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Multiplicity results for homoclinic solutions in nonsmooth dynamical p-Laplacian systems without compactness

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Abstract

This paper investigates a class of nonsmooth dynamical p-Laplacian systems and establishes the existence of infinitely many homoclinic solutions-solutions that vanish at infinity. The main novelty lies in overcoming the lack of the Palais-Smale compactness condition by employing a refined variant of Clarke's critical point theory, adapted to the nonsmooth variational framework.

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Keywords: *Nonsmooth dynamical systems, Variational methods.*

1. Introduction

Dynamical systems provide a powerful mathematical framework for analyzing the evolution of states over time under deterministic or rule-based processes. Over the past decades, the field has expanded to encompass a wide variety of systems, including time-dependent (non-autonomous), discrete, and geometrically complex models.

A recent advancement in this direction is the introduction of Lorentzian Anosov families (LA-families) by Molaei and Khajoei [1]. In their work, they develop a geometric theory of dynamical systems on sequences of Lorentzian manifolds, where each manifold is endowed with a unique splitting of the tangent space that varies continuously via a semi-Riemannian connection. They further explore new concepts such as the Lorentzian shadowing property (types I and II), contributing to the broader understanding of structural stability in time-varying Lorentzian geometries.

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On the discrete side, Al-Rahim and Al-Sharaa [2] examine general non-autonomous discrete dynamical systems (g −NDS) and study how core dynamical properties like transitivity and sensitivity can be inherited from these systems to their autonomous counterparts (ADS). Notably, their work identifies fundamental differences between g-NDS and ADS in terms of minimality, offering new insight into how non-autonomy alters the long-term behavior of discrete systems.

Motivated by these developments in both geometric and discrete non-autonomous dynamics, we turn our attention to a class of nonsmooth continuous-time dynamical systems governed by nonlinear differential operators. In particular, this paper focuses on the following class of nonsmooth dynamical p-Laplacian systems:

$$\frac{d}{dt}(|\dot{y}(t)|^{p-2}\dot{y}(t)) + M|\dot{y}(t)|^{p-2}\dot{y}(t) + \nabla_y F(t, y(t)) = 0, \tag{1.1}$$

where $p > 1$, $t \in \mathbb{R}$, $y(t) \in \mathbb{R}^n$, M is a constant, and $F: \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is a continuous function. Here, $\nabla_y F$ denotes the gradient of $F(t, y)$ with respect to the spatial variable y.

We are particularly interested in homoclinic solutions of (1.1), that is, classical solutions $u \in C^2(\mathbb{R}, \mathbb{R}^n)$ satisfying $u(t) \to 0$ and $\dot{u}(t) \to 0$ as $|t| \to \infty$. If $u \not\equiv 0$, we refer to it as a nontrivial homoclinic solution.

The aim of this work is to establish the existence of infinitely many such nontrivial homoclinic solutions, even in the absence of compactness conditions such as the Palais-Smale condition. Our approach leverages variational methods combined with refined nonsmooth critical point theory, offering new results in the study of nonsmooth, non-autonomous dynamical systems with a p-Laplacian structure.

Laplacian systems, characterized by their linearity, are commonly employed in applications such as heat conduction, fluid dynamics, image processing (e.g., image denoising), and geometric modeling. In contrast, p-Laplacian systems are nonlinear and particularly pertinent in contexts where nonlinear effects are significant. These include modeling the flow of non-Newtonian fluids, phase transitions in materials, and the analysis of complex biological systems.

