Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 29 (2026), No. 6, pp. 1991–2009
DOI : 10.47974/JIM-2445

Application of neutrosophic over soft orbit topological spaces in basketball team player selection

R. Narmada Devi *
Department of Mathematics
Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology
Avadi
Chennai 600062
Tamil Nadu
India

Yamini Parthiban †
Department of Mathematics
SRM Institute of Science and Technology
Ramapuram
Chennai 600089
Tamil Nadu
India
and
Department of Mathematics and Science Education
Faculty of Education
Harran University
Sanliurfa
Turkey

S. Padmapriya §
Department of Mathematics
Mahendra College of Engineering
Minnampalli
Salem 636106
Tamil Nadu
India

This article has been corrected with minor changes. These changes do not impact the academic content of the article.

* *E-mail:* narmadadevi23@gmail.com (Corresponding Author)
† *E-mail:* yaminiparthiban.msc@gmail.com
§ *E-mail:* priyasathi17@gmail.com

G. Muthumari [‡]
Department of Mathematics
R. M. D Engineering College
Kavaraipettai
Chennai 601206
Tamil Nadu
India

Abstract

This paper gives a detail exploration of neutrosophic over soft orbit Open Set using neutrosophic over soft topological space by introducing its fundamental concepts and theorem-purveying. The framework extended the soft set theory as well as that of neutrosophic sets for the management of uncertainty, indeterminacy, and fuzziness in topological spaces as a strong tool in decision-making under complex conditions. Practical application A key feature of the paper is the practicality of the framework using the tangent similarity measure designed to assess relationships between alternatives. For example, an illustration of the real utility of the paper is a numerical example aimed at showing how the framework of the neutrosophic over soft orbit Open Set can be applied in order to select the corresponding player for a basketball match. This example illustrates the systematic decision process provided through the framework, which is supposed to ensure that the optimum team is prepared while considering multiple criteria and uncertainty. The paper fills the gap between the theoretical progress within set theory and real-world applications towards handling uncertainty in a broad decision-making.

Subject Classification: *54A05, 03E72, 90B50.*

Keywords: *Neutrosophic over soft topological space, Neutrosophic over soft orbit open set, Neutrosophic over soft orbit topological space.*

1. Introduction

Uncertainty is a natural and consistent phenomenon in everyday life. As an example, when you toss a coin or roll a die on a slope, the final results become uncertain. In such a scenario, Zadeh [29] enlarged the classical set theory to a fuzzy set theory that not only represented imprecise information and degrees of truth, but also provided non-crisp membership. Later on, Bellman and Zadeh [3] depicted decision-making under uncertainty as a fuzzy scenario which helped them to scientifically discuss and promote further through the mathematic framework of fuzzy set theory that the uncertainty modeling in decision making processes has become an effective way with respect to operations and management. Zadeh [30] achieved further refinement in the year 1978 when he put forward fuzziness as the conceptual basis of the so-called possibility theory. This process not only changed the comprehension of the decision-making process by prioritizing 'which is allowed' over 'which is certain', but also opened the

[‡] *E-mail:* **muthumarig@saec.ac.in**

floodgate to the usage of various theoretical tools for the handling of uncertainty in scientific and engineering applications. Simultaneously, Bustince and Burillo [5] confronted the issue of correlations among interval-valued intuitionistic fuzzy sets and proposed a systematic technique for the analysis of relations among fuzzy sets defined by degrees of membership represented by intervals.

Atanassov and Shannon [1] took Zadeh's original fuzzy set model to the next level by presenting the intuitionistic fuzzy set theory, which represents uncertainty through three different and independent aspects: the membership degree, non-membership degree, and hesitation (i.e. not being sure). The flexibility of modeling incomplete and imprecise information is greatly improved by this enriched representation. Later on, Smarandache [24] suggested neutrosophic logic as a common ground in logical systems. Neutrosophic logic can be interpreted as a generalization of both classical and fuzzy logics, as it does so by incorporating explicitly the dimensions of truth, indeterminacy, and falsity as independent ones, thus, presenting a more expressive framework for addressing the issues of uncertainty, inconsistency, and ambiguity in complex areas like decision making and data analysis.

Simultaneously, Molodtsov [19] suggested the concept of soft set theory together with its principal results, thus giving a novel mathematical approach for handling uncertainty. The use of soft set theory allows for a very flexible and advanced structure which is particularly required in economic and decision-making problems where classical mathematical models might not work. Based on this, Maji, Biswas, and Roy [17] took one more step and compounded the soft set model by introducing a parameterized set-framework which, in turn, made it more applicable in decision-support systems dealing with uncertainties.

Wang et al. [27] introduced single-valued neutrosophic sets, which were the simplest yet very efficient approximations of neutrosophic sets where an element is symbolized by a single triplet for its truth, indeterminacy, and falsity. Broumi [4] then brought forth the notion of generalized neutrosophic soft sets that merge neutrosophic and soft set theories together and thus form a powerful framework for uncertainty and imprecision to be dealt with in a wide scope of applications. Moreover, Ye [28] invented a new correlation coefficient for single-valued neutrosophic sets and proved its utility in multiple attribute decision-making (MADM) problems. Recently, Smarandache and Pramanik [25] mentioned developments and trends in the field of neutrosophic theory and its applications.

Smarandache [26] was the one that made neutrosophic concepts more comprehensive by providing, along with other developments, such as neutrosophic oversets, undersets, and offsets generalizations in neutrosophic logic, probability, and statistics. Following a somewhat similar track, Dhavaseelan et al. [11] proposed the idea of neutrosophic α-m-continuity which is indeed a generalization of the classical topology continuity since it makes use of neutrosophic logic; thus, the treatment of uncertainties and indeterminacies in topological spaces is easier now.

Moreover, Jansi, Mohana, and Smarandache [13] unveiled a fresh correlation measure for Pythagorean neutrosophic sets, granting interdependence to truth and falsity components and thus providing a more sophisticated tool for the evaluation of relations among neutrosophic elements. Besides, Majeed and El-Sheikh [22] made a step further and analyzed fuzzy orbit topological spaces, opening up a new area where fuzzy set theory is applied to topological structures and helping to define the dynamic systems under uncertainty more accurately. Mhavarkar [14] presents a detailed financial analysis of bankruptcy risk using Altman's Z-score model applied to India Cement Ltd., demonstrating its effectiveness in predicting corporate financial distress. The study emphasizes the role of statistical and management models in enhancing decision-making processes in uncertain economic environments. Alemayehu and Ageze [31] develop the theory of fuzzy ideals and fuzzy congruences in distributive demi-p-algebras. Shirina et al. [32] investigate the relationship between employee performance, retention, and engagement in higher education institutions, highlighting key factors influencing organizational effectiveness. The study offers optimization-based insights that support strategic human resource management and institutional sustainability.

Christianto, Boyd, and Smarandache [6] highlighted the wide-ranging application of neutrosophic logic and its different theoretical aspects in many scientific fields which was a direct indication of its applicability in the portrayal of the complex system comprising uncertainty, indeterminacy, and contradiction. Moreover, Mallick and Pramanik [18] were the pioneers who introduced the concept of pentapartitioned neutrosophic sets, which is the format of the conventional neutrosophic sets that allows five different types of information.

In addition, Saeed et al. [23] put forward the idea of multi-polar neutrosophic soft sets and their application in the medical field for diagnosis and decision-making, thus illuminating their role in dealing with multidimensional uncertainty. Later on, Madhumathi and Irudayam [15] suggested the creation of neutrosophic orbit topological spaces which led to the development of the classical topological structures by introducing the elements of uncertainty, vagueness, and contradiction through the neutrosophic framework. In yet another stride, Radha et al. [21] were the first to illuminate the field of neutrosophic Pythagorean sets with dependent neutrosophic components and they especially strived for the creation of new and better correlation coefficients.

