Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 29 (2026), No. 5-B, pp. 1433–1439
DOI : 10.47974/JIM-2530

Using FP-transforms to solve a system of Canes blood of skin

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Abstract

Partial differential equations are used profoundly in scientific and engineering fields. This paper presents the application of the FP-transform in solving FPDEs. The first and second-order FPD formulas have been derived using highly generalized H-differentiability definitions. The FP-transform is introduced first, and then, based on its results, a class of fuzzy partial differential equations is constructed. An application has also been presented in the last section to demonstrate that the intended method will work perfectly.

Subject Classification: *Primary 93A30, Secondary 49K15.*

Keywords: *Mathematical modeling, Ordinary differential equations, A duopoly economy, Market share.*

1. Introduction

The study of physical systems requires deriving equations using differential calculus. Developing an excellent solution to differential equations demands too much work to be realistic for these types of problems. Using fuzzy math provides the most suitable method for this problem. People have successfully used FDEs to solve science and engineering problems for many years. The core concepts of fuzzy sets and their mathematical operations are described in [1]. The study of fuzzy derivatives and fuzzy integration progressed from [2–9] and [10], [11], respectively. Engineers and physicists use PDEs in physics and engineering, as well as in scientific fields related to chemistry and

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biology. The mathematical formula for partial differential equations applies to systems with multiple variables and their derivatives. The basic functions of differential equations do not adequately handle real-world physical issues. Watching events demands handling different factors facing each other. Modeling heat transfer within a wire requires handling spatial and temporal dimensions in the simulation. Several methods have been developed to solve partial differential equations, both analytically and computationally [3–5]. When solving linear partial differential equations, the fuzzy integral simplifies the original function, thereby making the problem easier to solve, as demonstrated in [10–19] and [8]. The later parts of this study material follow the design outlined below. Here we present FP-transform results for FP-derivatives of first and second order, along with basic principles of fuzzy numbers and fuzzy functions. These formulas help solve systems of partial differential equations.

Definition 1: Assume that $cot = cot(\varpi, s)$ be a continuous If s be the real parameters & fuzzy valued functions, the function's FP-transform follows cot. Denote by $\tilde{Y}_s(\varpi, tan(g))$ is defined as follows:

$$\tilde{\mathrm{Y}}_{\mathrm{s}}(\varpi, \tan(\mathrm{g})) = \mathrm{Q}_s[\cot(\varpi, \mathrm{s})] = \tan(\mathrm{g})^2 \int_0^{\infty} e^{-\left(i\sqrt[2\mathrm{N}]{\tan(\mathrm{g})}\right)\mathrm{s}} \cot(\varpi, \mathrm{s}) ds$$

$$= \lim_{\tau \to \infty} \tan(\mathrm{g})^2 \int_0^{\tau} e^{-\left(i\sqrt[2\mathrm{N}]{\tan(\mathrm{g})}\right)\mathrm{s}} \cot(\varpi, \mathrm{s}) ds$$

$$\tilde{\mathrm{Y}}_{\mathrm{s}}(\varpi, \tan(\mathrm{g})) = \begin{bmatrix} \lim\limits_{\tau \to \infty} \tan(\mathrm{g})^2 \int_0^{\tau} e^{-\left(i\sqrt[2\mathrm{N}]{\tan(\mathrm{g})}\right)\mathrm{s}} \underline{\cot}(\varpi, \mathrm{s}) ds\,, \\ \lim\limits_{\tau \to \infty} \tan(\mathrm{g})^2 \int_0^{\tau} e^{-\left(i\sqrt[2\mathrm{N}]{\tan(\mathrm{g})}\right)\mathrm{s}} \overline{\cot}(\varpi, \mathrm{s}) ds \end{bmatrix}$$

where there is a limit. so ϑ −cut representations for $\tilde{Y}_s(\varpi, tan(g))$ is given as:

$$\tilde{Y}_s(\varpi, tan(g); \vartheta) = Q_s[cot(\varpi, s; \vartheta)] = \left[\gamma\left(\underline{cot}(\varpi, s)\right), \gamma\left(\overline{cot}(\varpi, s)\right)\right],$$

Theorem 1: *Assume that* $cot: (0, \infty) \times (0, \infty) \to E$ *C-function with fuzzy values, and* cot_s *the PD of with respect to* s. *suppose that* $tan(g)^2 e^{-\left(i\sqrt[2N]{tan(g)}\right)s} cot(\varpi, s)$ *and* $tan(g)^2 e^{-\left(i\sqrt[2N]{tan(g)}\right)s} cot_s(\varpi, s)$ *are inadequate FRiemann integrals in* $[0, \infty]$, *so:*