When $p = 2$, the p-Laplacian Hamiltonian system (1.1) takes the simple form:

$$\ddot{y}(t) + M\dot{y}(t) + \nabla_y F(t, y(t)) = 0, \tag{1.2}$$

which is a special form of the extensively studied Hamiltonian systems

$$\ddot{y} + \nabla_y F(t, y(t)) = 0. \tag{1.3}$$

The theory of differential equations is an expansive field that can be approached from various perspectives, as discussed in [3]. In particular, the existence of homoclinic solutions for system (1.3) is a central problem that has been extensively studied, as seen in [4, 5, 6, 7, 8, 9]. These studies generally assume that the nonlinearity F is in $C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, and they rely on the associated action functional $G: Y \to \mathbb{R}$, defined as

$$G(y) = \frac{1}{2}\int_{\mathbb{R}} |\dot{y}|^2\, dt - \int_{\mathbb{R}} F(t,y)\, dt, \quad y \in X := W^{1,2}(\mathbb{R}),$$

where

$$W^{1,2}(\mathbb{R}) = \left\{ y: \mathbb{R} \to \mathbb{R}^n \mid \int_{\mathbb{R}} |\dot{y}|^2\, dt + \int_{\mathbb{R}} |y|^2\, dt < \infty \right\}.$$

This functional satisfies the Palais-Smale condition (see definition 1.1).

Definition 1.1: A sequence (y_n) is called a Palais-Smale sequence for G if $G(y_n)$ is bounded and the gradient $G'(y_n)$ converges to zero in the dual space X^*. The functional G is said to satisfy the Palais-Smale condition (denoted (PS)) if every Palais-Smale sequence possesses a subsequence that converges strongly.

A common challenge in all of these problems is that the underlying space is not compact, and the associated functional does not satisfy the Palais-Smale condition, even when the nonlinearity $F \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$.

In 1990, Rabinowitz [8] demonstrated the existence of homoclinic solutions (that vanish at infinity) as a limit of $2kT$-periodic solutions for the system (1.3) when $F \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, and $F(t,y) = -\frac{1}{2}L(t)y \cdot y + W(t,y)$, with $L \in C(\mathbb{R}, \mathbb{R}^{n\times n})$ and $W(t,y)$ satisfying the well-known Ambrosetti-Rabinowitz type condition:

(AR) There exists a constant $\mu > 2$ such that $0 < \mu W(t,y) \leq y \cdot \nabla_y W(t,y)$ for all $y \in \mathbb{R}^n \backslash \{0\}$.

The (AR) condition plays a crucial role in proving the Palais-Smale condition. Using similar techniques, many authors have studied the existence of solutions that vanish at infinity for the system (HS) using critical point theory and variational methods, as seen in [5, 8, 9]. Afterwards, in 1991 Rabinowitz and Tanaka [9] introduced a coercivity condition on the matrix L:

$$l(t) = \inf_{|\xi|=1} L(t)\xi \cdot \xi \to +\infty \quad \text{as} \quad |t| \to +\infty \qquad (L_0).$$

They established a compactness theorem under the nonperiodic case and proved the existence of homoclinic solutions for the system (1.3) under the Ambrosetti-Rabinowitz condition. In fact, the condition (L_0) helps recover the compactness of the Sobolev embedding. Since then, it has been widely applied in the study of the existence of homoclinic solutions for second-order nonperiodic systems, as seen in [8, 9, 10, 11, 12, 13, 14, 15, 16].

In 2005, Izydorek and Janczewska [6] studied the system (1.3) using the classical mountain pass theorem by Rabinowitz and proved the existence of at least one homoclinic solution, when the potential $F(t,y)$ is T-periodic in t and takes the form:

$$F(t,y) = -F_1(t,y) + F_2(t,y), \tag{1.4}$$

where $F_1, F_2 \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, and satisfy the following main assumptions:

(A_1) There exist constants $b_1, b_2 > 0$ such that

$$b_1|y|^2 \leq F_1(t,y) \leq b_2|y|^2 \quad \text{for all} \quad (t,y) \in \mathbb{R} \times \mathbb{R}^n;$$

(A_2) $F_1(t,y) \le \nabla_y F_1(t,y) \cdot y \le 2F_1(t,y)$ for all $(t,y) \in \mathbb{R} \times \mathbb{R}^n$;

(A_3) $\nabla_y F_2(t,y) = o(|y|)$ as $|y| \to 0$, uniformly with respect to t;

(A_4) F_2 satisfies the Ambrosetti-Rabinowitz (AR) condition.

