

## **Hybrid nanofluid flow of blood between two rotating plates embedded in a porous medium**

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### **Abstract**

This study investigates the magnetohydrodynamic (MHD) flow of a hybrid nanofluid between two rotating plates within a porous medium, using blood as the base fluid with gold and silver nanoparticles. The governing partial differential equations are transformed into a set of ordinary differential equations via similarity transformation and then solved using shooting technique along with the 4th-order Runge-Kutta method in MATLAB. Graphical and numerical results are presented to illustrate the effects of key engineering parameters. The important findings are: each of the normal velocity, axial velocity and transverse velocity decreases with increasing values of porous medium parameter, magnetic parameter (M)

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is to increase resistive forces in the fluid, suppressing motion in all directions. This model offers valuable insights for applications in thermal management and the dynamics of arterial blood flow, with potential implications for controlling characteristics of the microcirculatory system, such as drug delivery processes.

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## 1. Introduction

A nanofluid is a specially engineered fluid created by dispersing metallic or non-metallic nanoparticles—typically smaller than 100 nm—into a conventional base liquid. The incorporation of these nanoparticles significantly enhances the thermal performance of the fluid because heat transfer depends on factors such as particle volume fraction, particle geometry, Brownian motion, thermophoretic movement, surface area enhancement and interfacial layering between particles and liquid molecules. Since nanoparticles exhibit unique nanoscale thermal and chemical properties, their inclusion can markedly increase the thermal conductivity and heat transfer capability of the base fluid. Nanofluids have therefore become attractive for a wide range of applications, including electronic cooling, power generation, automotive thermal systems, aerospace engineering and biomedical devices. The concept of nanofluids was first introduced by Choi and Eastman [1] in 1995, who described them as highly conductive fluids formed by dispersing nanosized particles in a host liquid. Following this development, extensive research has explored how these fluids behave under different flow configurations, geometries, particle types and concentrations using both Newtonian and non-Newtonian formulations. Many studies have also examined their thermodynamic behavior under external influences, such as magnetic fields, buoyancy forces and thermal radiation, to optimize their performance in engineering systems. Hybrid nanofluids—containing more than one type of nanoparticle—have recently gained attention because they can outperform conventional nanofluids due to synergistic thermal effects among the mixed particles. Higher nanoparticle concentrations typically lead to improved heat transfer and smaller particle sizes further enhance conductivity because of increased surface area. For example, Tayebi and Chamkha [2] explored buoyancy-driven flow and thermal behavior of hybrid nanofluids exposed to a uniform transverse magnetic field, and employed a finite-volume based computational method. Ali et al. [3] performed a simulation on carbon nanotube (CNT) and hybrid nanofluids mixed with swimming microorganisms over a stretching surface. Additionally, Das et al. [4] examined the combined effects of temperature-induced buoyancy, gravity and magnetic forces on non-Newtonian hybrid nanofluid flow inside an enclosure, incorporating Hall current and thermal radiation influences. Das et al. [5] studied magneto-buoyancy-driven hybrid nanofluid flow inside a rotating channel. These studies collectively demonstrate

the growing importance of hybrid nanofluids in advanced heat transfer applications. Thermal transport and fluid motion of nano/hybrid nanofluids have also been examined in rotating and stretching geometries due to their relevance in engineering and industrial processes. Kumar et al. [6] investigated such a system around a rotating sphere with heat generation and absorption, demonstrating how particle shape strongly influences thermal behavior. Hussain et al. [7] used the finite element method to explore the flow and thermal features of a non-Newtonian hybrid nanofluid inside a wavy enclosure. Sethy et al. [8] studied heat transfer characteristics in a hybrid nanoliquid moving within an enclosure containing a heated circular obstacle, emphasizing the effects of the Lorentz force. Similarly, Kumar et al. [9] analyzed mixed convection MHD hybrid nanofluid flow over a spinning sphere, incorporating nonlinear thermal radiation and employing the Keller–Box method for numerical treatment. Tayebi et al. [10] examined convection and heat transfer inside a circular annulus filled with a water-based hybrid nanofluid containing copper and alumina nanoparticles. In another contribution, Kumar et al. [11] looked at the behavior of a hybrid nanofluid around a rotating wedge, focusing on how particle shape affects the system. Negi et al. [12] evaluated Maxwell fluid flow in a rotating and stretching configuration, highlighting applications in spinning disk reactors.

