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Analysis of Casson fluid flow over a semi-infinite flat plate with leading edge accretion

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Abstract

This manuscript focuses on the leading-edge accretion and heat absorption performance of Casson fluids in rolling mills. Casson fluid flow associated with leading-edge accretion is used to understand and improve the performance of coolant for heat exchangers. We investigate the unsteady Casson fluid flow over a flat surface with a moving slot. Using the Blasius-Rayleigh-Stokes variable, the governing equations are converted to ODE system and solved by the bvp4c procedure in MATLAB. The results examine the effects of accretion rate on Casson fluid flow and compare both non-Newtonian and Newtonian fluids. This analysis demonstrates that non-Newtonian fluid flow has better heat transfer efficiency with increasing leading-edge accretion. Increasing the Prandtl number and Casson fluid parameter reduces the heat transfer efficiency near the surface. By accounting for nonlinear velocity, momentum, thermal interactions, Blasius-Rayleigh-Stokes conditions, and conduction, this research delves into under explored aspects of fluid dynamics, highlighting its originality and contribution to the field.

Subject Classification: *76D05, 80A05, 80M15.*

Keywords: *Casson fluid flow, Thermal conduction, Flat surface, Accretion.*

1. Introduction

Heat exchangers in power plants faces the unwanted material accumulation and leading-edge accretion on heat transfer surfaces. This accumulation minimize the thermal transfer efficiency, and reduced the system lifespan,

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energy consumption. Sun et al. [1] and Yukawa, Abe, and Fujiwara [2] investigated the decrease in the lifespan of the system due to the increase in surface thickness and oxide deposition on the rolling surface. Recently in fluid dynamics, non-Newtonian fluids promising coolant for enhancing heat transfer rate. Casson non-Newtonian fluid has unique rheological properties make it particularly advantageous in environments with high thermal stresses.

Stewartson [3] studied the boundary layer motion of compressible fluids in motion from rest. Their findings indicated that the outflow increases, potentially forming two compression waves. Todd [4] extended this work by presenting solutions for accretion in laminar boundary-layer equations. The study revealed that extreme cases of these solutions, and highlighting key differences between boundary-layer behaviors under ablation and accretion conditions. Further exploration of unstable flow was conducted by Fang [5], Fang, Zhang, and Yao [6], Dzulkifli et al. [7] and Hussanan et al. [8].

Ilias et al. [9] conducted a theoretical analysis of magnetic nanofluid's leading-edge accretion, focusing on unsteady MHD boundary layer. This research demonstrated that aligning magnetic fields with fluid flow enhances the control of thermal transfer with leading-edge accretion processes. Ahmad et al. [10], Reddy et al. [11], Basha, Kumar, and Reddy, [12], and Lu et al. [13] examined the inclusion of magnetic particles improved heat transfer efficiency, a result of enhanced thermal conductivity and the alignment of magnetic dipoles under an external magnetic field.

Ali et al. [14] explored the finite element simulations, and analyzed the intricate interactions between flow, heat transfer, and species concentration. Kumar et al. [15] and Mabood and Khan [16] investigated the Rayleigh-Stokes and Blasius flows, where electrical resistance generates heat. Their studies revealed that these flow dynamics maximize energy transport in thermal management systems.

More recently, Adebayo et al. [17], Ishaq and Ahmad [18], and Nuwairan and Souayeh [19] used Blasius-Rayleigh-Stokes variables to theoretically solve the convective nanofluid flow. Gangadhar et al. [20] focused on the combination of collocation and finite difference methods to solve transformed boundary value problems. Lastly, Animasaun et al. [21] analyzed ternary-hybrid nanofluid flow over horizontal surfaces, revealing how convective and unsteady accelerations influence thermal behavior. Their findings provide valuable insights into heat transfer mechanisms for engineering applications. Nisar et al. [22] used the Atangana Baleanu fractional derivative to examined the uncontrolled growth of cancer with reproductive.

The literature highlights the extensive applications of Newtonian fluids in heat transfer processes, particularly in scenarios involving leading-edge accretion and ablation. These studies have been instrumental in optimizing heat transfer mechanisms, showcasing advancements that incorporate shear-dependent viscosity and sophisticated thermal conductivity models.