1. *where* cot *the function represents a first form that is differentiable relative to s.*

$$Q_s[cot_s(\varpi, s)] = \left(i\sqrt[2N]{tan(g)}\right) Q_s[cot(\varpi, s)].\, tan(g)^2 cot(\varpi, 0),$$

2. *If* cot *is function represents the second form which is differentiable with respect to s.*

$$Q_s[cot_s(\varpi, s)] = -tan(g)^2 cot(\varpi, 0).\left(-i\sqrt[2\mathrm{N}]{tan(g)}\right) Q_s[cot(\varpi, s)].$$

Proof: Since $cot_s(\varpi, s)$ be C-Fuzzy valued functions, so we have:

Case 1:

$$Q_s[cot_s(\varpi)] = \gamma_s\left[\underline{cot}_s(\varpi,s;\vartheta)\right], \gamma_s[\overline{cot}_s(\varpi,s;\vartheta)]$$

$$\gamma_s\left[\underline{cot}_s(\varpi,\vartheta)\right] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)\gamma_s\left[\underline{cot}(\varpi,s;\vartheta)\right] - tan(g)^2\underline{cot}(\varpi,0;\vartheta),$$

$$\gamma_s[\overline{cot}_s(\varpi,\vartheta)] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)\gamma_s[\overline{cot}(\varpi,s;\vartheta)] - tan(g)^2\overline{cot}(\varpi,0;\vartheta).$$

$$Q_s[cot_s(\varpi)] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)\gamma_s\left[\underline{cot}(\varpi,s;\vartheta)\right] - \tan(g)^2\,\underline{cot}(\varpi,0;\vartheta), \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)\gamma_s[\overline{cot}(\varpi,s;\vartheta)] - tan(g)^2\overline{cot}(\varpi,0;\vartheta).$$

$$Q_s[cot_s(\varpi,s)] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)Q_s[cot(\varpi,s)].\,tan(g)^2cot(\varpi,0).$$

Case 2: Function differentiability applies to cot as the second type of function because any arbitrary value works here. $\vartheta \in [0,1]$

$$Q_s[cot_s(\varpi)] = \gamma_s[\overline{cot}_s(\varpi,s;\vartheta)], \gamma_s\left[\underline{cot}_s(\varpi,s;\vartheta)\right].$$

$$Q_s[cot_s(\varpi)] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)\gamma_s[\overline{cot}(\varpi,s;\vartheta)] - \tan(g)^2\,\overline{cot}(\varpi,0;\vartheta),$$

$$\left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)\gamma_s\left[\underline{cot}(\varpi,s;\vartheta)\right] - tan(g)^2\underline{cot}(\varpi,0;\vartheta).$$

$$Q_s[cot_s(\varpi,s)] = -tan(g)^2cot(\varpi,0).\left(-i^{2\mathrm{N}}\sqrt{tan(g)}\right)Q_s[cot(\varpi,s)].$$

Theorem 2: *Let $u: (0,\infty) \times (0,\infty) \to E$ be C-Fuzzy - valued functions & cot, cot_s FP-Transforms represent functions valued at C-Fuzzy level, taking evaluations about the second order with respect to s.*

1. If cot & cot_s for such that be differentiable for the first form relative to s.

$$Q_s[cot_{ss}(\varpi,s)] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)^2 Q_s[cot(\varpi,s)].\,tan(g)^2$$
$$\left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)cot(\varpi,0).\,tan(g)^2cot_s(\varpi,0)$$

2. If cot for such that are differentiable of the first form on s. and let s and cot_s the second form is differentiable on s, so we have:

$$Q_s[cot_{ss}(\varpi,s)] = -tan(g)^2\left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)cot(\varpi,0).$$
$$\left(-i^{2\mathrm{N}}\sqrt{tan(g)}\right)^2 Q_s[cot(\varpi,s)].\,tan(g)^2cot_s(\varpi,0)$$

3. If cot is for those that are a first form differentiable on s. with cot_s first differentiable on s, so:

$$Q_s[cot_{ss}(\varpi,s)] = -tan(g)^2\left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)cot(\varpi,0).$$
$$\left(-i^{2\mathrm{N}}\sqrt{tan(g)}\right)^2 Q_s[cot(\varpi,s)] - tan(g)^2cot_s(\varpi,0),$$

4. If cot and cot_s are second differentiable on s, so we have

$$Q_s[cot_{ss}(\varpi,s)] = \left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)^2 Q_s[cot(\varpi,s)].$$
$$tan(g)^2\left(i^{2\mathrm{N}}\sqrt{tan(g)}\right)cot(\varpi,0) - tan(g)^2cot_s(\varpi,0).$$

Proof: We have two cases:

Case 1: For any arbitrary$\vartheta \in [0,1]$

$Q_s[cot_{ss}(\varpi)] = \gamma_s[\underline{cot}_{ss}(\varpi, s; \vartheta)], \gamma_s[\overline{cot}_{ss}(\varpi, s; \vartheta)]$.

