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Neutrosophic S^F_β-T_i spaces in neutrosophic topological spaces

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Abstract

The conception belonging to fuzzy sets was proposed by Zadeh [25]. Subsequently, the fuzzy sets along with the concept of fuzzy logic are used extensively in various applications implicating uncertainty. Although, it is examined that there still endure few situations which are not enclosed by fuzzy sets. Later, the fuzzy set exhibits as the conception based on the intuitionistic fuzzy set by Atannassov [5]. The fuzzy set is characterized by the membership function. Still, it is not possible to construct a membership function for each state. To meet this need, Al-Nafee, Al-Hamido, and Smarandache [3] characterized the neutrosophic set approach for solving problems connecting imprecise, ambiguous together with inconsistent data. Neutrosophic topological space exhibits an expansion based on intuitionistic topological space along with each triplet set in the neutrosophic topological space consists of membership, indeterminacy along with non- membership values. In this article, a unique conception based on the neutrosophic S^F_β- T_i ($i = 0, 1, 2, 3, 4$) spaces in neutrosophic topological spaces are proposed. Further, many characterizations of neutrosophic S^F_β- T_i (i = 0, 1, 2, 3, 4) spaces are studied and discussed. In addition, many theorems are proved with the counter

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examples. In Figure 1, the interrelationship between neutrosophic S_β^F- T_i (i = 0, 1, 2, 3, 4) spaces are studied and discussed with suitable illustration.

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Keywords: *Neut op st, Neut cs st, Neut S_β^F-op st, Neut S_β^F-cs st, Neut S_β^F-T_i (i = 0, 1, 2, 3, 4) spcs.*

1. Introduction

Zadeh [25] proposed the conception based on fuzzy set which appoints each object as a grade of membership ranging among zero along with one. In addition, the concepts based on inclusion, union, intersection and complement were defined in the fuzzy set and some of its properties had been studied. Smarandache [23] proposed the conception of neut st as a generalization based on intuitionistic fuzzy set. From then, it is inclined as very profitable in the fields based on decision making, artificial intelligence, etc. Atanassov [5] originated the conception based on intuitionistic fuzzy set together with few of its important inheritances were discuss in detail. Salama and Alblowi [22] acquainted the novel conception of the neut st theory. In addition, several operations were defined and several properties were discussed. Further, the interrelationship between neut sts and other known actual set was studied and discussed. Maji [16] acquainted the conception based on soft set in neut tpl spc. Furthermore, some operations were established in neut soft set and some of its properties had been studied. Iswarya and Bageerathi [14] acquainted the conception based on neut semi op and neut semi cs sets in neut tpl spcs. Further, the operators like neut semi interior and neutrosophic semi closure was characterized and discussed. Rao and Rao [19] proposed the notion of neut semi op and pre op sets in neut tpl spcs. In addition, many characterization of above mentioned concepts had been inspected along with discussed.

Arokiarani et al. [4] proposed the theory based on neut semi op fts in neut tpl spcs. In addition, some characterizations of neut semi op fts with other existing fts were studied with the counter examples. Imran et al. [13] proposed the notion of neut semi-α-op sts in neut tpl spcs. In addition, many characterizations of neut semi-α-op sts was studied and discussed with the counter examples. Bera and Mahapatra [6] constructed a topology on a neut soft set. In addition, the concept of neut soft interior, neut soft closure and neut soft boundary had been defined and some of its properties were investigated. Al-Nafee, Al-Hamido, and Smarandache [3] prefaced the conception of neut crisp points in the neut tpl spcs. Further, the conception of neut crisp limit point was defined using the concept of neut crisp points and few of its inheritances were examined in detail. Dhavaseelan et al. [10] originated the concept based on neut α^m-cs st in a neut tpl spc. In addition, the concepts of neut α^m-cont ft, strongly neut α^m-cont ft and neut α^m-irresolute ft were defined together with inspected few of its characterizations. Gündüz, Ozturk, and Bayramov [11] acquainted the approach based on neut soft point set in the neut tpl spc. In addition, the concepts of neut soft T_i-spcs were defined and some interrelationship among them was studied in detail.

Das and Pramanik [7-9] proposed the perception of generalized neut b-op st in a neut tpl spc. In addition, the operators like generalized neut b-interior and generalized neut b-closure was defined and investigated few of its important inheritances. In addition, the perception of neut φ-op st and neut φ-op mapp in a neut tpl spc was proposed. Furthermore, some properties of the above mentioned concepts were investigated with the counter examples. Maheswari and Chandrasekar [15] introduced the conception of neut generalized b-continuity in a neut tpl spc and many characterizations had been studied in detail. Hanif and Imran [12] originated the conception of neut generalized homeomorphism in a neut tpl spc. In addition, the concept of neut generalized op and cs mapps were examined and discussed. Mehmood et al. [18] proposed the conception of neut soft p-separation axioms in neut soft tpl spcs. Further, some properties of neut soft p^i-spaces $(i = 0,1,2,3)$ were studied with the counter examples. Acikgoz and Esenbel [2] proposed the concepts of separation axioms in neut tpl spcs. In addition, some notion such as neut quasi-coincidence and neut cluster point was defined and studied with the counter examples.