Atanassov [2] provided a theoretical boost when he looked into intuitionistic fuzzy modal topological structures. Madhumathi and Irudayam [16] on the other hand, made a further contribution to the neutrosophic set concept by coining the term 'neutrosophic orbit continuous mappings' which enables one to draw a parallel between the idea of continuity for functions defined in orbit topological spaces and the one in the vacuum.

In quite a similar vein, Murugesan et al. [20] culminated their research in a multi-dimensional analytical comparison between fuzzy and neutrosophic cognitive mapping models through the pandemic of COVID–19 as the ultimate case study that clearly loses no sight of their modeling potentials. Hussein and Al-Nafie [12] investigate the concepts of regularity and normality in nano topological spaces using pre-open set structures and extends classical topological properties into nano topology, providing new insights into generalized space structures. Deshpande et al. [7] explore the applications of topology and knot theory within the framework of quantum field theory. The study highlights how topological structures contribute to understanding complex physical phenomena in modern theoretical physics. Devi and Parthiban [8, 9] significantly impinged upon the area by postulating the amalgamation of neutrosophic logic with soft topological frameworks, thus paving the way for a topology-based decision analysis under parameterized uncertainty. Devi and Parthiban [10] carried on with the existing firmament and sought to evaluate drug compounds by means of neutrosophic over soft hyperconnected spaces that inextricably linked the contribution of the neutralsophic topology to biomedical decision-support systems.

The current study focuses on the analysis of the neutrosophic over soft orbit Open Sets or the neutrosophic over soft topological spaces. The first part includes a clear and thorough presentation of the topic with accurate definitions of the concept's basic components. The whole process marks a major step towards the merging of soft set theory with neutrosophic set theory by means of topology, thus the working through of the threefold uncertainty, indeterminacy, and fuzziness becomes more effective.

The proposed scheme has as its cornerstone a new analytical instrument that simplifies the tough decision making without significantly sacrificing its practical relevance. The authors make it very clear through their research how the theoretical basis is closely undermined and the practical uses are in decision making under uncertainty. To be more precise, the focus is on the tangent similarity measure which is used for assessing how close the current alternatives are. This measure is a tool that provides a systematic and a standardized way of comparing the different solutions especially when the information is either lacking or not clear or is in doubt.

The method applied is supported by a numerical example which is added for the purpose of making things clearer and easier to understand. The example revolves around the process of selecting the best player for a basketball team and it highlights how the neutrosophic over soft orbit Open Set methodology allows for multi-criteria evaluation and informed decision-making. This formalized process thus offers the trifold backing of preparing strategically, being resilient, and selecting the best option in tough situations.

In this sense, the actual showing of the technique not only relates the theoretical ideas with the actual decision problems but also demonstrates the advantages of the modern mathematical theories like neutrosophic and soft set theories in different application fields, with sports analytics being one of them. The performance of the tangent similarity measure also serves as an indicator of the proposed method's power to handle complex decision-making scenarios characterized by vagueness and uncertainty.

2. Preliminary Concepts

This section presents the essential concepts and definitions required to study *neutrosophic over soft topological spaces* [10].

Definition 1 (Neutrosophic Over Soft Set): [10] Let ϑ be a non-empty reference set, and let $\mathcal{E}$ be a parameter catalog linked to ϑ. A Neutrosophic Over Soft Set on ϑ (shortened as an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ -set) is generated through the parameter-driven correspondence: $\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}: \mathcal{E} \to \rho(\vartheta)$, where $\rho(\vartheta)$ denotes the ensemble of all neutrosophic over soft subcollections of ϑ.

The constructed $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is formalized as:

$$\xi = \left(\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E}\right) = \{(\mathrm{e}, \{\langle \varpi, \aleph_{\xi}(\varpi), \eth_{\xi}(\varpi), \varrho_{\xi}(\varpi)\rangle : \varpi \in \vartheta\}) : \mathrm{e} \in \mathcal{E}\}.$$

3. Neutrosophic Over Soft Orbit Topological Spaces

Definition 2 (Neutrosophic Orbit Components): [10] Let ϑ be a non–empty base set and let $(\vartheta, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ denote an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$–topological structure. For an arbitrary self–mapping $\varphi: \vartheta \to \vartheta$ and an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$–subset ξ of ϑ.

Definition 3 (Neutrosophic Orbit): [10] Let ξ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$–subset of the universe ϑ under the transformation $\varphi: \vartheta \to \vartheta$. The global orbit assembly of ξ is the union of its neutrosophic orbit components and is expressed as: $\mathfrak{O}(\xi) = \{\mathfrak{O}_{\aleph}(\xi), \mathfrak{O}_{\eth}(\xi), \mathfrak{O}_{\varrho}(\xi)\}$, where the component orbits evolve as:

$$\mathfrak{O}_{\aleph}(\xi) = \{\xi_{\aleph} \sqcap \varphi^{1}(\xi_{\aleph}) \sqcap \varphi^{2}(\xi_{\aleph}) \sqcap \ldots \sqcap \varphi^{\mathfrak{p}}(\xi_{\aleph})\},$$

$$\mathfrak{O}_{\eth}(\xi) = \{\xi_{\eth} \sqcap \varphi^{1}(\xi_{\eth}) \sqcap \varphi^{2}(\xi_{\eth}) \sqcap \ldots \sqcap \varphi^{\mathfrak{p}}(\xi_{\eth})\},$$

$$\mathfrak{O}_{\varrho}(\xi) = \{\xi_{\varrho} \sqcup \varphi^{1}(\xi_{\varrho}) \sqcup \varphi^{2}(\xi_{\varrho}) \sqcup \ldots \sqcup \varphi^{\mathfrak{p}}(\xi_{\varrho})\},$$

for all iteration limits $\mathfrak{p} \in \mathbb{Z}^{+}$.

Definition 4 (Orbit–Open and Orbit–Closed Sets): [10] Let $(\vartheta, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$–topological system, and let $\varphi: \vartheta \to \vartheta$ be a self–mapping operator. Any orbit set of ϑ produced under φ that already resides in the topology $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is referred to as an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit open set.

The complementary orbit set within ϑ is designated as an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ *orbit closed set*.

Remark 1: Any $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit–open configuration generated via the self–map φ necessarily belongs to the class of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ open sets. The reverse implication fails in general. As illustrated in Example 2, the set $\mathcal{K}$ qualifies as $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ open in the topological sense, yet it cannot be interpreted as a neutrosophic orbit construction induced by φ.

Example 1: Let $\vartheta = \{\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3\}$ and $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, \xi, \mathcal{K}\}$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topology. Where,

$$\xi = \{\langle \mathfrak{h}_1, 1.3,0.5,0.6\rangle, \langle \mathfrak{h}_2, 1.4,0.6,0.8\rangle, \langle \mathfrak{h}_3, 1.1,0.3,0.7\rangle\}$$

$$\mathcal{K} = \{\langle \mathfrak{h}_1, 1.1,0.6,0.8\rangle, \langle \mathfrak{h}_2, 1.4,0.5,0.3\rangle, \langle \mathfrak{h}_3, 1.1,0.3,0.2\rangle\}$$

Define $\varphi: \vartheta \to \vartheta$ as $\varphi(\mathfrak{h}_1) = \mathfrak{h}_3$, $\varphi(\mathfrak{h}_2) = \mathfrak{h}_1$, $\varphi(\mathfrak{h}_3) = \mathfrak{h}_2$. The $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit set of ξ under the mapping φ is defined as

$$\mathfrak{O}(\xi) = \{\xi \sqcap \varphi^1(\xi) \sqcap \varphi^2(\xi) \sqcap \varphi^3(\xi) \sqcap \dots \sqcap \varphi^{\mathfrak{p}}(\xi)\}$$

$$= \sqcap \{\xi, \varphi^1(\xi), \varphi^2(\xi), \varphi^3(\xi)\}$$

$$\mathfrak{O}(\xi) = \{\langle \mathfrak{h}_1, 1.1, 0.3, 0.8\rangle, \langle \mathfrak{h}_2, 1.1, 0.3, 0.8\rangle, \langle \mathfrak{h}_3, 1.1, 0.3, 0.8\rangle\}$$

$\therefore$ $\mathcal{K}$ is not a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit open set under the mapping φ.