Dinarvand, Hosseini, and Pop [13] investigated three water-based nanofluids containing titanium, copper, and alumina nanoparticles and reported significant enhancements near the stagnation point. Further contributions include the work of Mabood et al. [14], who explored nanofluid flow with heat generation effects, while Ghadikolaei, Hosseinzadeh, and Ganji [15] considered glycol–water hybrid nanofluids within a rotating stretching channel, reporting notable temperature reductions. Tayebi, Öztop, and Chamkha [16] assessed natural convection in hybrid nanofluids subjected to internal heat sources and sinks, employing the finite volume method. Additionally, Tayebi and Chamkha [17] analyzed the role of magnetic fields in mixed convection inside an enclosure filled with hybrid nanofluid. Bhatti and Rashidi [18] applied the Buongiorno model to study thermophoretic diffusion in hybrid nanofluids and Turcu et al. [19] were among the first to experimentally test hybrid nanoparticles. Single and hybrid nano-additives improve thermal conductivity and heat transfer, while magnetic field, porosity and chemical reactions significantly affect flow and thermal behavior in porous rotating systems [20, 21]. Mahato et al. [22] further examined MHD hybrid nanofluid flow incorporating Hall current effects. Together, these studies highlight the expanding research interest in hybrid nanofluids under magnetic, rotational, convective and thermophoretic influences. Hybrid nanofluids, which are suspensions of two or more types of nanoparticles within a base fluid, exhibit enhanced thermal and rheological properties compared to traditional nanofluids [23–26]. Blood used as the base fluid, this enhancement is particularly beneficial for applications such as hyperthermia treatments, targeted drug delivery, and improved blood flow control, as hybrid nanofluids can offer more efficient heat transfer and fluid dynamics in physiological environments [27, 28]. Incorporating hybrid nanofluids into blood flow through porous media,

such as biological tissues, introduces additional complexity. Porous media create resistance that impacts both the motion and thermal characteristics of the nanofluid, which can be beneficial for controlled thermal and mass transport in the body. This configuration simulates natural conditions, where blood flows through porous structures, such as capillary beds and tissue interstices, allowing researchers to model real-world biomedical scenarios effectively [29-32]. The magnetohydrodynamic (MHD) effect arises when a magnetic field interacts with a moving conductive fluid, generating an opposing Lorentz force in electrically conductive fluids like water, saline solutions or plasma. This force reduces the flow rate while increasing temperature and concentration, potentially delaying the transition from laminar to turbulent flow. This phenomenon is widely studied in modern applications [33-36], including heat exchangers, electronic devices, as well as chemical and biological reactors, where enhancing heat transfer remains a challenge. Nanofluids, as a novel category of heat transfer fluids, enhance energy transport due to their altered flow dynamics and increased thermal conductivity. When multiple nanoparticles are combined within a base fluid, the transport characteristics become even more complex and often enhanced. Among them, silver and gold nanoparticles are widely recognized in biomedical engineering because of their exceptional stability, high thermal conductivity, catalytic behavior and strong antimicrobial activity. Gold-based nanomaterials, for example, exhibit notable effectiveness against a broad spectrum of microorganisms such as bacteria, fungi, protozoa and various viruses. In the present study, where blood is considered as the working fluid, special attention is given to biomedical significance-including how temperature and pressure influence blood viscosity. These physiological factors are incorporated into the analysis to ensure that the modeled flow behavior closely reflects realistic biological conditions.

The novelty of this study is that it uniquely combines the effects of hybrid nanofluid flow, porous medium parameters, and heat transfer dynamics in a rotating plate system with a focus on blood flow. The findings contribute to a deeper understanding of how velocity components and temperature are influenced by porosity and permeability, providing valuable insights for both biomedical applications and engineering systems requiring efficient fluid motion and thermal management in porous environments.

## 2. Mathematical modeling

In this work, we consider the steady and incompressible flow characteristics of hybrid nanofluids between two rotating plates with dispersed nanoparticles, as shown in Figure 1. The hydrodynamics of arterial blood flow and heat transfer processes may both be examined using this model. Gold and silver nanoparticles were employed in the study and  $u_w = ax$ , where 'a' is a positive constant, provides represents the linear motion of the vertical right wall. The fluid is assumed to be incompressible with a uniform flow field. Since the base fluid and nanoparticles are in equilibrium, slip effects are ignored. The induced magnetic

field is also disregarded in this study as it is believed that the magnetic Reynolds number is extremely low.

