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Some results via pre-feebly-open sets in topological spaces

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Abstract

In this research, we introduce a recent type of open set, called a pre-feebly-open set, obtained by accrediting the feebly open set. We have defined and studied this type and specified several of its characteristics. We found some features and connections with other sets, obtained results that explain them, and symbolized them with the symbol pre-feebly-open. Also, we introduced and defined the notions of pre-f-closure and pre-f-interior. As well as offer the properties, prove them, and others.

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1. Introduction

Esmaeel and Shahadhuh [1] award the definition semi-open shortly (s-o.) set its fine on $E \subseteq \overline{E^{\circ}}$. Das [2] defined the notion of semi-closure, and he knows

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of E the least s-c. set in topological spaces shortly (T. S.) W contained E and lessening by s-cl(E) or $\overline{\mathrm{E}}^{\mathrm{s}}$. The nation pre-open shortly (pre-o.) set to submit it via Mashhour [3]. He also defined the notion pre-closure, and he knows of a set E, the least pre-closed set in T. S. W, contained E and lessening by pre-cl(E) or $\overline{\mathrm{E}}^{\mathrm{pre}}$. The fact $\overline{\mathrm{E}}^{\mathrm{pre}}$ ($\overline{\mathrm{E}}^{\mathrm{s}}$ [2]) is the crossing of every pre-closed (semi-closed) set contained in E, also $\overline{\mathrm{E}}^{\mathrm{pre}} \subseteq \overline{\mathrm{E}}$, and defined the notion pre-interior set he knows a set E, the union in T. S. W of every pre-open subsets of W contained in E, lessening pre-int(E) or $\mathrm{E}^{\circ\mathrm{pre}}$, and it was defined in [4] feebly open shortly (f-o.), set, and feebly closed shortly (f-c.), set. We will use (pre-f-o.) to pre-feebly open a set, (pre-f-c.) to pre-feebly close a set.

2. Preliminaries

Definition 2.1: In a T. S. (W,τ), and $\mathrm{V} \subseteq \mathrm{W}$, then:

1- V is the term pre-open [3] if $\mathrm{V} \subseteq \overline{\mathrm{V}}^{\circ}$. and V is a term pre-closed [3] if $\overline{\mathrm{V}^{\circ}} \subseteq \mathrm{V}$.

2- Terms V is s-o.. [1] if $\mathrm{V} \subseteq \overline{\mathrm{V}^{\circ}}$. terms V is s-c. [1] if $\overline{\mathrm{V}}^{\circ} \subseteq \mathrm{V}$.

3- Terms V is α-open [5], [6] if $\mathrm{V} \subseteq \overline{\mathrm{V}^{\circ}}^{\circ}$. and V is a term β-open set [7] if $\mathrm{V} \subseteq \overline{\overline{\mathrm{V}}^{\circ}}$.

4- Terms V is b-open [8], [9] if $\mathrm{V} \subseteq \overline{\mathrm{V}}^{\circ} \cup \overline{\mathrm{V}^{\circ}}$.

Proposition 2.2: [1]. Let $\{\mathrm{E}_{\varepsilon}\}_{\varepsilon\in\Lambda}$ be a family of s-o. (resp. pre-o.) in a T. S. over W, then $\bigcup_{\varepsilon\in\Lambda} \mathrm{E}_{\varepsilon}$ is s-open (resp. pre-open).

Remark 2.3: [1]. The intersection of two s-open (resp. pre-open) in W does not s-open (resp. pre-open) by the next examples:

Example 2.4: Let $\mathrm{W} = \{\mathrm{h}, \mathrm{k}, \mathrm{r}\}$ and $\tau = \{\emptyset, \mathrm{W}, \{\mathrm{h}\}, \{\mathrm{k}\}, \{\mathrm{h}, \mathrm{k}\}\}$, then $\{\mathrm{h}, \mathrm{r}\}$ and $\{\mathrm{k}, \mathrm{r}\}$ are s-open, but $\{\mathrm{h}, \mathrm{r}\} \cap \{\mathrm{h}, \mathrm{r}\} = \{\mathrm{r}\}$ is not s-open.