In this paper, we study the system (1.1) and prove the existence of infinitely many homoclinic solutions (that vanish at infinity) when $F(t,y)$ takes the form (1.4), but $F \not\in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, and the associated action integral:

$$G(y) = \int_{\mathbb{R}} \frac{1}{p}[|\dot{y}(t)|^p + M|\dot{y}(t)|^{p-2}\dot{y}(t) \cdot y(t)] \\ dt - \int_{\mathbb{R}} F(t,y)\, dt, \quad y \in X := W^{1,p}(\mathbb{R}), \tag{1.5}$$

does not satisfy the Palais-Smale condition.

Specifically, we consider the following hypotheses:

(H_1) $\nabla_y F \in C(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$.

(H_2) There exists a constant $b > 0$ such that

$$F_1(t,y) \ge b|y|^p$$

for all $t \in \mathbb{R}$ and $y \in \mathbb{R}^n$.

(H_3) $\lim\limits_{|y| \to 0} \frac{F(t,y)}{|y|^p} = +\infty, \quad uniformlyint.$

(H_4) There exists $\vartheta \in (1,p)$ such that for all $(t,y) \in \mathbb{R} \times \mathbb{R}^n$,

$$|F_2(t,y)| \le d(t)|y|^{\vartheta},$$

where $d \in L^\rho(\mathbb{R}, \mathbb{R}^+)$ for some $1 \le \rho \le \frac{p}{p-\vartheta}$.

(H_5) $F(t,-y) = F(t,y)$ for all $(t,y) \in \mathbb{R} \times \mathbb{R}^n$.

(H_6) $|M| \le \min\{1, pb\}$.

The main result of this paper is the following theorem.

Theorem 1.1: *Under the hypotheses* $(H_1) - (H_6)$, *the system (1.1) possesses infinitely many nontrivial homoclinic solutions.*

Remark 1.1: Under the assumptions $(H_1) - (H_6)$, G is continuous on X but is not of class $C^1(X)$. A typical example is given by:

$$F_1(t,y) = \frac{1}{2}\left(1 + \frac{1}{2}\sin\left(\frac{1}{|y|^\alpha}\right)\right)|y|^2$$

and

$$F_2(t,y) = d(t)|y|^{\vartheta},$$

where $0 < \alpha < 1$, $1 < \vartheta < 2$, and $d \in C(\mathbb{R}; \mathbb{R}) \cap L^\rho(\mathbb{R})$ for some $1 \le \rho \le \frac{2}{2-\vartheta}$, with $d \ge 0$ and $d \ne 0$. Clearly, F_1 and F_2 satisfy the assumptions of Theorem 1.1 with $p = 2$, but do not satisfy the corresponding results in [5, 6, 13, 14, 15] and the references therein.

For $y \in X$ and $z \in W^{1,2}(\mathbb{R}) \cap L^1(\mathbb{R})$, the derivative of G at y in the direction of z is given by:

$$(G'(y), z) = \int_{\mathbb{R}} \dot{y} \cdot \dot{z}\, dt + \int_{\mathbb{R}} y \cdot z\, dt + \int_{\mathbb{R}} Ay \cdot \dot{z}\, dt + \frac{1}{2}\int_{\mathbb{R}} \sin\left(\frac{1}{|y|^{\alpha}}\right) y \cdot z\, dt$$
$$-\frac{\alpha}{4}\int_{\mathbb{R}} |y|^{-\alpha}\cos\left(\frac{1}{|y|^{\alpha}}\right) y \cdot z\, dt - \vartheta \int_{\mathbb{R}} d(t)|y|^{\vartheta-2} y \cdot z\, dt.$$

There exist $y, z \in X$ such that the integral

$$\int_{\mathbb{R}} |y|^{-\alpha}\cos(|y|^{-\alpha}) y \cdot z\, dt$$

diverges. Therefore, G is not a C^1-functional on X.