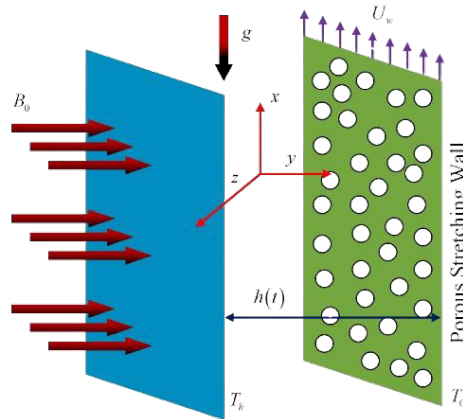


**Figure 1**  
**Flow diagram**

### 2.1. Model equation :

The governing equations are given below Mahanthesh, Gireesha, and Gorla [37]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.1)$$

$$\rho_{hnf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + 2\Omega w \right) = -\frac{\partial p}{\partial x} + \mu_{hnf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma_{hnf} B_0^2 u + g_0(\rho\beta)_{hnf}(T - T_0) - \frac{\mu_{hnf}}{Kp^*} u \quad (2.2)$$

$$\rho_{hnf} \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_{hnf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (2.3)$$

$$\rho_{hnf} \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} - 2\Omega u \right) = \mu_{hnf} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - \sigma_{hnf} B_0^2 w - \frac{\mu_{hnf}}{Kp^*} w \quad (2.4)$$

$$\begin{aligned} (\rho c_p)_{hnf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) &= k_{hnf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \\ &+ \mu_{hnf} \left( 2 \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right) + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right) + \\ &Q_0(T - T_0) - \frac{\partial q_r}{\partial y} \end{aligned} \quad (2.5)$$

The term  $q_r$  represents the heat flux, which is defined as:

$$q_r = \frac{16}{3} \frac{\sigma^*}{k^*} T_\infty^3 \frac{\partial T}{\partial y} \quad (2.6)$$

While

$$u = U_w = ax, v = -v_w, w = 0, T = T_w \text{ at } y = 0$$

$$u = nx, v = v_w, w = 0, T = T_o \text{ at } y = h$$

It should be noted that  $v_w < 0$  represents injection,  $v_w > 0$  represents suction.

## 2.2 Thermal and physical characteristics of a hybrid nanofluid:

Hybrid nanofluids thermal and physical characteristics are as follows Hussain et al. [38] and the properties of the base fluid and nanoparticles are presented in Table 1.

$$\frac{\rho_{hnf}}{\rho_f} = (1 - \varphi_{Ag} - \varphi_{Au})\rho_f + \frac{\varphi_{Au}\rho_{Au} + \varphi_{Ag}\rho_{Ag}}{\rho_f} \quad (2.8)$$

$$\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} = (1 - \varphi_{Ag} - \varphi_{Au}) + \varphi_{Ag}(\rho c_p)_{Ag} + \varphi_{Au}(\rho c_p)_{Au} \quad (2.9)$$

$$\frac{k_{hnf}}{k_f} = \frac{\left(\frac{\varphi_{Au}k_{Au} + \varphi_{Ag}k_{Ag}}{\varphi_{Ag} + \varphi_{Au}} + 2k_f + 2(\varphi_{Ag}k_{Ag} + \varphi_{Au}k_{Au}) - 2(\varphi_{Ag} + \varphi_{Au})k_f\right)}{\left(\frac{\varphi_{Au}k_{Au} + \varphi_{Ag}k_{Ag}}{\varphi_{Ag} + \varphi_{Au}} + 2k_f - (\varphi_{Ag}k_{Ag} + \varphi_{Au}k_{Au}) - (\varphi_{Ag} + \varphi_{Au})k_f\right)} \quad (2.10)$$

$$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1 - \varphi_{Ag} - \varphi_{Au})^{2.5}} \quad (2.11)$$

$$\frac{\sigma_{hnf}}{\sigma_f} = 1 + 3 \frac{\frac{\varphi_{Au}\sigma_{Au} + \varphi_{Ag}\sigma_{Ag}}{\sigma_f} - (\varphi_{Au} + \varphi_{Ag})}{\left(\frac{\varphi_{Au}\sigma_{Au} + \varphi_{Ag}\sigma_{Ag}}{\sigma_f} + 2\right) - \left(\frac{\varphi_{Au}\sigma_{Au} + \varphi_{Ag}\sigma_{Ag}}{\sigma_f} - (\varphi_{Au} + \varphi_{Ag})\right)} \quad (2.12)$$

Now, using the appropriate similarity variable Malik, Mallawi, and Haq [39]:

$$\eta = \frac{y}{h}, u = axf'(\eta), v = -ahf(\eta), w = axg(\eta), \theta(\eta) = \frac{T - T_o}{T_w - T_o} \quad (2.13)$$

Using these transformations, the provided PDEs are transformed into ODEs.