Example 2.5: Let $\mathrm{W} = \{\mathrm{h}, \mathrm{s}, \mathrm{v}, \mathrm{p}\}$ and $\tau = \{\emptyset, \mathrm{W}, \{\mathrm{s}\}, \{\mathrm{v}, \mathrm{p}\}, \{\mathrm{s}, \mathrm{v}, \mathrm{p}\}\}$, then $\{\mathrm{h}, \mathrm{s}, \mathrm{v}\}$ and $\{\mathrm{h}, \mathrm{s}, \mathrm{p}\}$ are pre-open, but $\{\mathrm{h}, \mathrm{s}, \mathrm{v}\} \cap \{\mathrm{h}, \mathrm{s}, \mathrm{p}\} = \{\mathrm{h}, \mathrm{s}\}$ is not pre-open.

Definition 2.6: [4]. A set E in a T. S. over W named f-o. in W if there occur open set M s.t. $\mathrm{M} \subseteq \mathrm{E} \subseteq \overline{\mathrm{M}}^{\mathrm{s}}$.

Theorem 2.7: [1]. *In T. S.* (W, τ)*, E is term f-o. set* $\Leftrightarrow$ *if* $E \subseteq \overline{(E^{0})}^{s}$.

Remark 2.9: [7]. In T. S. (W, τ) if E is open in W, then E is f-o. and E^{c} is f-c.. But the obverse is not true it can be viewed by next example.

Example 2.9: If $\mathrm{W} = \{\mathrm{h}, \mathrm{v}, \mathrm{r}, \mathrm{j}, \mathrm{q}\}$ and $\tau = \{\emptyset, \mathrm{W}, \{\mathrm{h}\}, \{\mathrm{h}, \mathrm{j}\}, \{\mathrm{h}, \mathrm{v}, \mathrm{j}\}\}$, then f-o. sets $= \{\emptyset, W, \{h, v, r, j\}, \{h, v, j\}, \{v, j\}, \{h\}\}$,

f-c. sets $= \{\emptyset, W, \{q\}, \{r, q\}, \{h, r, q\}, \{h, r, j, q\}\}$. Take $E = \{h, v, r, j\}$ is f-open but it's not open set, and $E^c = \{q\}$ is f-c., but it's not closed.

Proposition 2.10: [10]: Let W be a T. S. and $V, L \subseteq W$. Then:

1. $\overline{V}^f$ is f-c.
2. $V \subseteq \overline{V}^f$.
3. V is f-c. $\Leftrightarrow V = \overline{V}^f$.
4. If $V \subseteq L$ then $\overline{V}^f \subseteq \overline{L}^f$.
5. $\overline{\overline{V}^f}^f = \overline{V}^f$.
6. If $\{V_\varepsilon\}_{\varepsilon\epsilon\Lambda}$ be a combination of subsets of W, then $\overline{\cup_{\varepsilon\epsilon\Lambda} V_\varepsilon}^f = \cup_{\varepsilon\epsilon\Lambda} \overline{V}^f_\varepsilon$.
7. $\overline{V}^f = V \cup V'^f$
8. If $\{V_\varepsilon\}_{\varepsilon\epsilon\Lambda}$ be a collection of subsets of W, then $\overline{\cap_{\varepsilon\epsilon\Lambda} V_\varepsilon}^f = \cap_{\varepsilon\epsilon\Lambda} \overline{V}^f_\varepsilon$.
9. $\overline{V}^f \subseteq \overline{V}$.
10. $\overline{\overline{V}^f} = \overline{\overline{V}}^f = \overline{V}$.
11. $\overline{V}^f = V \cup \overline{\overline{V}^\circ}$.
12. $r \in \overline{V}^f \Leftrightarrow$ any f-o. M contained r, $M \cap V \neq \emptyset$.