Al-Rahman et al. [1] originated the conception based on MR-Metric spcs and explore their connection with b-Metric spcs. Further, several methods of constructing MR-Metric spcs and different types of convergence associated with these metric were discussed in detail. Chandel and Chauhan [21] proposed the conception based on bipolar b-metric spc which is an extension of b-metric spc and bipolar metric spc. In addition, the results of uniqueness and existence of bipolar b-metric spc was discussed in detail with the suitable illustration. Chauhan, Tyagi, and Mani [24] originated the conception based on rational type connections within the framework in F-modular b-metric spcs. Furthermore, many properties were defined with the proper examples. Soni and Hasmani [20] explored and studied the concept of Vaidya metric spc. Further, various geometrical quantities related to Vaidya metric spc was discussed in detail. Rani et al. [17] studied some geometric and algebraic properties in computer graphics and vision systems. In addition, many real time examples also given and many properties were discussed in detail.

2. Methodology

Definition 2.1: Consider S as a non-empty st. A neut st H exhibits an entity acquiring the pattern $H = \{< s, \mathcal{K}_H(s), \mathcal{L}_H(s), \mathcal{M}_H(s) >: s \in S\}$ where $\mathcal{K}_H(s)$, $\mathcal{L}_H(s)$, $\mathcal{M}_H(s)$ remain as the degree belonging to membership, degree belonging to indeterminacy and the degree belonging to non-membership subsequently of all component $s \in S$ to the set H.

The neut st $H = \{< s, \mathcal{K}_H(s), \mathcal{L}_H(s), \mathcal{M}_H(s) >: s \in S\}$ can be described as an organized triplet $< \mathcal{K}_H(s), \mathcal{L}_H(s), \mathcal{M}_H(s) \gg$ in] -0, 1 +[beside S.

Based on explicitness, the pattern $H =< \mathcal{K}_H, \mathcal{L}_H, \mathcal{M}_H >$ for the neut st $H = \{< s, \mathcal{K}_H(s), \mathcal{L}_H(s), \mathcal{M}_H(s) >: s \in S\}$ had been used.

Definition 2.2: Consider $E =< \mathcal{K}_E(s), \mathcal{L}_E(s), \mathcal{M}_E(s) >$ as a neut st around S, then the complement $C_n(E)$ is designated as $C_n(E) = \{< s, \mathcal{M}_E(s), \mathcal{L}_E(s), \mathcal{K}_E(s) >: s \in S\}$.

Definition 2.3: Considering the two neut sts $G = \{< s, \mathcal{K}_G(s), \mathcal{L}_G(s), \mathcal{M}_G(s) > : s \in S\}$ and $H = \{< s, \mathcal{K}_H(s), \mathcal{L}_H(s), \mathcal{M}_H(s) >: s \in S\}$. Then

1) $G \subseteq H \Leftrightarrow \mathcal{K}_G(s) \leq \mathcal{K}_H(s), \mathcal{L}_G(s) \leq \mathcal{L}_H(s)$ and $\mathcal{M}_G(s) \geq \mathcal{M}_H(s)\ \forall\ s \in S$
2) $G = H$ on condition $G \subseteq H$ along with $H \subseteq G$
3) $G \cap H = \{< s, \mathcal{K}_G(s) \wedge \mathcal{K}_H(s), \mathcal{L}_G(s) \vee \mathcal{L}_H(s), \mathcal{M}_G(s) \vee \mathcal{M}_H(s) >: s \in S\}$
4) $G \cup H = \{< s, \mathcal{K}_G(s) \vee \mathcal{K}_H(s), \mathcal{L}_G(s) \wedge \mathcal{L}_H(s), \mathcal{M}_G(s) \wedge \mathcal{M}_H(s) >: s \in S\}$

Definition 2.4: Consider $\{C_k : k \in J\}$ as a family based on neut st in S. Then,

1) $\cap C_k = \{< s, \wedge \mathcal{K}_{C_k}(s), \vee \mathcal{L}_{C_k}(s), \vee \mathcal{M}_{C_k}(s) >: s \in S\}$
2) $\cup C_k = \{< s, \vee \mathcal{K}_{C_k}(s), \wedge \mathcal{L}_k(s), \wedge \mathcal{M}_{C_k}(s) >: s \in S\}$.

Definition 2.5. A neut topo on a non-empty set S_n exhibits a class τ_{Neut} based on the neut subsets in S fascinating the consecutive conditions:

(NT1) $0_{Neut,}\ 1_{Neut} \in \tau_{Neut}$

(NT2) $E_1 \cap E_2 \in \tau_{Neut}$ for any $E_1, E_2 \in \tau_{Neut}$

(NT3) $\cup E_i \in \tau_N\ \forall \{E_i : i \in J\} \subseteq \tau_{Neut}$

In the aforesaid case the ordered duo (S_n, τ_N) or explicitly S_n is designated as neut tpl spc (concisely NTS) and any neut st in τ_{Neut} exhibits as neut op st (concisely NOS). The complement $\overline{V_n}$ based on a neut op st V_n in S_n is labeled as neut cs st (concisely NCS) in S_n.

Definition 2.6: Let F_n represent a neut st in neut tpl spc S_n. Then, $Nint(F_n) = \cup\{E_n : E_n$ is a neut op st in S_n and $E_n \subseteq F_n\}$ is termed as neut interior of F_n. $NCl(F_n) = \cap\{K_n : K_n$ exhibits a neut cs st in S_n together with $F_n \subseteq K_n\}$ is labelled as neut closure of F_n.