Indeed, define for $t \in \mathbb{R}$,

$$\kappa(t) = \frac{1}{1+|t|^{\frac{1}{2-\alpha}}}, \quad y(t) = (\kappa(t), 0, \ldots, 0), \quad v(t) = (\kappa(t)\cos(|\kappa(t)|^{-\alpha}), 0, \ldots, 0).$$

Calculations show that $y, z \in X$ and

$$\int_{\mathbb{R}} |y|^{-\alpha}\cos(|y|^{-\alpha}) y \cdot z\, dt = +\infty.$$

2. Proof of theorem 1.1

In this section, we denote by

$$W^{1,p}(\mathbb{R}) := \left\{ y: \mathbb{R} \to \mathbb{R}^n \,\middle|\, \int_{\mathbb{R}} |\dot{y}|^p\, dt + \int_{\mathbb{R}} |y|^p\, dt < \infty \right\},$$

and for $y \in W^{1,p}(\mathbb{R})$, $\|y\|^p := \int_{\mathbb{R}} [|\dot{y}|^p + |y|^p]\, dt$.

A key tool in this paper is the following corollary, which can be considered as a variant of the standard Theorem ([17], Theorem 1.1).

Corollary 2.1: ([17] Theorem 1.1) *Let X be a separable and reflexive Banach space with norm $P \cdot P$, and let E be a dense subspace of X. Assume that G is a continuous functional defined on X that is E-differentiable. Suppose that G satisfies the following conditions:*

(B_1) *G is an even functional, i.e., $G(-y) = G(y)$ for all $y \in X$, and it is bounded from below.*

(B_2) *If $\{y_n\} \subset X$, $y \in X$, and $y_n y$ weakly in X, then*
$$(G'(y_n), z) \to (G'(y), z) \quad \forall z \in E.$$

(B_3) *G is weakly sequentially lower semicontinuous.*

(B_4) *The set $\{y \in X | G(y) \leq G(0)\}$ is bounded in X.*

(B_5) *For any positive integer k, there exists a k-dimensional subspace X_k of X and $\rho_k > 0$ such that*
$$\sup_{X_k \cap S_{\rho_k}} G < G(0),$$
where $S_\rho = \{y \in X | \|y\| = \rho\}$.

Then, G has infinitely many pairs of critical points $\{\pm y_k | k = 1,2, \ldots\}$ satisfying $G(\pm y_k) \leq G(0)$, $y_k \neq 0$ for $k = 1,2, \ldots$, and $y_k 0$ as $k \to \infty$.

The proof of our main result is presented in four steps:

Step 1: G satisfies conditions (B_1) and (B_4). Indeed, let $E = X \cap C_0^\infty(\mathbb{R})$. Since X is continuously embedded into $L^\infty(\mathbb{R})$, we deduce that under the assumption (H_1), G is E-differentiable, and for any $y \in X$ and $z \in E$,

$$(G'(y).z) = \int_{\mathbb{R}} \left[|\dot{y}(t)|^{p-2}\dot{y} \cdot \dot{z} + M\,|\dot{y}|^{p-2}\dot{y} \cdot z + \nabla_y F_1(t,y) \cdot z - \nabla_y F_2(t,y) \cdot z\right]dt.$$

By (H_2) and (H_4):