Theorem 3.1: *Let $(\vartheta, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a neutrosophic over soft topological space, and let $\varphi: \vartheta \to \vartheta$ be a bijection. For any $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit–open set ξ generated under φ, the transformation preserves invariance, that is,*

$$\varphi(\xi) = \xi.$$

Proof: Let $(\vartheta, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a neutrosophic over soft topological system, and assume $\varphi: \vartheta \to \vartheta$ is bijective. The invariance claim is examined through the following three mutually independent scenarios:

Case 1: If $\varphi(\mathfrak{h}_i) = \mathfrak{h}_j$ for all $\mathfrak{h}_i, \mathfrak{h}_j \in \vartheta$ and $i \neq j \forall i, j \in \mathbb{Z}^+$

Suppose $\vartheta = \{\mathfrak{h}_1, \mathfrak{h}_2\}$ and $\varphi: \vartheta \to \vartheta$ defined as $\varphi(\mathfrak{h}_1) = \mathfrak{h}_2$, $\varphi(\mathfrak{h}_2) = \mathfrak{h}_1$. Let ξ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit open set under mapping φ.

$$\mathfrak{O}(\psi) = \xi.$$

Then there exists a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ set ψ such that

$$\mathfrak{O}(\psi) = \psi \sqcap \varphi^1(\psi) \sqcap \varphi^2(\psi) \sqcap \varphi^3(\psi) \sqcap \dots = \xi.$$

Let $\psi = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle\}$

$$\varphi(\psi) = \{\langle \mathfrak{h}_1, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle, \langle \mathfrak{h}_2, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle\}$$

$$\varphi^2(\psi) = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle\}$$

$$\begin{aligned}\mathfrak{O}(\psi) = \{&\langle \mathfrak{h}_1, [\inf\langle \aleph^1(\varpi_1), \aleph^2(\varpi_2), \aleph^1(\varpi_1), \aleph^2(\varpi_2) \dots \rangle],\\
&[\inf\langle \eth^1(\varpi_1), \eth^2(\varpi_2), \eth^1(\varpi_1), \eth^2(\varpi_2) \dots \rangle],\\
&[\sup\langle \varrho^1(\varpi_1), \varrho^2(\varpi_2), \varrho^1(\varpi_1), \varrho^2(\varpi_2) \dots \rangle]\rangle,\\
&\langle \mathfrak{h}_2, [\inf\langle \aleph^2(\varpi_2), \aleph^1(\varpi_1), \aleph^2(\varpi_2), \aleph^1(\varpi_1) \dots \rangle],\\
&[\inf\langle \eth^2(\varpi_2), \eth^1(\varpi_1), \eth^2(\varpi_2), \eth^1(\varpi_1) \dots \rangle],\\
&[\sup\langle \varrho^2(\varpi_2), \varrho^1(\varpi_1), \varrho^2(\varpi_2), \varrho^1(\varpi_1) \dots \rangle]\rangle\}\\
= \{&\langle \mathfrak{h}_1, [\min\langle \aleph^1(\varpi_1), \aleph^2(\varpi_2)\rangle], [\min\langle \eth^1(\varpi_1), \eth^2(\varpi_2)\rangle]\\
&[\max\langle \varrho^1(\varpi_1), \varrho^2(\varpi_2)\rangle]\rangle, \langle \mathfrak{h}_2, [\min\langle \aleph^1(\varpi_2), \aleph^2(\varpi_2)\rangle],\\
&[\min\langle \eth^1(\varpi_1), \eth^2(\varpi_2)\rangle][\max\langle \varrho^1(\varpi_1), \varrho^2(\varpi_2)\rangle]\rangle\}\\
= \xi&\end{aligned}$$

In general, if $\vartheta = \{\mathfrak{h}_1, \mathfrak{h}_2\}$ and ξ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit open set ξ under mapping φ. Then there exists a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ set

$$\psi = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle \dots\}$$

$$= \{\langle \mathfrak{h}_{\mathrm{i}}, \aleph^{\mathrm{i}}(\varpi_{\mathrm{i}}), \eth^{\mathrm{i}}(\varpi_{\mathrm{i}}), \varrho^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle\}$$

$$\mathfrak{O}(\psi) = \xi \tag{1}$$

That means $\mathfrak{O}(\psi) = \{\langle \mathfrak{h}_{\mathrm{i}}, [\inf\langle \aleph^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle], [\inf\langle \eth^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle], [\sup\langle \varrho^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle]\rangle\}$
$= \{\langle \mathfrak{h}_{\mathrm{i}}, [\mathfrak{r}, \mathfrak{s}, \mathfrak{t}]\rangle\}$

Where, $\mathfrak{r} = \inf\langle \aleph^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle$, $\mathfrak{s} = \inf\langle \eth^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle$ and $\mathfrak{t} = \sup\langle \aleph^{\mathrm{i}}(\varpi_{\mathrm{i}})\rangle$

Now for each $\varpi_{\mathrm{j}} \in \vartheta$, we have,

$$\varphi(\xi)(\varpi_{\mathrm{j}}) = \begin{cases} \sqcup_{\varphi(\varpi_{\mathrm{i}})=\varpi_{\mathrm{j}}} \xi(\varpi_{\mathrm{i}}) & if \quad \varphi^{\rightarrow}(\varpi_{\mathrm{j}}) \neq \odot \\ \langle \varpi, 0,0,\Omega\rangle & if \quad \varphi^{\rightarrow}(\varpi_{\mathrm{j}}) = \odot \end{cases}$$

From the hypothesis and the definition of φ, we get $\varphi(\xi)(\varpi_{\mathrm{j}}) = \xi(\varpi_{\mathrm{i}}) = \langle \mathfrak{h}_{\mathrm{i}}, [\mathfrak{r}, \mathfrak{s}, \mathfrak{t}]\rangle$ $\forall \varpi_{\mathrm{j}} \in \vartheta$ Hence $\varphi(\xi) = \xi$

Case 2: If $\varphi(\mathfrak{h}_{\mathrm{i}}) = \mathfrak{h}_{\mathrm{j}}; \mathfrak{h}_{\mathrm{i}}, \mathfrak{h}_{\mathrm{j}} \in \vartheta$ and $\mathrm{i} = \mathrm{j}$

In this case the least number of elements $(\vartheta, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ must be 3 element.

So, suppose that if $\vartheta = \{\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3\}$, then from the hypothesis and the definition of φ, the mapping $\varphi: \vartheta \rightarrow \vartheta$ can be defined as

$$\varphi(\varpi_{\mathrm{i}}) = \begin{cases} \varpi_{\mathrm{j}} & if \quad \mathrm{i} = \mathrm{j} = 1 \\ \varpi_{\mathrm{j}} & if \quad \mathrm{i} \neq \mathrm{j} \forall \mathrm{i} = \{2,3\}, \mathrm{j} = \{2,3\} \end{cases}$$

Let ξ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ orbit open set under mapping φ.