Definition 2.7: Consider $N(S)$ as the set of all neut sts over S. A neut st $P = \{< s, \mathcal{K}_P(s), \mathcal{L}_P(s), \mathcal{M}_P(s) >: s \in S\}$ is called a neut pt if and only if for any element $d \in S$, $\mathcal{K}_P(d) = \alpha, \mathcal{L}_P(d) = \beta, \mathcal{M}_P(d) = \gamma$ for $d = s$ and $\mathcal{K}_P(d) = 0, \mathcal{L}_P(d) = 1, \mathcal{M}_P(d) = 1$ for $d \neq s$, where $0 < \alpha \leq 1$, $0 \leq \beta < 1$, $0 \leq \gamma < 1$.

A neut pt $P = \{< s, \mathcal{K}_P(s), \mathcal{L}_P(s), \mathcal{M}_P(s) >: s \in S\}$ will be express as $P^s_{\alpha,\beta,\gamma}$ or $s_{\alpha,\beta,\gamma}$. For the neut pt $s_{\alpha,\beta,\gamma}$, s will be represented as its support.

The complement based on the neut pt $s_{\alpha,\beta,\gamma}$ will be expressed as $s^C_{\alpha,\beta,\gamma}$.

Definition 2.8: Let C be a neut st over S. Also consider $x_{\alpha,\beta,\gamma}$ and $y_{\theta,\lambda,\mu}$ as two neut pts in S. Then

(i) $x_{\alpha,\beta,\gamma}$ is forenamed to be enclosed in C, expressed as $x_{\alpha,\beta,\gamma} \subseteq C$, on condition $\alpha \leq \mathcal{K}_C(x), \beta \geq \mathcal{L}_C(x), \gamma \geq \mathcal{M}_C(x)$.
(ii) $x_{\alpha,\beta,\gamma}$ is said to belong to C, denoted by $x_{\alpha,\beta,\gamma} \in C$, in the case that $\alpha < \mathcal{K}_C(x), \beta > \mathcal{L}_C(x), \gamma > \mathcal{M}_C(x)$.
(iii) $x_{\alpha,\beta,\gamma}$ is said to be contained in $y_{\theta,\lambda,\mu}$, denoted by $x_{\alpha,\beta,\gamma} \subseteq y_{\theta,\lambda,\mu}$, if and only if $x = y$ and $\alpha \leq \theta, \beta \geq \lambda, \gamma \geq \mu$.
(iv) $x_{\alpha,\beta,\gamma}$ is said to belong to $y_{\theta,\lambda,\mu}$, denoted by $x_{\alpha,\beta,\gamma} \in y_{\theta,\lambda,\mu}$, on condition $x = y$ along with $\alpha < \theta, \beta > \lambda, \gamma > \mu$.

Definition 2.9: Consider (X, τ_{Neut}) as a neut tpl spc and $D \subseteq X$. Let H be a neut st over D such that

$$\mathcal{K}_H(g) = \begin{cases} 1, g \in D \\ 0, g \notin D \end{cases}, \mathcal{L}_H(g) = \begin{cases} 1, g \in D \\ 0, g \notin D \end{cases}, \mathcal{M}_H(g) = \begin{cases} 0, g \in D \\ 1, g \notin D \end{cases}$$

Consider $\tau_D = \{H \cap F : F \in \tau\}$, formerly (D, τ_Y) is called a neut subspace of (X, τ_{Neut}). If $H \in \tau$, then (D, τ_D) is termed as neut op subspace of (X, τ_{Neut}).

Definition 2.10: A subset C based on a neut tpl spc (B, τ_{Neut}) is said to be a neut semi-β^F cs st in case that $U \subseteq Ncl(C)$ whenever $C \subseteq U$ where U remains as an β-op st in (B, τ_{Neut}).

3. Results and Discussion

Neutrosophic S_β^F-T_i Spaces

Definition 3.1: A neut tpl spc (X_n, τ_{Neut}) is termed as a neut S_β^F-T_0 ($N\ S_\beta^F$-T_0) spc if for every pair of neut pts $x_{\alpha_N, \beta_N, \gamma_N}, y_{\theta_N, \lambda_N, \mu_N} (x \neq y)$ in X_n, there exists a neut S_β^F-op st R such that $x_{\alpha_N, \beta_N, \gamma_N} \in R$, $y_{\theta_N, \lambda_N, \mu_N} \notin R$ (or)$x_{\alpha_N, \beta_N, \gamma_N} \notin R$, $y_{\theta_N, \lambda_N, \mu_N} \in R$.

Example 3.1: Consider the set $X_n = \{a, b\}$. Consider a neut topo $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, H, I, H \cup I, H \cap I\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >, 1_{Neut} = < (1,0,0)\ (1,0,0) >$, $H =< (0.3, 0.5, 0.8)\ (0.2, 0.4, 0.7) >$ and $I =< (0.2, 0.4, 0.9)\ (0.1, 0.3, 0.7) >$. Clearly, (X, τ_{Neut}) is a neut S_β^F-T_0 spc.