In this manuscript, we investigate the Casson fluid flow over a horizontal surface emerged with a moving slot, influenced by unsteady boundary layers.

These phenomena provide critical insights into the dynamic behavior of the fluid. The numerical solutions are obtained using MATLAB's bvp4c procedure, ensuring convergence and accuracy. This study comprehensively analyze the accretion effects of Casson fluid flow over a flat stretched surface. By addressing this significant gap, our research contributes to a deeper understanding of unsteady fluid flow dynamics. Additionally, this work offers a thorough analysis of momentum and temperature profiles for Casson fluid flow, considering the impact of leading-edge accretion, moving slots, and thermal conduction on a horizontal surface.

2. Mathematical formulation

Consider the incompressible Casson fluid flow with velocity u, v over a flat semi-infinite plate. Let T_w is surface temperature, T_∞ is ambient temperature. The external magnetic force $B(x) = \frac{B_0}{\sqrt{\nu t \cos\alpha + \frac{\nu x}{U_w}\sin\alpha}}$ is applied normal to the sheet. The Schematic diagram of this flow model is shown in Figure 1.

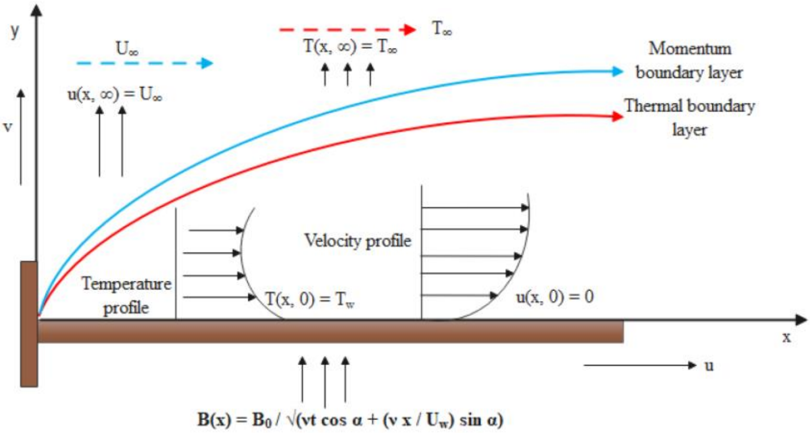


Figure 1
Schematic diagram

The boundary layer equations are characterized by Ahmad et al. [10]

$$u_x + v_y = 0 \tag{1}$$

$$u_t + uu_x + vu_y = \nu\left(1 + \frac{1}{\beta}\right)u_{yy} - \frac{\sigma}{\rho}B(x)^2(u - U_\infty) \tag{2}$$

$$T_t + uT_x + vT_y = k\ T_{yy}. \tag{3}$$

The relevant boundary conditions are

$$y = 0 \quad \Rightarrow \quad T(y) = T_w, \quad v(y) = u(y) = 0, \tag{4}$$

$$y \to \infty \quad \Rightarrow \quad T(y) = T_\infty, \quad u(y) = U_\infty, \tag{5}$$

where T_w is a specified constant value, and the velocity $u \to U_\infty$ and temperature $T \to T_\infty$ as $y \to \infty$.

Introducing the stream function and Blasius-Rayleigh-Stocks similarity variables [4]

$$= U_w\sqrt{\nu t\cos\alpha + \frac{\nu x}{U_w}\sin\alpha f(\eta)},\ \text{where}\ \eta = \frac{y}{\sqrt{\nu t\cos\alpha + \frac{\nu x}{U_w}\sin\alpha}}$$

$$T = T_\infty + \theta(\eta)(T_w - T_\infty). \tag{6}$$

Using (6), the equations (1)-(5) are transformed to the form

$$(1+\beta)f'''(\eta) + f''(\eta)\left[\frac{\beta}{2}(\cos\alpha\eta + \sin\alpha f(\eta))\right] - M\ f'(\eta) = 0 \tag{7}$$

$$\theta''(\eta) + \theta'(\eta)\left[\frac{Pr}{2}(\cos\alpha\eta + \sin\alpha f(\eta))\right] = 0, \tag{8}$$

and

$$\eta = 0 \Rightarrow \theta(0) = 1,\ f(0) = 0,\ f'(0) = 1;$$
$$\eta \to \infty \quad \Rightarrow \theta(\infty) = 0,\ f'(\infty) = 0. \tag{9}$$