$$f^{iv} - Re_x \frac{A_1}{A_2} (f'f'' - ff''') - 2K_r \frac{A_1}{A_2} g' - \frac{A_5}{A_2} Mf'' - Re_x \frac{A_1}{A_2} Gr_x \beta^* \theta' - \frac{A_1}{A_2} \frac{1}{Kp} f'' = 0 \quad (2.14)$$

$$g'' - Re_x \frac{A_1}{A_2} (f'g - fg') + 2K_r \frac{A_1}{A_2} f' - \frac{A_5}{A_2} Mg - \frac{A_1}{A_2} \frac{1}{Kp} g = 0 \quad (2.15)$$

$$\theta'' + Pr \left[ Re_x \frac{A_3}{A_4} f\theta' + Ec \frac{A_2}{A_4} (4f'^2 + g^2) + \frac{1}{A_4} Re_x S_1 \theta \right] / \left( 1 + \frac{4Rd}{3A_4} \right) = 0 \quad (2.16)$$

Dimensionless quantities are expressed as follows:

$$Re_x = \frac{ah^2}{\nu_f}, M = \frac{\sigma_f B_0^2 h^2}{\rho_f \nu_f}, Kr = \frac{\Omega h^2}{\nu_f}, Pr = \frac{\mu_f (\rho c_p)_f}{\rho_f k_f}, Ec = \frac{\rho_f a^2 h^2}{(\rho c_p)_f (T_o - T_w)}$$

$$A_1 = \frac{\mu_{hnf}}{\mu_f}, A_2 = \frac{\rho_{hnf}}{\rho_f}, A_3 = \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}, A_4 = \frac{k_{hnf}}{k_f}, A_5 = \frac{\sigma_{hnf}}{\sigma_f},$$

$$Gr_x = \frac{g_o \beta_f (T_h^* - T_o^*)}{a^2 x}, \beta^* = \frac{\beta_{hnf}}{\beta_f} = \frac{(1-\phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f}}{(1-\phi) + \phi \frac{(\rho_s)}{(\rho_f)}}, S_1 = \frac{Q_o a (\rho c_p)_f}{a^2}, \alpha = -\left(\frac{V_L}{ah}\right)$$

The following are the boundaries:

$$\begin{aligned} f(0) &= f_w, f'(0) = 1, g(0) = 0, \theta(0) = 1, \\ f(1) &= f_w, f'(1) = \epsilon_r, g(1) = 0, \theta(1) = 0. \end{aligned} \quad (2.17)$$

Where  $f_w$  is the suction/injection parameter for upper surface, i.e.  $f_w = \frac{v_w}{ah}$  representing injection and  $f_w = \frac{-v_w}{ah}$  representing suction parameter for lower surface.

The skin-friction coefficient and the Nusselt number, which characterize the shear stress and heat transfer rate at the surface, are defined as follows:

$$\text{The skin friction parameter } C_f = \frac{(-2\mu_{hnf})}{\rho_{hnf} u_w^2} \frac{\partial u}{\partial y}, \quad C_f Re_x^{1/2} = \frac{A_1}{A_2} f''$$

$$\text{Nusselt number } Nu_x = \left( \frac{L}{K_f(T_0 - T_w)} \right) K_{hnf} \frac{\partial T}{\partial y}, \quad Nu_x Re_x^{-1/2} = A_4 \theta'$$

**Table 1**  
**Characteristics of base fluids and nanoparticles.**

| Properties                    | Blood                | Au                    | Ag                  |
|-------------------------------|----------------------|-----------------------|---------------------|
| $k$ W(m/k))                   | $491 \times 10^{-3}$ | 310                   | 400                 |
| $C_p$ (Kg/k))                 | 3594                 | 129                   | 235                 |
| $\rho$ (Kg·m <sup>-3</sup> )  | 1063                 | 19282                 | 10500               |
| $\sigma$ (s·m <sup>-1</sup> ) | 0.05                 | $4.10 \times 10^{-7}$ | $63 \times 10^{-6}$ |

### 3. Methodology:

After transforming the governing flow equations into nonlinear, coupled ordinary differential equations (ODEs), these are solved using the shooting technique along with the 4th-order Runge-Kutta (RK-4) method. The RK-4 method is designed for first-order initial value problems, so the shooting technique is employed to convert the boundary value problem into an equivalent initial value problem. The R-K method, particularly higher-order versions (e.g., 4th-order), provides excellent accuracy for solving ordinary differential equations (ODEs). This ensures reliable numerical solutions for boundary layer flow problems. Furthermore, the nonlinear coupled ODEs are reduced into a system of first-order ODEs, as shown below.