Theorem 3.1: *Consider a one-one function $f: (X, \tau_1) \to (Y, \tau_2)$ which is a neut cont ft after a neut tpl spc (X, τ_1) to another neut tpl spc (Y, τ_2). If (Y, τ_2) is a neut S_β^F-T_0 spc, then (X, τ_1) is also a neut S_β^F-T_0 spc.*

Proof: Assume that (Y, τ_2) is a neut S_β^F-T_0 spc. Consider $x_{\alpha_N, \beta_N, \gamma_N}$ and $y_{\theta_N, \lambda_N, \mu_N}$ $(x \neq y)$ as any neut pts in (X, τ_1). Therefore, the ft $f: (X, \tau_1) \to (Y, \tau_2)$ is a one-one ft, it implies that $f(x_{\alpha_N, \beta_N, \gamma_N})$, $f(y_{\theta_N, \lambda_N, \mu_N})$ are distinct neut pts in (Y, τ_2). Since, (Y, τ_2) is a neut S_β^F-T_0 spc, then there exists a neut S_β^F-op st R in Y and satisfying the condition $f\left(x_{\alpha_N, \beta_N, \gamma_N}\right) \in R, f(y_{\theta_N, \lambda_N, \mu_N}) \notin R$

(or) $f\left(x_{\alpha_N,\beta_N,\gamma_N}\right) \notin R, f(y_{\theta_N,\lambda_N,\mu_N}) \in R$. Therefore, it implies that $x_{\alpha_N,\beta_N,\gamma_N} \in f^{-1}(R), y_{\theta_N,\lambda_N,\mu_N} \notin f^{-1}(R)$ (or) $x_{\alpha_N,\beta_N,\gamma_N} \notin f^{-1}(R), y_{\theta_N,\lambda_N,\mu_N} \in f^{-1}(R)$. Since, f is a neut cont ft, so $f^{-1}(R)$ is a neut S_β^F op st in (X,τ_1). Therefore, for each pair of distinct neut pts $x_{\alpha_N,\beta_N,\gamma_N}$, $y_{\theta_N,\lambda_N,\mu_N}$ in (X,τ_1), there exists a neut S_β^F-op st $f^{-1}(R)$ aforesaid that $x_{\alpha_N,\beta_N,\gamma_N} \in f^{-1}(R), y_{\theta_N,\lambda_N,\mu_N} \notin f^{-1}(R)$ (or) $x_{\alpha_N,\beta_N,\gamma_N} \notin f^{-1}(R), y_{\theta_N,\lambda_N,\mu_N} \in f^{-1}(R)$. Hence, it is proved that (X,τ_1) is a neut S_β^F-T_0 spc.

Definition 3.2: A neut tpl spc (X,τ_{Neut}) is called a neut S_β^F-T_1 ($N\ S_\beta^F$-T_1) spc if for any pair of neut pts $x_{\alpha_N,\beta_N,\gamma_N}$, $y_{\theta_N,\lambda_N,\mu_N}$ $(x \neq y)$ in X, there exists two neut S_β^F-op sts R and S aforesaid that $x_{\alpha_N,\beta_N,\gamma_N} \in R, x_{\alpha_N,\beta_N,\gamma_N} \notin S$ and $y_{\theta_N,\lambda_N,\mu_N} \notin R, x_{\alpha_N,\beta_N,\gamma_N} \in S$.

Example 3.2: Consider the set $X = \{a,b\}$. Define a neut topo $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, I, J, I \cup J, I \cap J\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >$, $1_{Neut} =< (1,0,0)\ (1,0,0) >$, $I =< (0.2, 0.4, 0.6)\ (0.1, 0.3, 0.9) >$ and $J =< (0.2, 0.5, 0.8)\ (0.1, 0.4, 0.8) >$. Clearly, (X,τ_{Neut}) is a neut S_β^F-T_1 spc.

Theorem 3.2: *Assume that $f: (X,\tau_1) \to (Y,\tau_2)$ is both one-one and neut cont ft from a neut tpl spc (X,τ_1) to another neut tpl spc (Y,τ_2). If (Y,τ_2) is a neut S_β^F-T_1 spc, then (X,τ_1) is also a neut S_β^F-T_1 spc.*

Proof: Let (Y,τ_2) be a neut S_β^F-T_1 spc. Consider $x_{\alpha_N,\beta_N,\gamma_N}$, $y_{\theta_N,\lambda_N,\mu_N}$ $(x \neq y)$ as any two neut pts in X. Meanwhile, $f: (X,\tau_1) \to (Y,\tau_2)$ is a one-one ft, which implies $f\left(d_{\alpha_N,\beta_N,\gamma_N}\right), f(e_{\theta_N,\lambda_N,\mu_N})$ are also distinct neut pts in Y. Subsequently, (Y,τ_2) is a neut S_β^F-T_1 spc, there occurs two neut S_β^F- op sts R and S in Y aforesaid that $f\left(d_{\alpha_N,\beta_N,\gamma_N}\right) \in R, f(d_{\alpha_N,\beta_N,\gamma_N}) \notin S$ and $f\left(e_{\theta_N,\lambda_N,\mu_N}\right) \notin R, f(e_{\theta_N,\lambda_N,\mu_N}) \in S$. Therefore, it implies that $d_{\alpha_N,\beta_N,\gamma_N} \in f^{-1}(R), d_{\alpha_N,\beta_N,\gamma_N} \notin f^{-1}(S)$ and $e_{\theta_N,\lambda_N,\mu_N} \notin f^{-1}(R), e_{\theta_N,\lambda_N,\mu_N} \in f^{-1}(S)$. Since, f is a neut cont ft, both $f^{-1}(R)$ and $f^{-1}(S)$ are neut S_β^F-op sts in X. Consequently, for every pair belonging to distinct neut pts $d_{\alpha_N,\beta_N,\gamma_N}$, $e_{\theta_N,\lambda_N,\mu_N}$ in X, there exists two neut S_β^F-op sts $f^{-1}(R)$ and $f^{-1}(S)$ aforesaid that $d_{\alpha_N,\beta_N,\gamma_N} \in f^{-1}(R), d_{\alpha_N,\beta_N,\gamma_N} \notin f^{-1}(S)$ and $e_{\theta_N,\lambda_N,\mu_N} \notin f^{-1}(R)$, $e_{\theta_N,\lambda_N,\mu_N} \in f^{-1}(S)$. Therefore, it is proved that (X,τ_1) is a neut S_β^F-T_1 spc.