3. Result and discussion

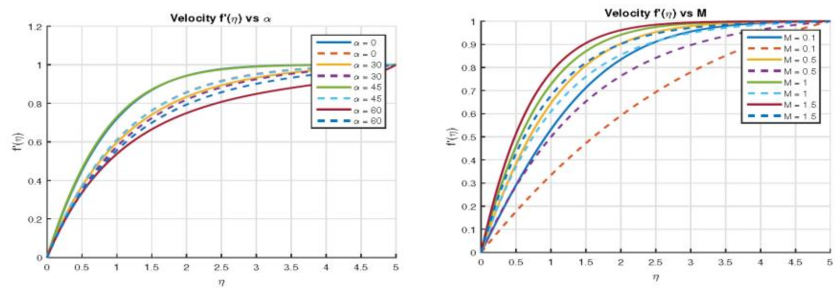
In 1997, Todd [4] explored solutions for a novel Blasius-Rayleigh-Stokes boundary layer within the range $0 < \alpha < \frac{\pi}{2}$. In this regime, accretion occurs at the leading edge, with the accretion rate monotonically increasing as α decreases. The non-dimensional governing equations (7)-(9) are solved by using MATLAB's bvp4c procedure. The governing equations that exclude the Casson fluid parameters (β) represent the Newtonian fluid model. The results, along with their implications, are discussed in the subsequent section, highlighting comparisons between Newtonian fluids (represented by solid lines) and non-Newtonian Casson fluids (represented by dashed lines).

Figure 2 (a) presents a comparison of non-Newtonian and Newtonian Casson fluid flow with increasing accretion rate α. As α increases, the velocity boundary layer expands, reaching its maximum velocity at $\alpha = 45°$. Beyond this point ($45° < \alpha < 90°$), the boundary layer thickness decreases. The non-Newtonian Casson fluid exhibits a lower velocity peak compared to the Newtonian fluid, reflecting reduced resistance to flow deformation. In Figure 2 (b), increasing external magnetic force M enhances the velocity of both non-Newtonian and Newtonian fluids. However, due to the higher inertial forces of Casson fluid, their momentum is lower than Newtonian. This property of non-Newtonian fluids aids in improving heat transfer efficiency.

Figures 3 (a) and 3 (b) illustrate the influences of the Pr and M on the temperature boundary layer. The maximum in Pr, and M minimize the temperature layer, leading to maximize thermal distribution in the Casson fluid. This uniformity indicates improved thermal stability. The characteristics of the Casson fluid contribute to more effective heat and mass transfer, as evidenced by the smoother and maximum heat transmission rate compared to Newtonian fluids.

From Figure 4 (a), increasing the accretion rate α expands the temperature boundary layer, reaching its maximum temperature distribution at $\alpha = 60^{\circ}$. Beyond this point, the temperature distribution steadily decreases for $30^{\circ} < \alpha < 45^{\circ}$. In Figure 4 (b), the higher values of β reduced the temperature layer, and enhancing thermal stability. This reflects the fluid's increased resistance to deformation, which dampens overall fluid flow.

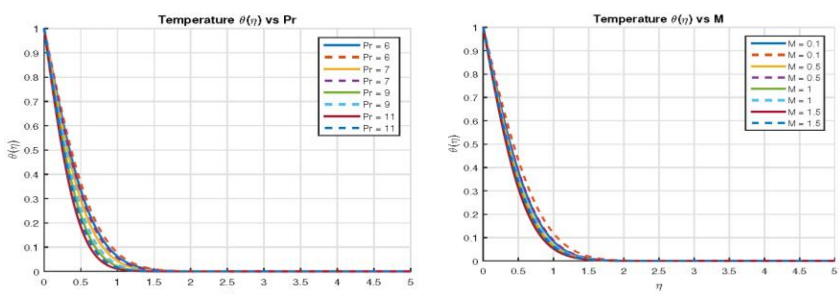
Figures 5 (a) and 5 (b) compare the streamlines of non-Newtonian and Newtonian fluids. The results indicate that Casson fluids exhibit a linear improvement in heat transfer efficiency with increasing external magnetic force. This increased resistance helps prevent material accumulation on surfaces, which is essential in addressing leading-edge accretion challenges. For applications where accretion is a concern, Casson fluids with higher β and accretion rate α values can offer improved control over material accumulation.