$Q[cot_{ss}(\varpi)] = \left(i^{2N}\sqrt{tan(g)}\right)^2 \gamma_s[\underline{cot}(\varpi, s; \vartheta)] -$
$tan(g)^2\left(i^{2N}\sqrt{tan(g)}\right)\underline{cot}(\varpi, 0; \vartheta) - tan(g)^2\underline{cot_s}\,(\varpi, 0; \vartheta)$,
$\left(i^{2N}\sqrt{tan(g)}\right)^2 \gamma_s[\overline{cot}(\varpi, s; \vartheta)] - tan(g)^2\left(i^{2N}\sqrt{tan(g)}\right)\overline{cot}(\varpi, 0; \vartheta) -$
$tan(g)^2\overline{cot}_s(\varpi, 0; \vartheta)$.

$Q_s[cot_{ss}(\varpi, s)] = \left(i^{2N}\sqrt{tan(g)}\right)^2 Q_s[cot(\varpi, s)]$.
$tan(g)^2\left(i^{2N}\sqrt{tan(g)}\right)cot(\varpi, 0). tan(g)^2 cot_s(\varpi, 0)$.

Case 2: For any arbitrary$\vartheta \in [0,1]$,
$Q_s[cot_{ss}(\varpi)] = \gamma_s[\overline{cot}_{ss}(\varpi, s; \vartheta)], \gamma_s[\underline{cot}_{ss}(\varpi, s; \vartheta)]$.
$Q_s[cot_{ss}(\varpi, s)] = -tan(g)^2\left(i^{2N}\sqrt{tan(g)}\right)cot(\varpi, 0)$.
$\left(-i^{2N}\sqrt{tan(g)}\right)^2 Q_s[cot(\varpi, s)]. tan(g)^2 cot_s(\varpi, 0)$.

Case 1: Here we get the following:

$$Q\left(\frac{\partial cot(\varpi,s)}{\partial\varpi}\right) = \left[\frac{\partial \underline{cot}(\varpi,s;\vartheta)}{\partial\varpi}, \frac{\partial \overline{cot}(\varpi,s;\vartheta)}{\partial\varpi}\right], 0 \leq \vartheta \leq 1.$$

Case 2: Here we get the following:

$$Q\left(\frac{\partial cot(\varpi,s)}{\partial\varpi}\right) = \left[\frac{\partial \overline{cot}(\varpi,s;\vartheta)}{\partial\varpi}, \frac{\partial \underline{cot}(\varpi,s;\vartheta)}{\partial\varpi}\right], 0 \leq \vartheta \leq 1.$$

Case 3: Here we get the following:

$$Q_s[cot_s(\varpi, s)] = \left(i^{2N}\sqrt{tan(g)}\right)Q_s[cot(\varpi, s)]. tan(g)^2 cot(\varpi, 0).$$

Case 4: Here we get the following:

$$Q_s[cot_s(\varpi, s)] = -tan(g)^2 cot(\varpi, 0). \left(-i^{2N}\sqrt{tan(g)}\right)Q_s[cot(\varpi, s)].$$

Example:

$cot_{\varpi} + w_s = 1$; I. C. (i) $cot(\varpi, 0) = (1 + \vartheta, 1 - \vartheta), w(\varpi, 0) = (2 + \vartheta, 4 - \vartheta)$
$w_{\varpi} + cot_s = 0$; B. C. (ii) $cot(0, s) = (3 + \vartheta, 5 - \vartheta)$ $\lim_{\varpi\to\infty}$ exist, $w(\varpi, 0)$

$$Q_s[cot_{\varpi}(\varpi, s)] + Q_s[w_s(\varpi, s)] = Q_s[1],$$
$$Q_s[w_{\varpi}(\varpi, s)] + Q_s[cot_s(\varpi, s)] = 0 .$$