Theorem 3.3: *If a neut tpl spc (X,τ_{Neut}) is a neut S_β^F-T_1 spc, then every neut pts in X is a neut S_β^F-cs sts.*

Proof: Suppose that (X, τ_{Neut}) is a neut S_β^F-T_1 spc. Accredit $x_{\alpha_N, \beta_N, \gamma_N}$ as an arbitrary neut pts in x. Now, let us consider a neut pt $y_{\theta_N, \lambda_N, \mu_N} \subseteq x^c_{\alpha_N, \beta_N, \gamma_N}$ $(y \neq x)$ in X. Therefore, (X, τ_{Neut}) exhibits a neut S_β^F-T_1 spc, then there occurs two neut S_β^F-op sts R_n and S_n aforesaid that $x^c_{\alpha_N, \beta_N, \gamma_N} \in R_n, x^c_{\alpha_N, \beta_N, \gamma_N} \notin S_n$ and $y_{\theta_N, \lambda_N, \mu_N} \notin R_n, y_{\theta_N, \lambda_N, \mu_N} \in S_n$. Therefore, $x^c_{\alpha_N, \beta_N, \gamma_N} = \cup_{y_{\theta_N, \lambda_N, \mu_N} \subseteq x^c_{\alpha_N, \beta_N, \gamma_N}} \{R_n, S_n : x^c_{\alpha_N, \beta_N, \gamma_N} \in R_n, x^c_{\alpha_N, \beta_N, \gamma_N} \notin S_n$ and $y_{\theta_N, \lambda_N, \mu_N} \notin R_n, y_{\theta_N, \lambda_N, \mu_N} \in S_n\}$. Since, $\cup_{y_{\theta_N, \lambda_N, \mu_N} \subseteq x^c_{\alpha_N, \beta_N, \gamma_N}} \{R_n, S: x^c_{\alpha_N, \beta_N, \gamma_N} \in R_n$, $x^c_{\alpha_N, \beta_N, \gamma_N} \notin S_n$ and $y_{\theta_N, \lambda_N, \mu_N} \notin R_n, y_{\theta_N, \lambda_N, \mu_N} \in S_n\}$ is a neut S_β^F-op st in X, hence $x^c_{\alpha_N, \beta_N, \gamma_N}$ is a neut S_β^F-op st in X. Therefore, it is proved that $x_{\alpha_N, \beta_N, \gamma_N}$ is a neut S_β^F-cs st in X.

Remark 3.1: Consider (X_n, τ_{Neut}) as a neut tpl spc. Then, X_n is a neut S_β^F-T_1 spc if and only if $x_{\alpha_N, \beta_N, \gamma_N} = \cap\{Ncl(R_n): x_{\alpha_N, \beta_N, \gamma_N} \in Ncl(R_n)\}$.

Definition 3.3: A neut tpl spc (X_n, τ_{Neut}) is said to be a neut S_β^F-T_2 ($N\ S_\beta^F$-T_2) spc or neut S_β^F-Hausdorff space if for every pair of neut pts $x_{\alpha_N, \beta_N, \gamma_N}, y_{\theta_N, \lambda_N, \mu_N}$ $(x \neq y)$ in X_n, there exists two neut S_β^F-op sts R_n and S_n aforesaid that $x_{\alpha_N, \beta_N, \gamma_N} \in R_n, x_{\alpha_N, \beta_N, \gamma_N} \notin S_n$ and $y_{\theta_N, \lambda_N, \mu_N} \notin R_n, y_{\theta_N, \lambda_N, \mu_N} \in S_n$ with $R_n \subseteq {S_n}^C$.

Example 3.3: Consider the set $X = \{a, b\}$. Define a neut topo as $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, A\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >, 1_{Neut} =< (1,0,0)\ (1,0,0) >$ and $A =< (0.1, 0.4, 0.7)\ (0.2, 0.4, 0.9) >$. Hence, it is clear that (X, τ_{Neut}) is a neut S_β^F-T_2 spc.

Theorem 3.4: *Let $f: (X_n, \tau_1) \to (Y_n, \tau_2)$ be a 1-1 and neut cont ft from a neut tpl spc (X_n, τ_1) to another neut tpl spc (Y_n, τ_2). If (Y_n, τ_2) is a neut S_β^F-T_2 spc, then (X_n, τ_1) is also a neut S_β^F-T_2 spc.*

Proof: Consider f as a neut cont ft, then the inverse image of a neut op st in (Y, τ_2) is also a neut op st in (X, τ_1). Also, it is known that the complement of neut op st is neut cs st in a neut tpl spc. Since, (Y, τ_2) is a neut S_β^F-T_2 spc, so every neut S_β^F-op st in (Y, τ_2) exhibits a neut S_β^F-cs st in (X, τ_1). Since, f exhibits a neut cont ft, it implies that $f(X) = Y$ is a neut op st in τ_2. It follows that $f^{-1}(Y) = X$ is a neut op st in τ_1. Therefore, (Y, τ_2) is a neut S_β^F-T_2 spc. This implies that $(f^{-1}(Y), \tau_1)$ is a neut S_β^F-T_2 spc. Hence, it is proved that (X, τ_1) is a neut S_β^F-T_2 spc.