Then there exists a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ set ψ such that

$$\mathfrak{O}(\psi) = \xi \tag{2}$$

$$\mathfrak{O}(\psi) = \psi \sqcap \varphi^1(\psi) \sqcap \varphi^2(\psi) \sqcap \varphi^3(\psi) \sqcap \dots = \xi.$$

Let

$$\psi = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle, \langle \mathfrak{h}_3, \aleph^3(\varpi_3), \eth^3(\varpi_3), \varrho^3(\varpi_3)\rangle\}$$

Then from the definition of φ, we get

$$\varphi(\psi) = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^3(\varpi_3), \eth^3(\varpi_3), \varrho^3(\varpi_3)\rangle, \langle \mathfrak{h}_3, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle\}$$

$$\varphi^2(\psi) = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle, \langle \mathfrak{h}_3, \aleph^3(\varpi_3), \eth^3(\varpi_3), \varrho^3(\varpi_3)\rangle\}$$

$$\therefore \mathfrak{O}(\psi) = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle,$$

$$\langle \mathfrak{h}_2, [\inf\langle \aleph^2(\varpi_2), \aleph^3(\varpi_3), \aleph^2(\varpi_2), \aleph^3(\varpi_3) \dots\rangle],$$

$$[\inf\langle \eth^2(\varpi_2), \eth^3(\varpi_3), \eth^2(\varpi_2), \eth^3(\varpi_3) \dots\rangle],$$

$$[\sup\langle \varrho^2(\varpi_2), \varrho^3(\varpi_3), \varrho^2(\varpi_2), \varrho^3(\varpi_3) \dots\rangle]\rangle,$$

$$\langle \mathfrak{h}_3, [\inf\langle \aleph^3(\varpi_3), \aleph^2(\varpi_2), \aleph^3(\varpi_3), \aleph^2(\varpi_2) \dots \rangle], [\inf\langle ð^3(\varpi_3), ð^2(\varpi_2), ð^3(\varpi_3), ð^2(\varpi_2) \dots \rangle], [\sup\langle \varrho^3(\varpi_3), \varrho^2(\varpi_2), \varrho^3(\varpi_3), \varrho^2(\varpi_2) \dots \rangle]\rangle\}$$

$$= \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), ð^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, [\min\langle \aleph^2(\varpi_2), \aleph^3(\varpi_3)\rangle], [\min\langle ð^2(\varpi_2), ð^3(\varpi_3)\rangle] [\max\langle \varrho^2(\varpi_2), \varrho^3(\varpi_3)\rangle], \langle \mathfrak{h}_3, [\min\langle \aleph^3(\varpi_3), \aleph^2(\varpi_2)\rangle], [\min\langle ð^3(\varpi_3), ð^2(\varpi_2)\rangle]\ [\max\langle \varrho^3(\varpi_3), \varrho^2(\varpi_2)\rangle]\rangle\}\ [\because 2]$$

$$= \xi$$

In general, if $\vartheta = \{\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3\}$ and ξ be a $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$ orbit open set under mapping φ, then there exists a $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$ set,

$$\psi = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), ð^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), ð^2(\varpi_2), \varrho^2(\varpi_2)\rangle, \langle \mathfrak{h}_3, \aleph^3(\varpi_3), ð^3(\varpi_3), \varrho^3(\varpi_3)\rangle \dots = \{\langle \mathfrak{h}_\mathrm{i}, \aleph^\mathrm{i}(\varpi_\mathrm{i}), ð^\mathrm{i}(\varpi_\mathrm{i}), \varrho^\mathrm{i}(\varpi_\mathrm{i})\rangle\}$$

$$\mathfrak{D}(\psi) = \xi$$

$$\mathfrak{D}(\psi) = \begin{cases} \langle \mathfrak{h}_\mathrm{i}, \aleph^\mathrm{i}(\varpi_\mathrm{i}), ð^\mathrm{i}(\varpi_\mathrm{i}), \varrho^\mathrm{i}(\varpi_\mathrm{i})\rangle & if \quad \varphi^{\rightarrow}(\varpi_\mathrm{i}) = \varpi_\mathrm{j}; \mathrm{i} = \mathrm{j} \\ \langle \mathfrak{h}_\mathrm{i}, [\mathfrak{r}, \mathfrak{s}, \mathfrak{t}]\rangle & if \quad \varphi^{\rightarrow}(\varpi_\mathrm{i}) = \varpi_\mathrm{j}; \mathrm{i} \neq \mathrm{j} \end{cases}$$

$$= \xi$$

Now for each $\varpi_\mathrm{i} \in \vartheta$, we have,

$$\varphi(\xi)(\varpi_\mathrm{j}) = \begin{cases} \sqcup_{\varphi(\varpi_\mathrm{i}) = \varpi_\mathrm{j}} \xi(\varpi_\mathrm{i}) & if \quad \varphi^{\rightarrow}(\varpi_\mathrm{j}) \neq \odot \\ \langle \varpi, 0,0, \Omega\rangle & if \quad \varphi^{\rightarrow}(\varpi_\mathrm{j}) = \odot \end{cases}$$

From the hypothesis and the definition of φ, we get

$$\mathfrak{D}(\psi) = \begin{cases} \langle \aleph^\mathrm{i}(\varpi_\mathrm{i}), ð^\mathrm{i}(\varpi_\mathrm{i}), \varrho^\mathrm{i}(\varpi_\mathrm{i})\rangle & if \quad \mathrm{i} = \mathrm{j} \\ \langle \mathfrak{h}_\mathrm{i}, [\mathfrak{r}, \mathfrak{s}, \mathfrak{t}]\rangle & if \quad \mathrm{i} \neq \mathrm{j} \end{cases}$$

Hence $\varphi(\xi) = \xi$

Case 3: If φ acts as the identity self–map on ϑ.

Under this assumption, every $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$–open subset of the space $(\vartheta, \tau_{\mathcal{N}_\mathfrak{s}^\mathfrak{o}})$ automatically satisfies the orbit–open criterion induced by φ. Since the mapping leaves each element of ϑ unchanged, the image of any $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$–set ξ remains invariant, yielding $\varphi(\xi) = \xi$.

Thus, the invariance property holds for all $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$–sets ξ, completing the verification.

Theorem 3.2: *Let $(\vartheta, \tau_{\mathcal{N}_\mathfrak{s}^\mathfrak{o}})$ be a $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$-topological space and $\varphi: \vartheta \to \vartheta$ be any constant mapping. Then $\varphi(\xi) = \xi$ for any $\mathcal{N}_\mathfrak{s}^\mathfrak{o}$ orbit open set ξ under mapping φ.*

Proof: Let $(\vartheta, \tau_{\mathcal{N}_s^{\mathfrak{o}}})$ be a $\mathcal{N}_s^{\mathfrak{o}}$-topological space.

Let ξ be a $\mathcal{N}_s^{\mathfrak{o}}$ orbit open set under mapping φ.

Then there exists $\mathcal{N}_s^{\mathfrak{o}}$ set $\psi = \{\langle \mathfrak{h}_i, \aleph^i(\varpi_i), \eth^i(\varpi_i), \varrho^i(\varpi_i)\rangle\}$ such that

$$\mathfrak{O}(\psi) = \xi \tag{3}$$

Since φ is constant mapping, there exists a fixed element $\varpi_{\mathfrak{f}} \in \vartheta$ such that

$$\varphi(\varpi_i) = \varpi_{\mathfrak{f}}$$

Then we have,

$$\varphi(\psi)(\varpi_j) = \begin{cases} \sqcup_{\varphi(\varpi_i)=\varpi_j} \psi(\varpi_i) & if\ \varphi^{\rightarrow}(\varpi_j) \neq \odot \\ \langle \varpi, 0,0,\Omega\rangle & Otherwise \end{cases}$$

Thus $\varphi(\psi)(\varpi_j) = \begin{cases} \langle sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\}, inf\{\psi(\varpi_i)\}\rangle & if\ \varpi_j = \varpi_{\mathfrak{f}} \\ \langle \varpi, 0,0,\Omega\rangle & if\ \varpi_j \neq \varpi_{\mathfrak{f}} \end{cases}$

$$\therefore \varphi(\psi) = \{\langle \varpi_{\mathfrak{f}}, sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\}, inf\{\psi(\varpi_i)\}\rangle\}$$

This means $\varphi(\psi)$ is a $\mathcal{N}_s^{\mathfrak{o}}$ point in $(\vartheta, \tau_{\mathcal{N}_s^{\mathfrak{o}}})$ with support $\varpi_{\mathfrak{f}}$ and degree $sup\{\psi(\varpi_i)\}$, degree $sup\{\psi(\varpi_i)\}$ and degree $inf\{\psi(\varpi_i)\}$.