- For $\rho = 1$, we have

$$\begin{aligned} G(y) &= \frac{1}{p}\int_{\mathbb{R}} |\dot{y}|^p\,dt + \frac{M}{p}\int_{\mathbb{R}} |\dot{y}(t)|^{p-2}\dot{y}(t) \cdot y(t)\,dt \\ &\quad + \int_{\mathbb{R}} F_1(t,y)\,dt - \int_{\mathbb{R}} F_2(t,y)\,dt \\ &\geq \frac{1}{p}\int_{\mathbb{R}} |\dot{y}|^p\,dt - \frac{|M|}{p}\|y\|^p + b\int_{\mathbb{R}} |y|^p\,dt - c_1\|y\|_{L^\infty(\mathbb{R})}^{\vartheta}\int_{\mathbb{R}} |d(t)|\,dt \\ &\geq \left(\min\left\{\frac{1}{p}, b\right\} - \frac{|M|}{p}\right)\|y\|^p - c_p\|y\|^{\vartheta}\int_{\mathbb{R}} |d(t)|\,dt. \end{aligned} \tag{2.1}$$

- For $1 < \rho \leq \frac{p}{p-\vartheta}$, then $\frac{\vartheta\rho}{\rho-1} \geq p$ and we have

$$\begin{aligned} G(y) &\geq \frac{1}{p}\int_{\mathbb{R}} |\dot{y}|^p\,dt - \frac{|M|}{p}\|y\|^p + b\int_{\mathbb{R}} |y|^p\,dt \\ &\quad - \left(\int_{\mathbb{R}} |d(t)|^\rho\right)^{1/\rho}\left(\int_{\mathbb{R}} |y|^{\frac{\vartheta\rho}{\rho-1}}\,dt\right)^{\frac{\rho-1}{\rho}} \\ &\geq \left(\min\left\{\frac{1}{p}, b\right\} - \frac{|M|}{p}\right)\|y\|^p - \|y\|^{\vartheta}\left(\int_{\mathbb{R}} |d(t)|^\rho\,dt\right)^{1/\rho}. \end{aligned} \tag{2.2}$$

Since G is even, then by (2.1) and (2.2), G is coercive and bounded from below.

Step 2: Let $\{y_n\} \subset X$ such that $y_n \rightharpoonup y$ in X then, by the Lebesgue convergence theorem, we have:

$$(G'(y_n), z) \quad \to \quad (G'(y), z), \quad \forall z \in E. \tag{2.3}$$

Step 3: Let $\{y_n\} \subset X$ such that $y_n y$ in X, then by ([18], Theorem 1.6), we have

$$\liminf_{n\to\infty}\int_{\mathbb{R}} |\dot{y}_n|^p\,dt \quad \geq \quad \int_{\mathbb{R}} |\dot{y}|^p\,dt. \tag{2.4}$$

Applying Fatou's lemma and using (H_2) leads to

$$\liminf_{n\to\infty}\int_{\mathbb{R}} F_1(t,y_n)\,dt \quad \geq \quad \int_{\mathbb{R}} F_1(t,y)\,dt. \tag{2.5}$$

While (H_4) implies

$$\lim_{n\to\infty}\int_{\mathbb{R}} F_2(t,y_n)\,dt \quad = \quad \int_{\mathbb{R}} F_2(t,y)\,dt. \tag{2.6}$$

By (2.4)-(2.6), we see that G satisfies the condition (B_3).

Step 4: For $C > 0$, it follows from (H_3) that there exists $T > 0$ such that $F_1(t, y) \geq C|y|^p$ for all $t \in \mathbb{R}$ and $|y| < T$, we have

$$G(y) \quad \leq \quad (1 + |M|)\|y\|^p - C\|y\|_{L^p(\mathbb{R})}^p.$$

This implies, for any $k \in \mathbb{N}$, if X_k is a k-dimensional subspace of $C_0^\infty(\mathbb{R})$ and $\rho_k > 0$ is sufficiently small, then

$$\sup_{X_k \cap S_{\rho_k}} G < G(0),$$

with C large enough, where $S_\rho = \{y \in X | \|y\| = \rho\}$.

Finally, as in the proof of ([7], Theorem 3.1), by (H_3), we deduce that the functional (1.5) satisfies the condition (B_5). Then, by Corollary 2.1, the system (1.1) has infinitely of many homoclinic solutions.

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Author Contributions

The author developed the main idea of the paper and wrote the manuscript.

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