We get the following cases:

$$\frac{\partial}{\partial\varpi} Q_s[cot(\varpi, s)] + \left(i^{2N}\sqrt{tan(g)}\right)Q_s[w(\varpi, s)]. tan(g)^2 w(\varpi, 0) = \frac{tan(g)^2}{i^{2N}\sqrt{tan(g)}},$$
$$\frac{\partial}{\partial\varpi} Q_s[w(\varpi, s)] + \left(i^{2N}\sqrt{tan(g)}\right)Q_s[cot(\varpi, s)]. tan(g)^2 cot(\varpi, 0) = 0 .$$

Then:

$$\frac{\partial}{\partial\varpi}\gamma_s[\underline{cot}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\underline{w}(\varpi,s)]-tan(g)^2\underline{w}(\varpi,0)=\frac{tan(g)^2}{i^{2N}\sqrt{tan(g)}},$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\overline{cot}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\overline{w}(\varpi,s)]-tan(g)^2\overline{w}(\varpi,0)=\frac{tan(g)^2}{i^{2N}\sqrt{tan(g)}}.$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\underline{w}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\underline{cot}(\varpi,s)]-tan(g)^2\underline{cot}(\varpi,0)=0\,,$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\overline{w}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\overline{cot}(\varpi,s)]-tan(g)^2\overline{cot}(\varpi,0)=0.$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\underline{cot}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\underline{w}(\varpi,s)]=tan(g)^2(2+\vartheta)+\frac{tan(g)^2}{i^{2N}\sqrt{tan(g)}},$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\overline{cot}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\overline{w}(\varpi,s)]=tan(g)^2(4-\vartheta)+\frac{tan(g)^2}{i^{2N}\sqrt{tan(g)}}.$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\underline{w}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\underline{cot}(\varpi,s)]=tan(g)^2(1+\vartheta),$$
$$\frac{\partial}{\partial\varpi}\gamma_s[\overline{w}(\varpi,s)]+\left(i^{2N}\sqrt{tan(g)}\right)\gamma_s[\overline{cot}(\varpi,s)]=tan(g)^2(1-\vartheta).$$
$$\underline{cot}(\varpi,s;\vartheta)=(3+\vartheta)k(s-\varpi)-(1+\vartheta)k(s-\varpi)+(1+\vartheta),$$
$$\overline{cot}(\varpi,s;\vartheta)=(5-\vartheta)k(s-\varpi)-(1-\vartheta)k(s-\varpi)+(1-\vartheta).$$
$$\underline{w}(\varpi,s;\vartheta)=s+(2+\vartheta)+(3+\vartheta)k(s-\varpi)-(1+\vartheta)k(s-\varpi),$$
$$\overline{w}(\varpi,s;\vartheta)=s+(4-\vartheta)+(5-\vartheta)k(s-\varpi)-(1-\vartheta)k(s-\varpi).$$
$$\frac{\partial}{\partial\varpi}Q_s[cot(\varpi,s)]-tan(g)^2w(\varpi,0).\left(-i^{2N}\sqrt{tan(g)}\right)Q_s[w(\varpi,s)]=\frac{tan(g)^2}{i^{2N}\sqrt{tan(g)}},$$
$$\frac{\partial}{\partial\varpi}Q_s[w(\varpi,s)]-tan(g)^2cot(\varpi,0).\left(-i^{2N}\sqrt{tan(g)}\right)Q_s[cot(\varpi,s)]=0\,.$$

Then:

$$\underline{cot}(\varpi,s;\vartheta)=(1+\vartheta)+\vartheta k(s-\varpi)-sk(s-\varpi)-(2+\vartheta)k(s-\varpi),$$
$$\overline{cot}(\varpi,s;\vartheta)=(1-\vartheta)+(2-\vartheta)k(s-\varpi)-sk(s-\varpi)-(4-\vartheta)k(s-\varpi).$$
$$\underline{w}(\varpi,s;\vartheta)=\vartheta k(s-\varpi)-sk(s-\varpi)-(2+\vartheta)k(s-\varpi)+s+(4-\vartheta),$$
$$\overline{w}(\varpi,s;\vartheta)=(2-\vartheta)k(s-\varpi)-sk(s-\varpi)-(4-\vartheta)k(s-\varpi)+s+(2+\vartheta).$$

Conclusion

This study demonstrates the effectiveness of the FP-transform in solving fuzzy partial differential equations by utilizing generalized H-differentiability. The derived first- and second-order formulas provide a structured approach for handling FPDEs. The presented application confirms the accuracy and reliability of the proposed method, highlighting its potential as a useful tool for solving complex problems in scientific and engineering domains.

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Received November, 2025