Theorem 3.5: *Assume that* (X, τ_{Neut}) *is a neut* S_β^F-T_1 *spc with the condition that complement of each neut* S_β^F*-op st is also a neut* S_β^F*-op st, then* (X, τ_2) *is a neut* S_β^F-T_2 *spc.*

Proof: Assume that (X, τ_{Neut}) is a neut S_β^F-T_1 spc with the condition that complement of each neut S_β^F-op st is also a neut S_β^F-op st. That is for M, a neut S_β^F-op st in (X, τ_{Neut}), $M^C = M$. Suppose that M is a neut S_β^F-op st in (X, τ_{Neut}). Thus, M^C exhibits a neut S_β^F-cs st in (X, τ_{Neut}). Again, by equation (i), M^C is a neut S_β^F-op st in (X, τ_{Neut}). Therefore, $M^C = M$. It implies that $(M^C)^C = M^C = M$. Therefore, every neut S_β^F-cs st in (X, τ_{Neut}) is both neut S_β^F-op and neut S_β^F-cs st in (X, τ_{Neut}). Hence, by Remark 1, (X, τ_{Neut}) is a neut S_β^F-T_2 spc.

Definition 3.4: Consider (X, τ_{Neut}) as a neut tpl spc. Formerly, X is labeled as a neut S_β^F-regular spc in case that for any neut pt $x_{\alpha_N, \beta_N, \gamma_N}$ in X, and neut S_β^F-cs st Q with $x_{\alpha_N, \beta_N, \gamma_N} \in Q^C$, there exist two neut S_β^F-op sts R_n and S_n such that $x_{\alpha_N, \beta_N, \gamma_N} \in R_n, Q \subseteq S_n$ and $R_n \subseteq {S_n}^C$.

Example 3.4: Consider the set $X = \{a, b\}$. Consider the neut topo $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, A\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >$, $1_{Neut} =< (1,0,0)\ (1,0,0) >$ and $A =< (0.1, 0.4, 0.7)\ (0.2, 0.5, 0.8) >$. Clearly, (X, τ_{Neut}) is a neut S_β^F-regular spc.

Definition 3.5: A neut tpl spc (X, τ_{Neut}) is said to be a neut S_β^F-T_3 spc if it is a neut S_β^F-T_1 spc and a neut S_β^F-regular spc.

Example 3.5: Consider the set $X = \{a, b\}$. Define a neut topo $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, A\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >, 1_{Neut} =< (1,0,0)\ (1,0,0) >$ and $A =< (0.1, 0.3, 0.7)\ (0.2, 0.4, 0.8) >$. Clearly, (X, τ_{Neut}) is a neut S_β^F-T_3 spc.

Theorem 3.6: *For any neut tpl spc* (X_n, τ_{Neut})*, then the consecutive results are the same:*

(i) X_n *exhibits a neut* S_β^F*-regular spc.*

(ii) *For any neut pt* $x_{\alpha_N, \beta_N, \gamma_N}$ *and any neut* S_β^F*-op st* R_n *containing* $x_{\alpha_N, \beta_N, \gamma_N}$*, there occurs a neut* S_β^F*-op st* S_n*aforesaid that*$x_{\alpha_N, \beta_N, \gamma_N} \in S_n \subseteq Ncl(S_n) \subseteq R_n$.

Proof:

(i)⟹(ii). Presume that (X_n, τ_{Neut}) is a neut S_β^F-regular spc. Formerly, for any neut pt $x_{\alpha_N, \beta_N, \gamma_N}$ in X_n, and a neut S_β^F-cs st Q_n with $x_{\alpha_N, \beta_N, \gamma_N} \in {Q_n}^C$, there exist two neut S_{β}^{F}-op sts S_n and P_n alike that $x_{\alpha_N, \beta_N, \gamma_N} \in S_n, Q_n \subseteq P_n$ and $S_n \subseteq {P_n}^C$. Since R^C is a neut S_{β}^{F}-cs st, there occurs a neut S_{β}^{F}-op st H_n such that $S_n \subseteq H_n$ and since $Ncl(S_n) \subseteq H_n$. And for a neut S_{β}^{F}-cs st H_n there occurs a neut S_{β}^{F}-op st R_n like that $H_n \subseteq R_n$. Hence, $x_{\alpha_N, \beta_N, \gamma_N} \in S_n \subseteq Ncl(S_n) \subseteq H_n \subseteq R_n$. This infers that, $x_{\alpha_N, \beta_N, \gamma_N} \in S_n \subseteq Ncl(S_n) \subseteq R_n$. Henceforth the proof.

(ii)⟹(i). The outcome is clear from the neut S_{β}^{F}-regular spc- Definition.

Definition 3.6: *A neut tpl spc (X_n, τ_{Neut}) is said to be a neut S_{β}^{F}-normal spc if for any pair of neut S_{β}^{F}-cs sts C and D with $C \subseteq D^C$ in X_n, there exist two neut S_{β}^{F}-op sts I and J in X_n aforesaid that $C \subseteq I, D \subseteq J$ and $I \subseteq J^C$.*

Example 3.6: Examine the set $X = \{a, b\}$. Define a neut topo $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, A\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >, 1_{Neut} =< (1,0,0)\ (1,0,0) >$ and $A =< (0.2, 0.4, 0.7)\ (0.2, 0.5, 0.8) >$. Hence, (X, τ_{Neut}) is a neut S_{β}^{F}-normal spc.