Taking  $f = y_1$ ,  $f' = y_2$ ,  $f'' = y_3$ ,  $f''' = y_4$ ,  $f^{iv} = y'_4$ ,  $g = y_5$ ,  $g' = y_6$ ,  $g'' = y'_6$ ,  $\theta = y_7$ ,  $\theta' = y_8$ ,  $\theta'' = y'_8$

$$y'_4 = -Re_x \frac{A_1}{A_2} (y_2 y_3 + y_1 y_4) + 2kr \frac{A_1}{A_2} y_6 + \frac{A_5}{A_2} M y_3 - Re_x \frac{A_1}{A_2} Gr_x \beta^* y_8 + \frac{A_1}{A_2} \frac{1}{Kp} y_3$$

$$y'_6 = Re_x \frac{A_1}{A_2} (y_2 y_5 - y_1 y_6) - 2kr \frac{A_1}{A_2} y_2 + \frac{A_5}{A_2} M y_5 + \frac{A_1}{A_2} \frac{1}{Kp} y_5$$

$$y'_8 = -Pr \left[ Re_x \frac{A_3}{A_4} y_1 y_8 + Ec \frac{A_1}{A_4} (4y_2^2 + y_5^2) + \frac{1}{A_4} Re_x S_1 y_7 \right] / (1 + \frac{4Rd}{3A_4})$$

Reduced form of boundary condition are as follows:

$$y_1 = f_w, y_2 = 1, y_5 = 0, y_7 = 1 \text{ at } \eta = 0$$

$$y_1 = f_w, y_2 = \epsilon_r, y_5 = 0, y_7 = 0 \text{ as } \eta = 1.$$

The solution of the system is obtained through the shooting technique coupled with the fourth-order RK scheme implemented in MATLAB.

#### 4. Validation:

The current numerical scheme was verified by comparing its results with those of previous studies under the specific conditions  $\varphi_{Ag} = \varphi_{Au} = 0$  and  $M = 0.5$ , as presented in Table 2. The comparison showed good consistency, indicating reliable agreement.

**Table 2**

**Comparison of present results with previously published result (Hussain et al. [40]).**

| $\alpha$ | $\lambda$ | Hussain et al. [40] |            | Present study |          |
|----------|-----------|---------------------|------------|---------------|----------|
|          |           | $f'(1)$             | $g'(1)$    | $f'(1)$       | $g'(1)$  |
| 0        | 2         | -4.582505           | -1.22921   | -4.58302      | -1.23042 |
| 0.3      |           | -2.622768           | -0.8058921 | -2.62389      | -0.80324 |
| 0.6      |           | -0.731301           | -0.361201  | -0.73653      | -0.36349 |
| 0.9      |           | 1.092487            | 0.105889   | 1.09252       | 0.10673  |
| 1.2      |           | 2.849166            | 0.597586   | 2.84194       | 0.59937  |
| 0.5      | -1        | 7.703171            | 3.025059   | 7.70434       | 3.02674  |
|          | 0         | 4.829804            | 1.403201   | 4.29437       | 1.40421  |
|          | 1         | 1.808905            | 0.296781   | 1.80671       | 0.29983  |
|          | 2         | -1.354212           | -0.511823  | -1.35943      | -0.51498 |
|          | 3         | -4.670059           | -1.132798  | -4.67001      | -1.13327 |

#### 5. Graphical Result and Discussion:

The effects of various parameters, such as porosity, suction/injection and magnetic field, on MHD hybrid nanofluid flow between two vertically rotating plates have been analyzed in detail. Throughout the discussion, the following parameters were maintained at constant values:  $Ec=0.01$ ,  $M=1.4$ ,  $Re_x=1$ ,  $Kr=0.5$ ,  $Gr_x=0.5$ ,  $Q=0.1$ ,  $Rd=0.5$ ,  $Kp=1$ ,  $\alpha=0.4$  unless otherwise stated.

In Figures 2, 3, and 4, with an increasing porous medium parameter, the decrease in the normal velocity, axial velocity, and transverse velocity (often referred to as the velocity components in the perpendicular, axial and tangential directions) can be explained by the complex interaction between the porous medium and the fluid dynamics. The porous medium parameter typically represents the permeability and porosity of the medium. As this parameter increases, the fluid faces more resistance to flow within the medium. This resistance means that the fluid's motion becomes more restricted, and the fluid has to exert more pressure to overcome the resistance of the porous material. As a result, the overall flow rate (including the normal, axial and transverse components) decreases.