By the same way we have,

$$\varphi^2(\psi) = \{\langle \varpi_{\mathfrak{f}}, sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\}, inf\{\psi(\varpi_i)\}\rangle\}$$

$$\varphi^3(\psi) = \{\langle \varpi_{\mathfrak{f}}, sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\}, inf\{\psi(\varpi_i)\}\rangle\}$$

For more clearing we have the following:

$$\psi = \{\langle \mathfrak{h}_1, \aleph^1(\varpi_1), \eth^1(\varpi_1), \varrho^1(\varpi_1)\rangle, \langle \mathfrak{h}_2, \aleph^2(\varpi_2), \eth^2(\varpi_2), \varrho^2(\varpi_2)\rangle, ...,$$
$$\langle \mathfrak{h}_{\mathfrak{f}}, \aleph^3(\varpi_{\mathfrak{f}}), \eth^{\mathfrak{f}}(\varpi_{\mathfrak{f}}), \varrho^{\mathfrak{f}}(\varpi_{\mathfrak{f}})\rangle ... \}$$

$$\varphi(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 0,0,\Omega\rangle, ...$$
$$\langle \varpi_{\mathfrak{f}}, sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\}, inf\{\psi(\varpi_i)\}\rangle ... \}$$

$$\varphi^2(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 0,0,\Omega\rangle, ... \langle \varpi_{\mathfrak{f}}, sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\},$$
$$inf\{\psi(\varpi_i)\}, inf\{\psi(\varpi_{\mathfrak{f}})\}\rangle ... \}$$

$$\varphi^3(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 0,0,\Omega\rangle, ...$$
$$\langle \varpi_{\mathfrak{f}}, sup\{\psi(\varpi_i)\}, sup\{\psi(\varpi_i)\}, inf\{\psi(\varpi_i)\}\rangle ... \}$$

Thus

$$\mathfrak{O}(\psi) = \psi \sqcap \varphi^1(\psi) \sqcap \varphi^2(\psi) \sqcap \varphi^3(\psi) \sqcap ... = \xi.$$

$$\mathfrak{O}(\psi) = \{\langle \mathfrak{h}_1, 0,0,\Omega\rangle, \langle \mathfrak{h}_2, 0,0,\Omega\rangle, ... \langle \mathfrak{h}_{\mathfrak{f}}, [\min\langle \aleph^3(\varpi_{\mathfrak{f}}), sup\{\psi(\varpi_i)\}\rangle],$$
$$[\min\langle \eth^{\mathfrak{f}}(\varpi_3), sup\{\psi(\varpi_i)\}\rangle]$$
$$[\max\langle \varrho^{\mathfrak{f}}(\varpi_3), inf\{\psi(\varpi_i)\}\rangle]\rangle\} \ [\because 3]$$

$$\mathfrak{O}(\psi) = \xi$$

This yield $\mathfrak{O}(\psi) = \xi$ is a $\mathcal{N}_s^{\mathfrak{o}}$ point in $(\vartheta, \tau_{\mathcal{N}_s^{\mathfrak{o}}})$ with support $\varpi_{\mathfrak{f}}$ and degree $\min\langle \aleph^3(\varpi_{\mathfrak{f}}), sup\{\psi(\varpi_i)\}\rangle$, degree $\min\langle \eth^{\mathfrak{f}}(\varpi_3), sup\{\psi(\varpi_i)\}\rangle\}$ and degree $\max\langle \varrho^{\mathfrak{f}}(\varpi_3), inf\{\psi(\varpi_i)\}\rangle$.

Hence from the definition of φ, we get $\varphi(\xi) = \xi$.

Example 2: Let $\vartheta = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5\}$ and $\tau_{\mathcal{N}_s^{\mathfrak{o}}} = \{\odot, \circledast, \xi\}$
where

$\xi = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 1.1,0,0.9\rangle, \langle \varpi_3, 0,0,1.1\rangle, \langle \varpi_4, 1.1,0,0\rangle, \langle \varpi_5, 1.1,0,0\rangle\}$

Define $\varphi: \vartheta \to \vartheta$ as $\varphi(\varpi_1) = \varpi_2, \varphi(\varpi_2) = \varpi_1, \varphi(\varpi_3) = \varpi_2, \varphi(\varpi_4) = \varpi_5, \varphi(\varpi_5) = \varpi_4$

φ is not bijective.

Let ψ is any $\mathcal{N}_s^{\mathfrak{o}}$-set.

$$\psi = \{\langle \varpi_1, 1.1,0.3,0.9\rangle, \langle \varpi_2, 1.2,0.4,0.8\rangle, \langle \varpi_3, 0,0.8,1.1\rangle, \langle \varpi_4, 1.6,0.2,0.4\rangle, \langle \varpi_5, 1.7,0.1,0.3\rangle\}$$

Then the $\mathcal{N}_s^{\mathfrak{o}}$ orbit of ψ is $\mathfrak{O}(\psi) = \psi \sqcap \varphi(\psi) \sqcap \varphi^2(\psi) \sqcap \varphi^3(\psi) \ldots = \xi$
where

$$\varphi(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 1.1,0.3,0.9\rangle, \langle \varpi_3, 1.1,0.4,0.8\rangle, \langle \varpi_4, 1.7,0.1,0.3\rangle, \langle \varpi_5, 1.7,0.1,0.3\rangle\}$$

$$\varphi^2(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 1.1,0.4,0.8\rangle, \langle \varpi_3, 1.1,0.3,0.9\rangle, \langle \varpi_4, 1.7,0.1,0.3\rangle, \langle \varpi_5, 1.7,0.1,0.3\rangle\}$$

$$\varphi^3(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 1.1,0.3,0.9\rangle, \langle \varpi_3, 1.1,0.4,0.8\rangle, \langle \varpi_4, 1.7,0.1,0.3\rangle, \langle \varpi_5, 1.7,0.1,0.3\rangle\}$$

$$\mathfrak{O}(\psi) = \{\langle \varpi_1, 0,0,\Omega\rangle, \langle \varpi_2, 1.1,0,0.9\rangle, \langle \varpi_3, 0,0,1.1\rangle, \langle \varpi_4, 1.1,0,0\rangle, \langle \varpi_5, 1.1,0,0\rangle\}$$

$$\mathfrak{O}(\psi) = \xi$$

Thus the $\mathcal{N}_s^{\mathfrak{o}}$-open set ξ is a $\mathcal{N}_s^{\mathfrak{o}}$ orbit open set under the mapping φ. But $\varphi(\xi) \neq \xi$.

Proposition 1: Let $(\vartheta, \tau_{\mathcal{N}_s^{\mathfrak{o}}})$ be a $\mathcal{N}_s^{\mathfrak{o}}$-topological space and $\varphi: \vartheta \to \vartheta$ be a mapping. If ξ is a $\mathcal{N}_s^{\mathfrak{o}}$ orbit open set under mapping φ, then $\mathfrak{O}(\xi) = \xi$.

Proof: $\mathfrak{O}(\xi) = \xi \sqcap \varphi(\xi) \sqcap \varphi^2(\xi) \sqcap \varphi^3(\xi) \ldots$

From remark 1, we have $\varphi(\xi) = \xi$,

$$\varphi^2(\xi) = \varphi(\varphi(\xi)) = \varphi(\xi) = \xi, \varphi^3(\xi) = \varphi(\varphi^2(\xi)) = \varphi(\xi) = \xi$$

Hence $\mathfrak{O}(\xi) = \xi$

Theorem 3.3: *Let $(\vartheta, \tau_{\mathcal{N}_s^{\mathfrak{o}}})$ be a $\mathcal{N}_s^{\mathfrak{o}}$-topological space and $\varphi: \vartheta \to \vartheta$ be a mapping. If ξ_1 and ξ_2 are $\mathcal{N}_s^{\mathfrak{o}}$ orbit open set under mapping φ, then $\mathfrak{O}(\xi_1 \sqcap \xi_2) = \mathfrak{O}(\xi_1) \sqcap \mathfrak{O}(\xi_2)$.*

Proof: Let us prove this theorem using theorem3.1.

