_e , μ_p and temperature ratios σ_+ and σ_- . We observed that the phase speed supports both slow and fast acoustic mode. The trapping of electron and positron is responsible for obtaining the fractional nonlinear term, which enhance the shock wave amplitude. Based on this observation, we infer that the potential (width) of the oscillatory shock wave becomes stronger (wider) as magnetic field

increases. The positive ions temperature ratio and negative ions temperature ratio controlling the behaviour of the shock waves for slow acoustic mode is studied. We have observed that, for the slow acoustic mode the shock structure is depend on positive ion viscosity, while it depends on negative ion viscosity for fast acoustic mode.

Conflict of Interest

The authors declare that they have no financial or personal connections that could have influenced the research presented in this paper.

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Data Availability

The authors declare that all the data related to the present work is provided in the manuscript. Our manuscript has no other associated data.

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