Again, with a higher porous medium parameter, the permeability of the medium increases. However, increased permeability does not necessarily lead to faster flow. Instead, it causes a redistribution of the velocity components, where the fluid flow is now more evenly distributed across the available porous space. This leads to a decrease in each of the velocity components because the flow is

"spread out" and faces greater frictional resistance within the porous matrix. The resistance to motion within the porous medium leads to a decrease in the overall velocity of the fluid in all directions (normal, axial and transverse).

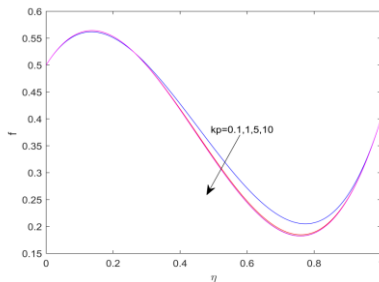


**Figure 2**  
Effects of normal velocity on  $K_p$ .

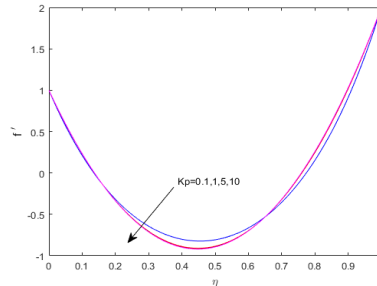


**Figure 3**  
Effects of axial velocity on  $K_p$ .

Figure 5: The temperature increases with increasing values of the porous medium parameter. When the porosity increases (higher porous medium parameter), it allows the fluid to flow more freely through the medium. As a result, the resistance to heat transfers decreases, allowing the fluid to absorb or retain more heat, which leads to an increase in temperature. Essentially, higher porosity creates pathways for better thermal conductivity, and the hybrid nanofluid absorbs heat more effectively from the surrounding surfaces or heat sources, leading to a higher temperature.

Figure 6: Higher values of suction parameter result in higher normal velocity. Suction involves drawing fluid away from the surface, which creates a motion in the normal direction (perpendicular to the plates). As suction increases, the fluid near the surface is removed more rapidly, which increases the velocity component in the normal direction. This is a direct result of the suction effect: fluid is being drawn into the porous surface, causing it to flow away from the surface in the perpendicular direction.

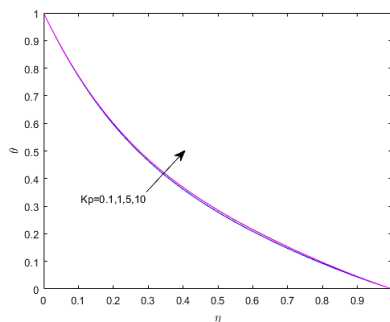


**Figure 4**  
Effects of transverse velocity on  $K_p$ .

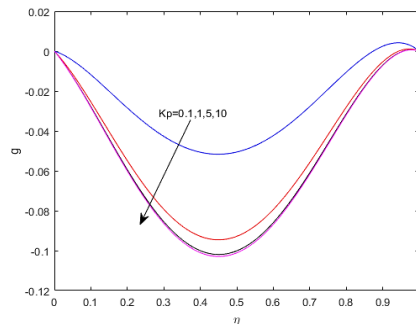


**Figure 5**  
Effects of temperature on  $K_p$ .

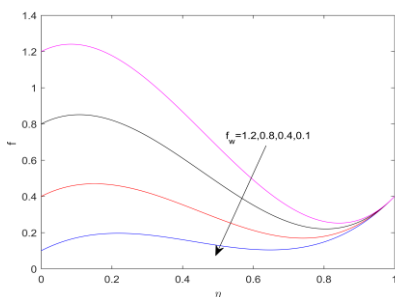


**Figure 6**  
Effects of normal velocity on suction.

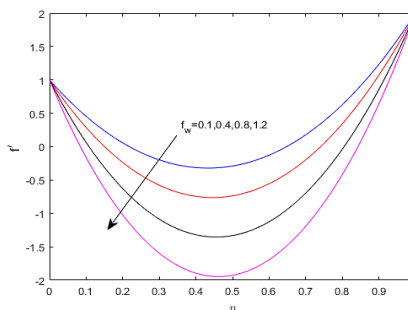


**Figure 7**  
Effects of axial velocity on suction

Figure 7: The axial velocity (the velocity in the direction of the flow) decreases as the suction parameter increases. Suction refers to the process of drawing fluid (in this case, blood or hybrid nanofluid) away from the surface or into the porous medium. When the suction parameter increases, it enhances the drainage effect on the flow, which tends to oppose the axial flow of the fluid. As the fluid is sucked away from the region between the rotating plates, it reduces the amount of fluid available to move in the axial direction. This reduces the local velocity near the plates and slows down the overall axial flow.