Case 1: Suppose that φ is bijective mapping and $\varphi(\mathfrak{h}_i) = \mathfrak{h}_j$ $\mathfrak{h}_i, \mathfrak{h}_j \in \vartheta$ and $\mathfrak{i} \neq \mathfrak{j}$

Let ξ_1 and ξ_2 are $\mathcal{N}_s^o$ orbit open set under mapping φ.

Then there exist $\mathcal{N}_s^o$-set ψ_1 and ψ_2 defined as

$\psi_1 = \{\langle \mathfrak{h}_i, \aleph^1(\varpi_i), \eth^1(\varpi_i), \varrho^1(\varpi_i)\rangle\}$ and

$\psi_2 = \{\langle \mathfrak{h}_i, \aleph^2(\varpi_i), \eth^2(\varpi_i), \varrho^2(\varpi_i)\rangle\}$ such that

$\mathfrak{O}(\psi_1) = \xi_1$, $\mathfrak{O}(\psi_2) = \xi_2$. From theorem 3.1 Case 1,we have, $\mathfrak{O}(\psi_1) = \{\langle \mathfrak{h}_i, \aleph^1(\varpi_i), \eth^1(\varpi_i), \varrho^1(\varpi_i)\rangle\} = \xi_1$

$\mathfrak{O}(\psi_2) = \{\langle \mathfrak{h}_i, \aleph^2(\varpi_i), \eth^2(\varpi_i), \varrho^2(\varpi_i)\rangle\} = \xi_2$.

Thus

$$\xi_1 \sqcap \xi_2 = \{\langle \mathfrak{h}_i, [\min\langle \aleph^1(\varpi_i), \aleph^2(\varpi_i)\rangle], [\min\langle \eth^1(\varpi_i), \eth^2(\varpi_i)\rangle] \\ [\max\langle \varrho^1(\varpi_i), \varrho^2(\varpi_i)\rangle]\rangle\}$$

Let $\mathfrak{a} = \min\langle \aleph^1(\varpi_i), \aleph^2(\varpi_i)\rangle$, $\mathfrak{b} = \min\langle \eth^1(\varpi_i), \eth^2(\varpi_i)\rangle$ and

$\mathfrak{c} = \max\langle \varrho^1(\varpi_i), \varrho^2(\varpi_i)\rangle$ $\forall \varpi_j \in \vartheta$

$$\varphi(\xi_1 \sqcap \xi_2)(\varpi_j) = \begin{cases} \sqcup_{\varphi(\varpi_i)=\varpi_j} (\xi_1 \sqcap \xi_2)(\varpi_i) & if \quad \varphi^{\rightarrow}(\varpi_j) \neq \odot \\ \langle \varpi, 0,0, \Omega\rangle & if \quad \varphi^{\rightarrow}(\varpi_j) = \odot \end{cases}$$
$$= \langle \mathfrak{a}, \mathfrak{b}, \mathfrak{c}\rangle$$

Hence $\varphi^1(\xi_1 \sqcap \xi_2) = \xi_1 \sqcap \xi_2$, $\varphi^2(\xi_1 \sqcap \xi_2) = \xi_1 \sqcap \xi_2$, $\varphi^3(\xi_1 \sqcap \xi_2) = \xi_1 \sqcap \xi_2$.

Then by the theorem 3.1, $\mathfrak{O}(\xi_1 \sqcap \xi_2) = \mathfrak{O}(\xi_1) \sqcap \mathfrak{O}(\xi_2)$.

Case 2: Suppose that φ is bijective mapping and $\varphi(\varpi_i) = \varpi_j; \varpi_i, \varpi_j \in \vartheta$ where $\mathfrak{i} = \mathfrak{j}$.

Let ξ_1 and ξ_2 are $\mathcal{N}_s^o$ orbit open set under mapping φ.

Then there exist $\mathcal{N}_s^o$-set ψ_1 and ψ_2 defined as

$$\psi_1 = \{\langle \mathfrak{h}_i, \aleph^1(\varpi_i), \eth^1(\varpi_i), \varrho^1(\varpi_i)\rangle\}$$

and $\psi_2 = \{\langle \mathfrak{h}_i, \aleph^2(\varpi_i), \eth^2(\varpi_i), \varrho^2(\varpi_i)\rangle\}$ such that $\mathfrak{O}(\psi_1) = \xi_1$, $\mathfrak{O}(\psi_2) = \xi_2$.

From theorem 3.1 Case 1, we have,

$$\mathfrak{O}(\psi_1) = \{\langle \mathfrak{h}_i, \aleph^1(\varpi_i), \eth^1(\varpi_i), \varrho^1(\varpi_i)\rangle\} = \xi_1$$

$$\mathfrak{O}(\psi_2) = \{\langle \mathfrak{h}_i, \aleph^2(\varpi_i), \eth^2(\varpi_i), \varrho^2(\varpi_i)\rangle\} = \xi_2.$$

Thus

$$\xi_1 \sqcap \xi_2 = \{\langle \mathfrak{h}_i, [\min\langle \aleph^1(\varpi_i), \aleph^2(\varpi_i)\rangle], [\min\langle \eth^1(\varpi_i), \eth^2(\varpi_i)\rangle] \\ [\max\langle \varrho^1(\varpi_i), \varrho^2(\varpi_i)\rangle]\rangle\}$$

Let $\mathfrak{a} = \min\langle \aleph^1(\varpi_i), \aleph^2(\varpi_i)\rangle$, $\mathfrak{b} = \min\langle \eth^1(\varpi_i), \eth^2(\varpi_i)\rangle$ and

$\mathfrak{c} = \max\langle \varrho^1(\varpi_i), \varrho^2(\varpi_i)\rangle$ $\forall \varpi_j \in \vartheta$

$$\varphi(\xi_1 \sqcap \xi_2)(\varpi_j) = \begin{cases} \sqcup_{\varphi(\varpi_i)=\varpi_j} (\xi_1 \sqcap \xi_2)(\varpi_i) & if \quad \varphi^{\rightarrow}(\varpi_j) \neq \odot \\ \langle \varpi, 0,0, \Omega\rangle & if \quad \varphi^{\rightarrow}(\varpi_j) = \odot \end{cases}$$

$$= \langle \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \rangle$$

Hence $\varphi^1(\xi_1 \sqcap \xi_2) = \xi_1 \sqcap \xi_2$ and $\varphi^2(\xi_1 \sqcap \xi_2) = \xi_1 \sqcap \xi_2$, $\varphi^3(\xi_1 \sqcap \xi_2) = \xi_1 \sqcap \xi_2$.

Then by the theorem 3.1, $\mathfrak{O}(\xi_1 \sqcap \xi_2) = \mathfrak{O}(\xi_1) \sqcap \mathfrak{O}(\xi_2)$.

Case 3: If φ is constant mapping

Then the case is obvious.