Proposition 2.11: [10]. A T. S. W, and E⊆W is called:

1) Non-dense or nowhere dense in W $\Leftrightarrow \overline{(E)}^\circ = \emptyset$.
2) Every dense or Dense in W $\Leftrightarrow \overline{E} = W$.

Lemma 2.12: [10]. *Every singleton {r}in T. S. is either nowhere dense or pre-open.*

3. On pre-feebly open sets.

Definition 3.1: In a T.S. (W,τ), and E is a subset in (W, τ), then E is term pre-f-o. set in W if $E \subseteq M$ where M is pre-o. set in W then $\overline{E}^f \subseteq M$, The complement of pre-f-o. is term pre-f-c. It follows $M \subseteq E^{o^f}$.

Note 3.2:

1- For each T.S. $\emptyset, W$ are pre-f-o.

2- Every subset of discrete or indiscrete T.S. is pre-f-o.

3- In (R, U) wherever U is usual Topology, then every closed interval is pre-f-o.

Example 3.3: If W={1,2,3} and $\tau = \{W, \emptyset, \{1\}\}$ then closed set is: $\{W, \emptyset, \{2,3\}\}$ pre-f-o. = $\{ W, \emptyset, \{2\}, \{3\}, \{2,3\}\}$. We nots that {{1, {1,2,{1,3}} are not pre-f-o.

Proposition 3.4: Each f-c. is pre-f-o.

Proof: Since V be f-c. set in T.S. W, If $V \subset N$, where N is pre-open, V is f-closed set then $V = \overline{V}^f$ and $\overline{V}^f = V \subseteq N$, so V is pre-f-o. set.

The opposite is generally not true; it can be viewed in an example.

Example 3.5: Let $W = \{1,2,3,4,5\}$, and $\tau = \{\emptyset, W, \{2,3,4,5\}, \{1,3,4\}, \{1\}, \{3,4\}\}$ where $E = \{1,2,4,5\}$, then the closed set = $\{\emptyset, W, \{2,3,4,5\}, \{1,2,5\}, \{2,5\}, \{1\}\}$ pre open

$= \{\emptyset, W, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}, \{1,3\}, \{3\}, \{4\}, \{1,4\}, \{2,3\}, \{2,4\},$ $\{3,4\}, \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4,5\}, \{1,3,5\}, \{2,3,4\}, \{2,3,5\}, \{3,4,5\}\}$

Let E={1,2,4,5} and $M = W$ which is a pre-open set, then $E \subseteq M$ since $\overline{E}^s = E \cup \overline{\overline{E}}^{\circ} = \{1,2,4,5\} \cup \overline{\{1,2,4,5\}}^{\circ}$, $\overline{E}^f$={1, 2, 4, 5} $\cup W = W \subseteq M = W\, \overline{\{1,2,4,5\}}^{\circ}$, then E is pre-f-o., but not f-closed because $\overline{\overline{E}}^{\circ} = W \subsetneq \{1,2,4,5\}$.

Corollary 3.6: *Every closed set is pre-f-o. set.*

Proof: Since E closed set, then E is f-c. [from proposition (3.4)], E is pre-f-o.set.

But the obverse of (corollary (3.6)) is not true it can be viewed by example (3.5) shows $E = \{1,2,4,5\}$ is pre-f-o., $\overline{E} = W \neq E \Longrightarrow \overline{E} \neq E$ Thus, E is not closed.

Proposition 3.7: The union of pre f-o. sets in T. S. overW is also pre f-o. set.

Proof: If $\cup_{\rho\in\mu} S_\rho \subseteq V$, V pre-o. in a T. S. W then $S_\rho \subseteq \cup_{\rho\in\mu} S_\rho \subseteq V \Longrightarrow S_\rho \subseteq V$

$\Longrightarrow \overline{S}^f \subseteq V \Longrightarrow \cup_{\rho\in\mu} \overline{S}^f \subseteq V$, Since $\{S_\rho\}_{\rho\in\mu}$ be a compilation of every subset in W, then $\overline{\cup_{\rho\in\mu} S_\rho}^f = \cup_{\rho\in\mu} \overline{S}^f \Longrightarrow \overline{\cup_{\rho\in\mu} S_\rho}^f = \cup_{\rho\in\mu} \overline{S}^f \subseteq V \Longrightarrow \overline{\cup_{\rho\in\mu} S_\rho}^f \subseteq V$, then $\cup_{\rho\in\mu} S_\rho$ is pre-open set.