Suction can be used in blood circulation models to control the flow rate and velocity of blood in medical devices. Hybrid nanofluids, with their improved heat transfer and flow characteristics, can help in reducing the workload of pumps or assist in controlling temperature and viscosity for better circulation during treatments like hyperthermia or targeted drug delivery.

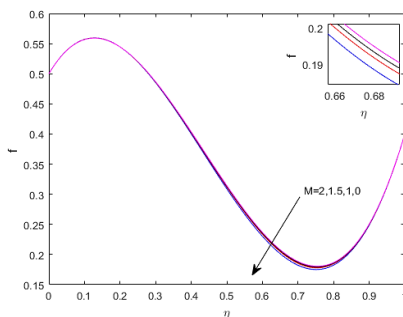


**Figure 8**  
Effects of transverse velocity on M.

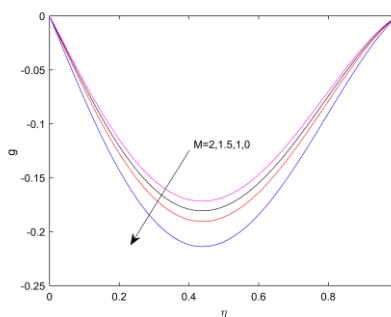


**Figure 9**  
Effects of normal velocity on M.

In Figures 8, 9, and 10, the magnetic parameter 'M' represents the strength of the applied magnetic field. For electrically conducting fluids like hybrid nanofluids, the interaction with the magnetic field induces a Lorentz force. This force acts as a resistance (or drag force) to the motion of the fluid. An increase in the magnetic parameter (M) intensifies the Lorentz force and the fluid's motion is

increasingly resisted, leading to a reduction in velocity components. Normal velocity decreases because the magnetic field suppresses fluid motion perpendicular to the plates. This suppression is stronger near the plates where velocity gradients are higher. The axial velocity, which represents the flow along the axis parallel to the plates, is directly affected by the drag force induced by the magnetic field. The higher  $M$ , the more significant the damping effect, reducing the fluid's momentum in the axial direction. The transverse velocity, which is influenced by the rotation of the plates and the fluid's inertial forces, also decreases due to the magnetic damping effect. The magnetic field resists the rotational-induced motion, slowing down the transverse velocity component.

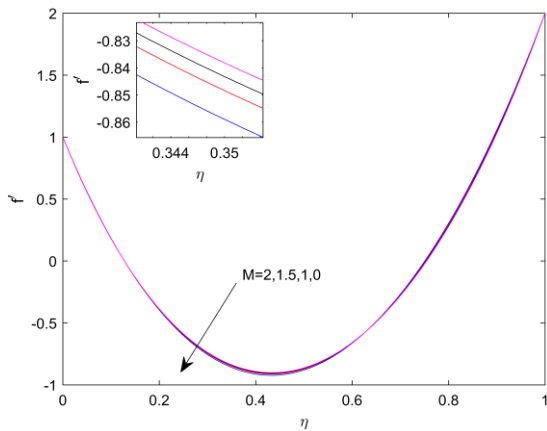


**Figure 10**  
**Effects of axial velocity on  $M$ .**

**Table 3**

**This shows how different parameters affect changes in the skin friction coefficient.**

| $f_w$      | $Kp$ | $M$ | $-f''(0)$ | $g'(0)$  |
|------------|------|-----|-----------|----------|
| <b>0.1</b> |      |     | 2.42758   | 0.21993  |
| <b>0.4</b> |      |     | 2.73167   | 0.15743  |
| <b>0.8</b> |      |     | 3.21980   | 0.03399  |
| <b>1.2</b> |      |     | 3.79064   | -0.14467 |
|            | 0.1  |     | 4.49726   | 0.13502  |
|            | 1    |     | 2.84572   | 0.13116  |
|            | 10   |     | 2.62706   | 0.03017  |
|            | 100  |     | 2.60772   | 0.00801  |
|            |      | 0   | 2.55286   | -0.12135 |
|            |      | 1   | 2.75221   | 0.10581  |
|            |      | 1.5 | 2.86896   | 0.13553  |
|            |      | 2   | 2.98319   | 0.15020  |

Table 3 represents the shear stresses in normal direction ( $f''(0)$ ) and in transverse direction ( $g'(0)$ ). Increase in suction ( $f_w$ ) increases shear stress in normal direction whereas decreases shear stress in transverse direction. When suction is applied at the surface of the rotating plates, the fluid is drawn towards the surface, which causes the boundary layer to thin.