(a) Impact of moving slot parameter α **(b) Impact of magnetic parameter M**

Figure 2

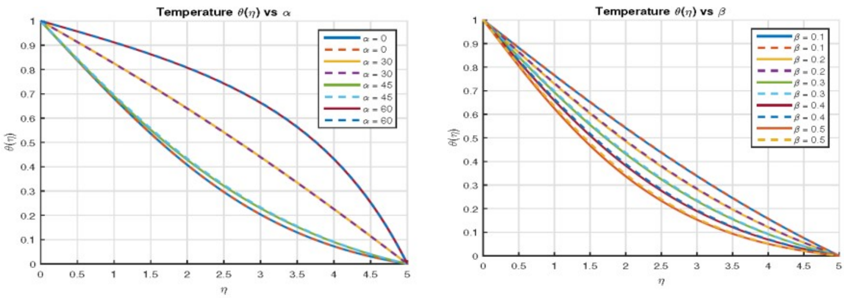
Comparison of non-Newtonian and Newtonian fluid while increasing moving slot and magnetic parameters



(a) Impact of Prandtl number Pr **(b) Impact of magnetic parameter M**

Figure 3

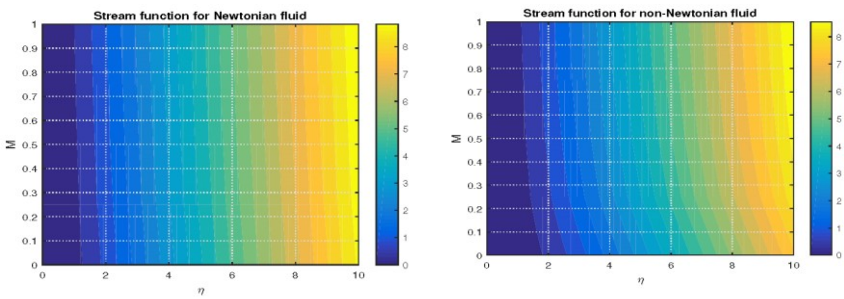
Comparison of non-Newtonian and Newtonian fluid while increasing Prandtl number and magnetic parameters



(a) Impact of moving slot parameter α **(b) Impact of Casson parameter β**

Figure 4

Comparison of non-Newtonian and Newtonian fluid while increasing moving slot parameter and Casson parameters



(a) Impact of Newtonian stream lines vs M **(b) Impact of non-Newtonian stream lines vs M**

Figure 5

Comparison of stream lines for non-Newtonian and Newtonian fluid while increasing magnetic parameters

4. Conclusion

In this manuscript, we investigated thermal transfer performance of Casson fluid flow with accretion. Using the Blasius-Rayleigh-Stokes variable, we identified key distinctions between the behaviors of non-Newtonian and Newtonian Casson fluids. The analysis highlights the impact of accretion rates on the Casson fluid flow, as demonstrated through streamline visualizations.

- The Casson fluid demonstrates superior performance in thermal transmission with increasing accretion rates.
- The increased accretion rate reduces the velocity near the surface except $\alpha = 45°$. Conversely, the Casson fluid has better the heat transfer rate at $\alpha = 60°$.
- Higher Pr and β reduces the temperature distribution. Higher Pr slows fluid motion and diminishes heat transfer efficiency, resulting in lower temperatures near the surface. A higher β promotes thermal stability through more uniform temperature profiles.

Future directions

Our future study will focus on the impacts on convection, diffusion, and concentration when accretion is present at the surface.

Nomenclature

u - fluid velocity in horizontal direction (m/s)
U_w - fluid velocity at the surface (m/s)
v - fluid velocity in vertical direction (m/s)
α - moving slot parameter (s^{-1})
U_∞ - ambient velocity (m/s)
T_∞ - ambient temperature (K)
T_w - temperature at the surface (K)
ν - kinematic viscosity (m^2/s)
β - Casson number
k - thermal diffusivity (m^2/s)
Pr - Prandtl number

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