Definition 3.7: A neut tpl spc (X, τ_{Neut}) is said to be a neut S_{β}^{F}-T_4 spc if it is both neut S_{β}^{F}-T_1 spc and neut S_{β}^{F}- normal spc.

Example 3.7: Consider the set $X = \{a, b\}$. Define a neut topo $\tau_{Neut} = \{0_{Neut}, 1_{Neut}, A\}$, where $0_{Neut} =< (0,0,1)\ (0,0,1) >, 1_{Neut} =< (1,0,0)\ (1,0,0) >$ and $A =< (0.1, 0.5, 0.9)\ (0.3, 0.6, 0.8) >$. Clearly, (X, τ_{Neut}) is a neut S_{β}^{F}-T_4 spc.

Theorem 3.7: *For any neut tpl spc (X_n, τ_{Neut}), then the consecutive conditions are identical:*

(i) *X_n exhibits a neut S_{β}^{F}-normal spc*

(ii) *For every neut S_{β}^{F}-cs st K and neut S_{β}^{F}-op st U_n with $K \subseteq U_n$, there exists a neut S_{β}^{F}-op st V such that $K \subseteq V \subseteq Ncl(V) \subseteq U_n$.*

Proof: **(i)⟹(ii).** Consider X_n as a neut S_{β}^{F}-normal spc. According to the concept of neut S_{β}^{F}-normal spc, there exists two neut S_{β}^{F}-op sets U_n and V_n in X_n, for any two neut S_{β}^{F}-cs sts K_n and H with $K_n \subseteq {H_n}^C$ in X_n aforesaid that $K_n \subseteq U_n$, $H_n \subseteq V_n$ and $U_n \subseteq {V_n}^C$. For every neut S_{β}^{F}-cs sts K_n and H with $K_n \subseteq {H_n}^C$, the result $K_n \cap H = \emptyset$ is obtained (for any neut S_{β}^{F}-cs sts K_n and H_n with $K_n \subseteq {H_n}^C$). Consider the neut S_{β}^{F}-op st $U_n$that comprises K_n and V_n that comprises H_n (i.e)$K_n \subseteq U_n$ and $H \subseteq V_n$. Therefore, there exists two neut S_{β}^{F}-op sts P_n along with Q_n that are $U_n P_n$ and $V_n\, Q_n$, where P_n along with Q_n may or may not be disjoint neut S_{β}^{F}-op sts. In terms of the first part, this implies that $K_n \subseteq U_n$ and

$\overline{U_n} \subseteq P_n$ which implies $K_n \subseteq U_n \subseteq \overline{U_n} \subseteq P_n$. Assuming $U_n = V_n$ and $P_n = U_n$ and are both neut S_β^F-op st, the result is obtained.

(ii)⟹(i). To prove that X_n is a neut S_β^F-normal spc. It is given that, for every neut S_β^F-cs sts K_n and U_n with $K_n \subseteq U_n$, there exists a neut S_β^F-op st V_n such that $K_n \subseteq V_n \subseteq Ncl(V_n) \subseteq U_n$. This implies that there exist two neut S_β^F-op sts U_n and V_n and for any two neut S_β^F-cs sts K_n and H_n with $K_n \subseteq {H_n}^C$, such that $K_n \subseteq U_n, H_n \subseteq V_n$ as $K_n \cap H_n = \emptyset$. As a result, any two neut S_β^F-cs sts K_n and $H_n$with $K_n \subseteq {H_n}^C$ in X_n, there exists two neut S_β^F-op sts U_n and V_n aforesaid that$K_n \subseteq U_n, H_n \subseteq V_n$ and $U_n \subseteq {V_n}^C$. Therefore, it is proved that X_n is a neut S_β^F-normal spc.

Theorem 3.8: *A neut subspace (Y, τ_Y) of a neut S_β^F-T_i spc (X_n, τ_{Neut}) is a neut S_β^F-T_i spc ($i = 0, 1, 2$).*

Proof: (For the case $i = 0$). Consider (X_n, τ_{Neut}) as a neut S_β^F-T_0 spc and (Y, τ_Y) is a neut subspace of (X, τ_{Neut}). Taking any two neut pts $x_{\alpha_N, \beta_N, \gamma_N}$ and $y_{\theta_N, \lambda_N, \mu_N}$ in (Y, τ_Y) with different supports. Then, $x_{\alpha_N, \beta_N, \gamma_N}$ and $y_{\theta_N, \lambda_N, \mu_N}$ are also neut pts in (X_n, τ_{Neut}). Since, (X, τ_{Neut}) is a neut S_β^F-T_0 spc, there exist neut S_β^F-op sts F and G such that $x_{\alpha_N, \beta_N, \gamma_N} \in F, y_{\theta_N, \lambda_N, \mu_N} \in F^C$ or $y_{\theta_N, \lambda_N, \mu_N} \in G, x_{\alpha_N, \beta_N, \gamma_N} \in G^C$. Consider a neut st H. Then, $F \cap H$ and $G \cap H$ are neut S_β^F-op sts in (Y, τ_Y) aforesaid that $x_{\alpha_N, \beta_N, \gamma_N} \in F \cap H, y_{\theta_N, \lambda_N, \mu_N} \in (F \cap H)^C$ or $y_{\theta_N, \lambda_N, \mu_N} \in G \cap H, x_{\alpha_N, \beta_N, \gamma_N} \in (G \cap H)^C$. This implies that (Y, τ_Y) is a neut S_β^F-T_0 spc. In other cases, in which $i = 1$ and $i = 2$, the proof is identical as above.