Proposition 3.8: If E be non-dense in a T. S. W then it is f-c.

Proof: Assume that E is a non-dense of W, then $\overline{E}^{\circ} = \emptyset$ and $\overline{\overline{E}}^{\circ} = \emptyset$, but $\emptyset \subseteq E \Longrightarrow$ E is f-c.

Proposition 3.9: Let W be T.S. then the crossing of pre-f-o. sets in W is pre-f-o.

Proof: Clear.

Proposition 3.10: In T. S. over W and $E \subseteq W$ then $\overline{E}^f$ is pre-f-o. set.

Proof: If $\overline{E}^{f} \subseteq M$ where M is pre-o., and $\overline{E}^{\overline{f}f} = \overline{E}^{f} \subseteq M \Longrightarrow \overline{E}^{\overline{f}f} \subseteq M \Longrightarrow \overline{E}^{f}$ pre-f-o. set.

Proposition 3.11: In T. S. over W and $E \subseteq W$ Then $\overline{E}$ is pre-f-o. set.

Proof: If $\overline{E} \subseteq U$, U is pre-o. set, since$\overline{\overline{E}}^{f} = \overline{\overline{E}}^{f} = \overline{E}$ (by proposition (2.10) (10)}, then $\overline{\overline{E}}^{f} = \overline{E} \Longrightarrow \overline{\overline{E}}^{f} \subseteq U$ where U is pre-o. $\Longrightarrow \overline{E}$ is pre-f-o. set.

Proposition 3.12: Let W be a T.S. and G⊆W, if G is pre-c., then G is pre-f-o. set.

Proof: If G is pre-c. then $G^{0} = \overline{G}^{0}$, as G is pre c. then $\overline{G}^{0} \subseteq G$, but $G^{0} = \overline{G}^{0}$, then $\overline{\overline{G}^{\circ}} \subseteq G \Longrightarrow G$ f-c. from (proposition (3.4)) G is pre-f-o. set.

Definition 3.13: In T. S. over W and $E \subseteq W$. Then the crossing of all pre-f-c. of W that includes E is the term pre-f-closure of E and shortly by $\overline{E}^{pre-f}$, or $\overline{E}^{pre-f} = \cap$ {L: L is pre-f-c. in W}.

Lemma 3.14: *Assume That W is a T.S., and $G \subseteq W$, then $\bar{G}^{pre-f}$ is the smallest pre-f-o. set include G.*

Proof: Since L be pre-f-c. set include G such that G⊆L, as $\overline{G}^{pre-f} = G \cup G'^{pre-f}$. and $\overline{G}^{pre-f} \subseteq \overline{L}^{pre-f}$, since $G \subseteq L$, then $\overline{G}^{pre-f} = G \cup G'^{pre-f} \subseteq L \cup L'^{pre-f} \subseteq L$, then $\overline{G}^{pre-f} \subseteq L$. Therefore $\overline{G}^{pre-f}$ is the smallest pre-f-o. set containing G.

Proposition 3.15: Let W be a T.S. and $G, L \subseteq W$. Then:

1. G is an Pre-f-c. set $\Longleftrightarrow G = \overline{G}^{pre-f}$. 2. $\overline{G}^{pre-f} \subseteq \overline{G}$. 3. $\overline{G}^{pre-f} = \overline{\overline{G}^{pre-f}}^{pre-f}$.
2. If $G \subseteq L$ then $\overline{G}^{pre-f} \subseteq \overline{L}^{pre-f}$.