As the boundary layer thins, the axial velocity increases near the surface, and this increase in velocity causes a sharper velocity gradient in the normal direction (the direction perpendicular to the plates). With suction, the gradient of the velocity becomes steeper because the thinning of the boundary layer leads to an increase in the velocity near the surface, which results in higher shear stress in the normal direction. In the other side, Suction tends to reduce the transverse velocity component  $g$  near the surface because it removes fluid from the boundary layer. With suction, the flow becomes more focused along the primary flow direction, and the transverse flow is suppressed. This reduction in transverse velocity leads to a decrease in the gradient of the transverse velocity, which is directly related to the shear stress in the transverse direction. As suction is applied, it diminishes the transverse velocity, reducing the velocity gradient in the transverse direction and therefore lowering the shear stress. Higher porosity ( $K_p$ ) reduces shear stress in both the normal and transverse directions. This occurs because increased porosity decreases the resistance to flow, which in turn leads to smaller velocity gradients in both directions. As velocity gradients decrease, so does the shear stress, as shear stress is directly related to the rate of change of velocity with respect to position. Higher magnetic parameter ( $M$ ) leads to increased shear stress in both the normal and transverse directions. This happens because the magnetic field causes the fluid to slow down in the axial direction, but the velocity gradients both normal and transverse to the surface become steeper as the field strength increases. This results in higher shear stress due to the greater velocity gradients near the surface.

## 6. Conclusion:

This research examines the thermal and flow characteristics of magnetohydrodynamic (MHD) hybrid nanofluids between two vertically rotating plates. Hybrid nanofluids, which integrate multiple nanoparticles, demonstrate enhanced heat transfer capabilities compared to nanofluids. The governing equations were transformed into a system of ODEs and solved numerically using the shooting technique coupled with the 4th-order RK method. The analysis explores the influence of key parameters, such as porosity, suction/injection and magnetic field, on velocity and temperature profiles. The findings highlight the potential of hybrid nanofluids to revolutionize heat transfer technologies and provide a foundation for future advancements in biomedical and engineering applications. highlights the following conclusions:

- The normal velocity, axial velocity and transverse velocity decrease with an increasing porous medium parameter.
- Temperature increases with increasing values of the porous medium parameter.

- The axial velocity (the velocity in the direction of the flow) decreases as the suction parameter increases.
- Higher values of suction parameter result in higher normal velocity.
- The overall effect of the magnetic parameter  $M$  is to increase resistive forces in the fluid, suppressing motion in all directions.

#### Practical Applications of the Study:

- The study provides insights into how hybrid nanofluids can enhance the control of drug delivery systems in microcirculatory environments, especially in porous biological tissues.
- Hyperthermia Treatments: Enhanced heat transfer capabilities of hybrid nanofluids (e.g., gold and silver nanoparticles) can improve the efficiency of hyperthermia-based cancer treatments.
- Understanding the effects of porosity and MHD on blood flow dynamics aids in modeling physiological scenarios, such as flow in capillaries and tissue interstices.
- Hybrid nanofluids, with superior thermal properties, are applicable in heat exchangers, cooling systems for electronic devices, and thermal energy storage systems.
- Understanding the interaction of hybrid nanofluids with porous media can guide the design of devices such as artificial kidneys or membranes used in dialysis.

#### Limitations of the Study:

- The study assumes laminar and steady-state conditions, which might not fully represent real-life turbulent or transient flow scenarios in biomedical or industrial applications.
- Only gold and silver nanoparticles are considered. Other combinations of nanoparticles might provide different insights or enhanced performance.
- The porous medium is modeled as homogeneous, whereas biological tissues and many industrial porous media are often heterogeneous.
- Potential effects like nanoparticle aggregation, Joule heating, and fluid-particle interaction dynamics are not accounted for, which could influence the accuracy of the results.

#### Nomenclature:

$Pr$  Prandtl number    $\Omega$  Rotation velocity    $\sigma$  Electrical conductivity  
 $Ec$  Eckert Number    $M$  Magnetic field    $\lambda$  Stretching parameter  
 $Re_x$  Reynolds Number    $Kr$  Rotation parameter    $hnf$  Hybrid nanofluid  
 $Gr_x$  Grashof Number    $S_1$  Heat source parameter    $f$  Base fluid  
 $Rd$  Radiation parameter    $K_p$  Porosity parameter  
 $\phi_{Ag}$  Volume fraction of silver    $\rho$  Density  
 $\phi_{Au}$  Volume fraction of gold    $\mu$  Viscosity  
 $\kappa$  Thermal conductivity    $(\rho C)_p$  Specific heat capacity

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