Hence $\mathfrak{O}(\xi_1 \sqcap \xi_2) = \mathfrak{O}(\xi_1) \sqcap \mathfrak{O}(\xi_2)$

4. Similarity Measure for Neutrosophic Over Soft Sets

Let $\mathcal{Q}_i$ and $\mathcal{S}_i$ be two neutrosophic over soft set aggregates defined on the shared universe ϑ. The goal is to develop a *similarity evaluator* that measures their degree of affinity solely through *component alignment* and *feature-level harmony*, intentionally omitting any influence of separation extent, deviation scale, or spatial distance between the sets. This resemblance index may be expressed in the following form:

Tangent–Based Similarity Index:

$$\rho(\mathcal{Q}_i, \mathcal{S}_i) = \frac{1}{n}\left(\sum_{i=1}^{n} 1 - tan\left[\frac{\pi[|\aleph_{Qi}(\varpi) - \aleph_{Si}(\varpi)| + |\eth_{Qi}(\varpi) - \eth_{Si}(\varpi)| + |\varrho_{Qi}(\varpi) - \varrho_{Si}(\varpi)|]}{12}\right]\right) \quad (4)$$

5. Flowchart

Figure 1 illustrates the step-by-step procedure for selecting the optimal alternative using the proposed method.

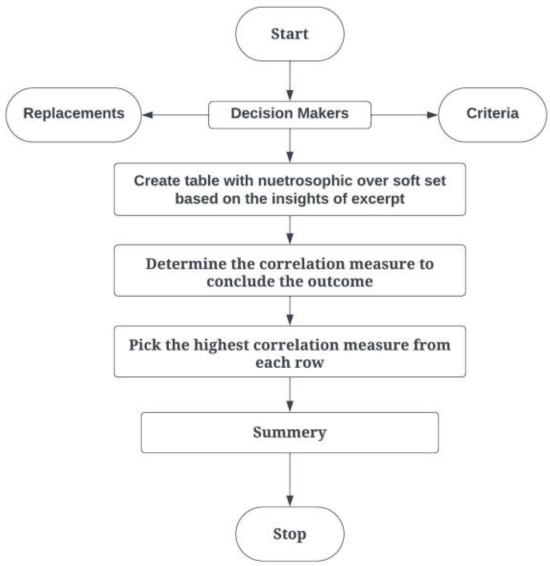

Figure 1

Flowchart of the proposed neutrosophic over soft decision-making model.

6. Computational Case Study

Choosing the correct player for the basketball match involves a structured and systematic way to ensure the team is equipped to face competition successfully. To begin with, one needs to have a clear view of the demands of the match, such as what the opposing team would play for, the competition level, and what roles they have to play within the team. By concentrating on essential objectives like enhancing offensive strategies, strengthening defense, or improving overall cooperation, the ground where effective decision-making takes place is laid out solidly.

Analyzing past games and performances is one of the most effective ways to find out what worked and what did not, thus, the team can get rid of the usual errors and get to the core of their approaches. This is a lengthy process involving the collection of player stats, video analysis, and the identification of previous performances' weak areas to be improved next time around. At this stage it is equally important to look at the team's resources; by this, it means the players' skills, physical condition, and their adaptability to different situations should all be considered.

After evaluation, the players should be ranked based on several criteria such as performance (shooting, ball handling, and defending), stamina (ability to run, cope with pressure), and cooperation (passing, communicating on the court). Practicing tactics is now possible during matches or simulations where the team is combining different players to find the best performance in the nearest thing to the real match scenario. If it continues to work on those combinations, then it guarantees that the team does not just select a player who is excellent individually but also one who has a positive contribution to the overall team morale.

The planning for emergencies must take place, such as there will be pre-arrangements or preparations in case of injuries, substitutions, or a shift in strategy by the opposing team. Flexibility and the capability to turn things around are characteristics that the selected player and the team must possess. Carrying that flexibility and adaptability that the team has to make any changes will only encourage the team to keep progressing with each movement throughout the match.

For example, a team that is going to play a championship basketball game will select three players for the event's best fit. Let the set of players be:

Players = $\{\mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3\}$,
Criteria = $\{Performance, Endurance, Teamwork\}$,
Position = {Selected,Non-Selected}

In this case, the skills of each player are evaluated based on the specified requirements. Performance metrics, endurance testing, and collaboration assessments provide both quantitative and qualitative information about the player's high level of performance. The main objective of this strategic process is to secure a player who not only meets the requirements of the match but also helps the team win throughout the tournament.

Table 1 presents the relationship between players and evaluation criteria, namely Performance, Endurance, and Teamwork, expressed in terms of neutrosophic components (truth, indeterminacy, and falsity values). It is observed that $\mathfrak{P}_1$ and $\mathfrak{P}_3$ have relatively higher truth membership values in Performance, whereas $\mathfrak{P}_2$ exhibits higher indeterminacy and falsity values across certain criteria.

Table 1

Relation between players and criteria

$\breve{Y}$	**Performance**	**Endurance**	**Teamwork**
$\mathfrak{P}_1$	(1.8,0.3,0.1)	(1.5,0.4,0.1)	(1.6,0.4,0.1)
$\mathfrak{P}_2$	(0.8,1.02,1.02)	(0.9,1.1,0.1)	(0.9,0.1,1.2)
$\mathfrak{P}_3$	(1.7,0.3,0.2)	(0.7,1.4,0.0)	(0.7,0.3,1.2)

Table 2 illustrates the interdependence of attributes with respect to the decision alternatives, categorized as Non-Selected and Selected. The neutrosophic values indicate how each criterion contributes differently to both categories, where Performance and Endurance show stronger association with the non-selected class, while Selected alternatives exhibit comparatively higher falsity components.

Table 2

Interdependence of Attributes and Ranking Order

$\breve{W}$	**Non-Selected**	**Selected**
Performance	(1.7,0.1,0.3)	(0.5,0.3,1.3)
Endurance	(1.6,0.3,0.2)	(0.6,0.3,1.2)
Teamwork	(0.7,1.2,0.1)	(0.7,1.3,0.1)

Table 3 provides the computed directional concordance similarity measures for each player under both Non-Selected and Selected categories. It is evident that player $\mathfrak{P}_2$ attains the highest similarity value (0.9909) in the Selected category, indicating its strong alignment with the desired selection criteria.

Table 3

Directional Concordance Similarity Measure

$\boldsymbol{\rho}$	**Non-Selected**	**Selected**
$\mathfrak{P}_1$	0.9963	0.9903
$\mathfrak{P}_2$	0.9902	0.9909
$\mathfrak{P}_3$	0.9933	0.9898

Figure 2 depicts the tangent similarity measure for machine (player) selection under the Selected category. The graph clearly shows that $\mathfrak{P}_2$ achieves the highest similarity value compared to $\mathfrak{P}_1$ and $\mathfrak{P}_3$, confirming its optimal selection.

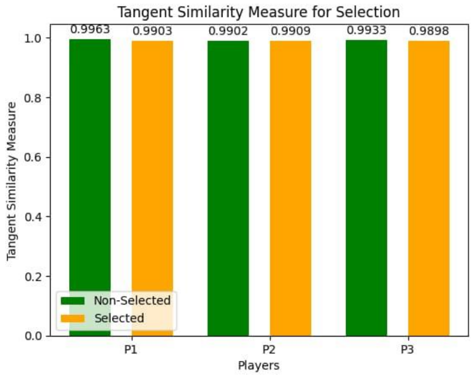

Figure 2
Graphical representation of tangent similarity measure for player selection.

Table 4 summarizes the final decision results based on the similarity measures. From the analysis, player $\mathfrak{P}_2$ is selected, while players $\mathfrak{P}_1$ and $\mathfrak{P}_3$ are not selected.

Table 4
Summary

Players	**Result**
$\mathfrak{P}_1$	Not selected
$\mathfrak{P}_2$	Selected
$\mathfrak{P}_3$	Not selected

Figure 3 presents a comparative analysis of tangent similarity measures for both Selected and Non-Selected categories. It can be observed that although all players have high similarity values, $\mathfrak{P}_2$ shows a relatively higher value in the Selected category, reinforcing the final decision outcome.