Proof: **1.** Since G be pre-f-c. set. And let $G \subseteq \overline{G}^{pre-f}$, then $\overline{G}^{pre-f} \subseteq G$ [since $\overline{G}^{pre-f}$ is the smaller pre-f-closed set containing G], then $G = \overline{G}^{pre-f}$. **Conversely** let $G = \overline{G}^{pre-f}$. Then $\overline{G}^{pre-f}$ be a Pre-f-c. set. As $G = \overline{G}^{pre-f} \Longrightarrow$ G is a Pre-f-c. set. **2.** clear **3.** Since $\overline{G}^{pre-f}$ is a pre-f-closed set, then $\overline{G}^{pre-f} = \overline{\overline{G}^{pre-f}}^{pre-f}$ by [2].

Let $G \subseteq L$ and Pre-f-int(G)L^{pre-f}, then $G \subseteq L^{pre-f}$, $\overline{L}^{pre-f}$ is the smaller pre-f-c. set include G. let $\overline{G}^{pre-f}$ be the smaller pre-f-c. set include G. Therefore $\overline{G}^{pre-f} \subseteq L^{pre-f}$.

Definition 3.16: Let W be a T. S. and $E \subseteq W$. The union of all pre-f-o. sets of W include in E is term pre-f-interior of E, symbolize by pre-f-int(E) or $E^{\circ\, pre-f}$ we means pre-f-int(E) $= \bigcup\{M: M$ pre-f-o. in W and $M \subseteq E\}$.

Proposition 3.17: Assume that W is a T. S. and $E \subseteq W$. Then $r \in E^{\circ pre-f}$ iff there is a pre-f-o. set M containing r so $r \in M \subseteq E$.

Proof: Assume that $r \in E^{\circ pre-f} \Leftrightarrow r \in \bigcup\{M: M \subseteq E$ be a pre-f-o. in W$\}. \Leftrightarrow \exists$ M is pre-f-o. in W so that $r \in M \subseteq E$.

Proposition 3.19: Let W be a T.S. and $K, L \subseteq W$. Then:

1. K is a Pre-f-o. set $\Leftrightarrow K = K^{\circ pre-f}$. **2.** $K^{\circ pre-f}$ is a Pre-f-o. set.
3. $K^{\circ pre-f} = (K^{\circ pre-f})^{\circ pre-f}$. **4.** If $K \subseteq L$ then $K^{\circ pre-f} \subseteq L^{\circ pre-f}$.

Proof:

1. $K^{\circ pre-f} = \bigcup\{M: M$ be a pre-f-o. in W and $M \subseteq K\}$, by [proposition (3.7)]. Then $K^{\circ pre-f}$ be pre-f-o.

2. Let K be a pre-f-o. set, by definition $E^{\circ pre-f} \subseteq E$, $E^{\circ pre-f} = \bigcup\{M: M \subseteq K$ is pre-f-o. in W$\} \Longrightarrow$ K is pre-f-o. set in W. Then $\subseteq K^{\circ pre-f} \Longrightarrow K = K^{\circ pre-f}$.

Conversely Let $K = K^{\circ pre-f}$, Since $K^{\circ pre-f}$is the union of every pre-f-o. sets and $K = K^{\circ pre-f}$, then K is a pre-f-o. set.

3. Let $K^{\circ pre-f} = \bigcup\{M: M$ pre-f-o. in W and $M \subseteq K\}$. $(K^{\circ pre-f})^{\circ pre-f} = \bigcup\{M: M$ pre-f-o. in W and $M \subseteq K^{\circ pre-f}\}$, by (2) $K = K^{\circ pre-f}$. Therefore $K^{\circ pre-f} = (K^{\circ pre-f})^{\circ pre-f}$.

4. Conclusion

In this paper, we introduced pre-feebly-open sets as a new generalization of feebly open sets in topological spaces. We investigated their fundamental properties and established their relationships with other known set types. Additionally, the concepts of pre-f-closure and pre-f-interior were defined and their key properties were proved. These results contribute a useful extension to the existing framework of topological spaces.

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