Theorem 3.9: *A neut subspace (Y, τ_Y) of a neut S_β^F-T_3 space (X_n, τ_{Neut}) is a neut S_β^F-T_3 spc.*

Proof: Consider (X_n, τ_{Neut}) as a neut S_β^F-T_3 spc, $Y \subseteq X_n$ and (Y, τ_Y) is a neut subspace as described in the Definition 2.9. Let $x_{\alpha_N, \beta_N, \gamma_N}$ along with $y_{\theta_N, \lambda_N, \mu_N}$ in (Y, τ_{Neut}) be neut pts in (Y, τ_Y) aforesaid that $x_{\alpha_N, \beta_N, \gamma_N} \subseteq y^C_{\theta_N, \lambda_N, \mu_N}$. It is obvious that $x_{\alpha_N, \beta_N, \gamma_N}$ and $y_{\theta_N, \lambda_N, \mu_N}$ are also neut pts in (X_n, τ_{Neut}). Since (X_n, τ_{Neut}) is a neut S_β^F-T_1 spc, there exists neut S_β^F-op sts F and G in (X_n, τ_N) aforesaid that $x_{\alpha_N, \beta_N, \gamma_N} \in F, y_{\theta_N, \lambda_N, \mu_N} \in G$ and $F \subseteq G^C$. Then, there exists neut S_β^F-op sts H and K in (Y, τ_Y) such that $H = F \cap Y$ and $K = G \cap Y$. Clearly, $x_{\alpha_N, \beta_N, \gamma_N} \in H, y_{\theta_N, \lambda_N, \mu_N} \in K$ and $H \subseteq K^C$. This implies that (Y, τ_Y) is a neut S_β^F-T_1 spc. Now, it is to be proved that (Y, τ_Y) is a neut S_β^F-regular spc. Let G be a neut S_β^F-cs st in (Y, τ_Y) and $x_{\alpha_N, \beta_N, \gamma_N}$ represent a neut pt in (Y, τ_Y) aforesaid that $x_{\alpha_N,\,\beta_N,\gamma_N} \subseteq G^C$. It is obvious that $x_{\alpha_N,\,\beta_N,\gamma_N}$ is also a neut pt in (X_n, τ_{Neut}) and there exists a neut S_β^F-cs st F in $(X_n, \tau_N), G = F \cap Y$. It is obvious that $x_{\alpha_N,\,\beta_N,\gamma_N} \subseteq F^C$. Therefore (X_n, τ_{Neut}) exhibits a neut S_β^F-regular spc, there exists neut S_β^F-op sts K and M in (Y, τ_Y) such that $K = H \cap Y$ and $M = L \cap Y$. Clearly, $x_{\alpha_N,\,\beta_N,\gamma_N} \in K, G \subseteq M$ and $K \subseteq M^C$. This implies that (Y, τ_Y) is a neut S_β^F-regular spc. Hence, it is proved that (Y, τ_Y) is a neut S_β^F-T_3 spc.

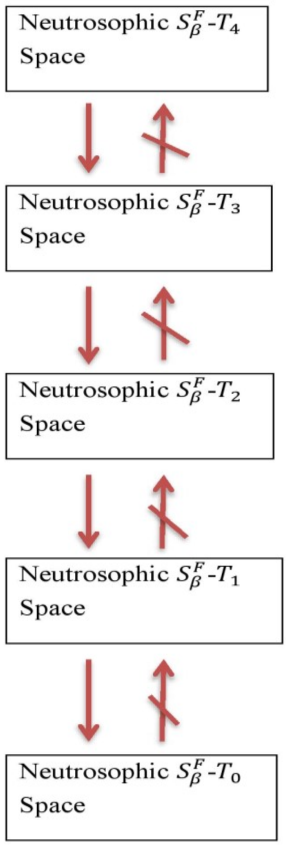

Figure 1

Relationship between neutrosophic S_β^F-T_i $(i = 0, 1, 2, 3, 4)$ spaces

Conclusion

In this article, the notion based on neut S_β^F-T_i $(i = 0,1,2,3,4)$ spcs are introduced via neut tpl spcs and some of its important properties have been reviewed. By introducing the concepts of neut S_β^F-T_i $(i = 0,1,2,3,4)$ spcs, many amusing results on neutrosophic separation axioms based on neut tpl spcs have been proved with the suitable illustration. Furthermore, many theorems related to the concept of neut S_β^F-regular spcs and neut S_β^F-normal spcs have been proved. The concept of neut S_β^F-T_i $(i = 0,1,2,3,4)$ spcs shall be used for originating the pair wise $s$eparation $a$xioms via neut bi-topol spcs. Furthermore, the proposed theory can be expanded in the concept of neut star topological environment.

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Abbreviation (The following notations have been used throughout this article)

S. No	Expansion	Notation
1)	Neutrosophic topological space	Neut tpl spc
2)	Neutrosophic topological spaces (Plural)	Neut tpl spcs
3)	Topological space	Topo spc
4)	Neutrosophic set	Neut st
5)	Open set	Op st
6)	Closed set	Cs st
7)	Neutrosophic point	Neut pt
8)	Function	Ft
9)	Continuous	Cont
10)	Mapping	Mapp
11)	Space	Spc
12)	Spaces (Plural)	Spcs

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