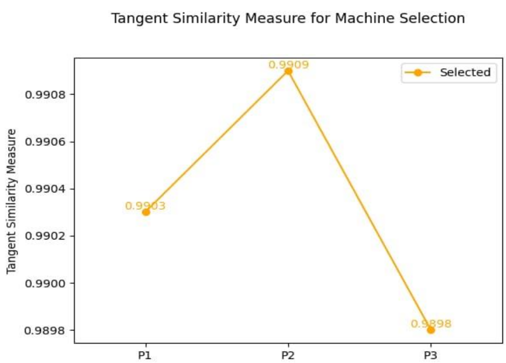

Figure 3
Graphical representation of tangent similarity measures for selected and non-selected players.

7. Conclusion

Neutrosophic Over Soft framework, which was investigated via the Neutrosophic Over Soft Topological Space, opens up a new area of soft computing that assists to make decisions in uncertain situations. The introduction of the neutrosophic over soft orbit Open Set was necessary for the proof of the previous assertion. The nonlinear similarity measure can be used to create a systematic way to choose the best player for specific match-each player's performance uncertainty and inter-team dynamics taken into consideration. Thus, this is the most appropriate method for real-world situations. The mathematical apparatus, including definitions and theorems concerning neutrosophic over soft sets, formed an unshakeable foundation for the secure and reliable decision-making process. Selecting Player 2 is a typical case of application in sports where the very fine distinctions have to be marked. This method not only makes the selection easier but also brings to the fore the broader benefit of the neutrosophic theory over the soft one in these diverse sectors.

Acknowledgment

I would like to express gratitude to Vel Tech Rangarajan Dr. Sagunthala R & D Institute of Science and Technology for Research Development Fund (VTU/RDF/FY 2025-2026/075).

References

[1] K. T. Atanassov and A. G. Shannon, "A note on intuitionistic fuzzy logics," *Acta Philosophica*, vol. 7, no. 1, pp. 121–125 (1998).

[2] K. T. Atanassov, "Intuitionistic fuzzy modal topological structure," *Mathematics*, vol. 10, no. 18, p. 3313 (2022).

[3] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Management Science,* vol. 17, no. 4, pp. B-141–B-164 (1970).

[4] S. Broumi, "Generalized neutrosophic soft set," *International Journal of Computer Science, Engineering and Information Technology*, vol. 3, no. 2, pp. 17-30 (2013).

[5] H. Bustince and P. Burillo, "Correlation of interval-valued intuitionistic fuzzy sets," *Fuzzy Sets Syst*., vol. 74, no. 2, pp. 237–244 (1995).

[6] V. Christianto, R. N. Boyd, and F. Smarandache, "Three possible applications of neutrosophic logic in fundamental and applied sciences," *International Journal of Neutrosophic Science*, vol. 1, no. 2, PP. 90-95 (2020).

[7] C. Deshpande, C. Ramtirthkar, T. A. Wani, and V. K. Bhosale, "Topology and knot theory applications in quantum field theory," *J. Interdiscip. Math.,* vol. 28, no. 6, pp. 2281–2287 (2025).

[8] R. N. Devi and Y. Parthiban, “Decision making by neutrosophic over soft topological space,” *Neutrosophic Sets Syst.*, vol. 66, pp. 76–94 (2024).

[9] R. N. Devi and Y. Parthiban, “Exploring neutrosophic over supra exterior modal topological structure: Theory and application in healthcare decision-making,” *Neutrosophic Sets Syst.*, vol. 72, pp. 272–285 (2024).

[10] R. N. Devi and Y. Parthiban, “Analyzing pharmaceutical compounds through neutrosophic over soft hyperconnected spaces,” *Orient. J. Chem.*, vol. 40, no. 6, pp. 1627–1633 (2024).

[11] R. Dhavaseelan, R. N. Devi, S. Jafari, and Q. H. Imran, “Neutrosophic alpha m-continuity,” *Neutrosophic Sets Syst.*, vol. 27, no. 1, pp. 171-179 (2019).

[12] G. A. Hussein and Z. D. Al-Nafie, “Regularity and normality of nano topological spaces modeled by pre-open sets structures,” *J. Interdiscip. Math.*, vol. 27, no. 4, pp. 667–673 (2024).

[13] R. Jansi, K. Mohana, and F. Smarandache, “Correlation measure for pythagorean neutrosophic sets with T and F as dependent neutrosophic components,” *Neutrosophic Sets Syst.*, vol. 30, no. 1, pp. 202-212 (2019).

[14] K. S. Mhavarkar, “Critical analysis of bankruptcy risk using Altman’s Z-score model of India Cement Ltd. (2020–2024),” *J. Stat. Manag. Syst.*, vol. 29, no. 4, pp. 399–412 (2026).

[15] T. Madhumathi and F. N. Irudayam, “Neutrosophic Orbit Topological Spaces,” *Trends Sci.*, vol. 18, no. 24, p. 1443 (2021).

[16] T. Madhumathi and F. N. Irudayam, “Neutrosophic Orbit Continuous Mappings,” *Neutrosophic Sets Syst*., vol. 50, p. 287 (2022).

[17] P. K. Maji, R. Biswas, and A. R. Roy, “Soft set theory,” *Comput. Math. Appl.*, vol. 45, no. 4–5, pp. 555–562 (2003).

[18] R. Mallick and S. Pramanik, “Pentapartitioned neutrosophic set and its properties,” *Neutrosophic Sets Syst.*, vol. 36, no. 1, pp. 184-192 (2020).

[19] D. Molodtsov, “Soft set theory First results,” *Comput. Math. Appl.*, vol. 37, no. 4, pp. 19–31 (1999).

[20] R. Murugesan, Y. Parthiban, R. N. Devi, K. R. Mohan, and S. K. Kumaravel, “A comparative study of fuzzy cognitive maps and neutro-

sophic cognitive maps on COVID variants," *Neutrosophic Sets and Systems,* vol. 55, no. 1, p. 20 (2023).

[21] R. Radha, A. S. A. Mary, R. Prema, and S. Broumi, "Neutrosophic pythagorean sets with dependent neutrosophic pythagorean components and its improved correlation coefficients," *Neutrosophic Sets Syst.,* vol. 46, pp. 77–86 (2021).

[22] R. N. Majeed and S. A. El-sheikh, "Fuzzy orbit topological spaces," *Mater. Sci. Eng.*, vol. 571, p. 012026 (2019).

[23] M. Saeed, M. Saqlain, A. Mehmood, and S. Yaqoob, "Multi-polar neutrosophic soft sets with application in medical diagnosis and decision-making," *Neutrosophic Sets Syst.*, vol. 33, pp. 183–207 (2020).

[24] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic.* Rehoboth, NM, USA: American Research Press (1999).

[25] F. Smarandache and S. Pramanik, *New Trends in Neutrosophic Theory and Applications,* vol. 1, I. Brussels, Belgium: Pons Editions (2016).

[26] F. Smarandache, *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset*. Brussels, Belgium: Pons Editions (2016).

[27] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Multispace & Multistructure*, vol. 12 (2010).

[28] J. Ye, "Another form of correlation coefficient between single valued neutrosophic sets and its multiple attribute decision-making method," *Neutrosophic Sets Syst.*, vol. 1, no. 1, pp. 8–12 (2013).

[29] L. A. Zadeh, "Fuzzy sets," *Inf. Control,* vol. 8, no. 3, pp. 338–353 (1965).

[30] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets Syst.*, vol. 1, no. 1, pp. 3–28 (1978).

[31] T. G. Alemayehu and Z. A. Ageze, "Fuzzy ideals and fuzzy congruences of distributive demi-p-algebras," *J. Discrete Math. Sci. Cryptogr.,* vol. 26, no. 8, pp. 2127–2137 (2022).

[32] S. Shirina, R. Bali, Y. Sharma, P. Singh, and S. Kadyan, "Exploring the connection among employee performance, employee retention, and employee engagement within higher education institutions (HEIs)," *J. Inf. Optim. Sci.,* vol. 44, no. 8, pp. 1529–1542 (2023).

Received November, 2025