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Some applications of nonlinear fractional differential equations by modified Elzaki transform

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Abstract

An major goal for this work is to propose a modified technique, the Elzaki Adomain Decomposition Method (ETADM), an iterative analytical approach that yields approximate analytical solutions for certain nonlinear problems. This technique relies on applying the Adomain Decomposition Method (ADM) to a nonlinear term that is amenable to solution using Adomain polynomials. ETADM has been used to break down the solution to a partial differential equation into an infinite number of components. Moreover, illustrative examples are provided, and the results obtained demonstrate the proposed method's accuracy, efficiency, and reliability. Results like this demonstrate how effective and efficient this method is in resolving problems of this kind. Therefore, using fractional differential equations as a model, our suggested approach can be used to analyze solutions to a broad range of genuine problems arising in the natural sciences and engineering.

Subject Classification: *35R11, 35R60.*

Keywords: *Decomposition method, Elzaki transform, Analytical method, Partial differential equations, Fractional differential equations.*

1. Introduction

One type of mathematical formula that employs fractional order derivatives is known as a fractional differential equation [1-5]. The fact that these equations can more effectively replicate the wide's ranges for the physicals and biologicals phenomena than traditional integer-order differentials equation has drawn

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considerable attention [6-9]. In many fields, including physics, engineering, biology, and many more, they are widely utilized. Numerical techniques were employed because the analytical methods proved ineffective for solving the nonlinear fractional differential equations. Nonlinear fractional differential equations can be solved numerically in a number of ways. Modified fractional Taylor series method, ADM, HPM, and VIM are among them [10-15].

2. Elzaki Transform Adomian Decompositions Methods of Solve NFPDEs

Consider non-linear differential equation with fractionals order as

$$cD_t^{\alpha}\Phi(\eta,t)+\text{Ɲ}\Phi(\eta,t)+W\Phi(\eta,t)=G(\eta,t)\,,n-1<\alpha\le n \tag{1}$$

And the initial conditions $\partial^{n-1}\left[\frac{\Phi(\eta,t)}{\partial^{n-1}}\right]_{t=0}=F_{n-1}(\eta), n=1,2,\ldots$

$$E[D_t^{\alpha}\Phi(\eta,t)]+E[\text{Ɲ}\Phi(\eta,t)]+E[W\Phi(\eta,t)]=E[G(\eta,t)]$$

Using the property of Elzaki transform, we obtain:

$$E[\Phi]=\sum_{k=0}^{n-1}\quad s^{k+2}\left[\frac{\partial^k s}{\partial t^k}\right]_{t=0}+s^{\alpha}\,E[G(\eta,t)]-s^{\alpha}E[\text{Ɲ}\Phi+W\Phi] \tag{2}$$

take an inverses ET on both side for Eq (2): so

$$\Phi(\eta,t)=H(\eta,t)-E^{-1}(s^{\alpha}E[\text{Ɲ}\Phi(\eta,t)+W\Phi(\eta,t)]) \tag{3}$$

$$\Phi(\eta,t)=\sum_{m=0}^{\infty}\quad \Phi_m(\eta,t) \tag{4}$$

$$\text{Ɲ}\Phi(\eta,t)=\sum_{m=0}^{\infty}\quad A_m(\Phi) \tag{5}$$

$$A_m(\Phi_0,\Phi_1,\Phi_2,\ldots,\Phi_m)=\frac{1}{m!}\frac{\partial^{\epsilon}}{\partial\epsilon^m}\left[\text{Ɲ}\left(\sum_{j=0}^{\infty}\quad \epsilon^j\Phi_j\right)\right]_{\epsilon=0}, m=0,1,2,\ldots$$

then $\sum_{m=0}^{\infty}\quad \Phi_m=H(\eta,t)-E^{-1}(s^{\alpha}E[\text{Ɲ}\sum_{m=0}^{\infty}\quad \Phi_m+W\sum_{m=0}^{\infty}\quad A_m(\Phi)])$

$\Phi_0(\eta,t)=H(\eta,t)\,,\Phi_1(\eta,t)=-E^{-1}(s^{\alpha}E[\text{Ɲ}\Phi_0(\eta,t)+A_0(\Phi)])$

$\Phi_2(\eta,t)=-E^{-1}(s^{\alpha}E[\text{Ɲ}\Phi_1(\eta,t)+A_1(\Phi)])$. The recursive relation is given by: $\Phi_{m+1}(\eta,t)=-E^{-1}(s^{\alpha}E[\text{Ɲ}\Phi_m(\eta,t)+A_m(\Phi)]), m=0,1,2,\ldots$

3. Solutions for some types of fractional PDEs by ETADM

3.1 Example 1: Consider $\frac{\partial^{\theta}X(\varkappa,\mathfrak{f},\hbar)}{\partial\hbar^{\theta}}+\frac{\partial^3X(\varkappa,\mathfrak{f},\hbar)}{\partial\varkappa^3}+\frac{\partial^3X(\varkappa,\mathfrak{f},\tau)}{\partial\mathfrak{f}^3}=0$, with $(\varkappa,\mathfrak{f},0)=X(\varkappa,\mathfrak{f},0)=Cos(\varkappa+\mathfrak{f}), 0<\theta\le 1$.

Solution: Taking ElZaki transform

$$E[\frac{\partial^{\theta}\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\hbar^{\theta}}]+E[\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\varkappa^3}]+E[\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\mathfrak{f}^3}]=0$$

$$E[\frac{X(\varkappa,\mathfrak{f},\hbar)}{s^{\theta}}]-s^{2-\theta}X(\varkappa,\mathfrak{f},0)=-E[\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\varkappa^3}+\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\mathfrak{f}^3}]=0$$

$$E[\,X(\varkappa,\mathfrak{f},\hbar)]=s^2Cos(\varkappa+\mathfrak{f})-s^{\theta}\,E[\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\varkappa^3}+\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\mathfrak{f}^3}]$$

Taking ElZaki transform

$$X(\varkappa,\mathfrak{f},\hbar)=Cos(\varkappa+\mathfrak{f})-\,E^{-1}[\,s^{\theta}\,E[\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\varkappa^3}+\frac{\partial^3\,X(\varkappa,\mathfrak{f},\hbar)}{\partial\,\mathfrak{f}^3}]]$$

$X_0(\varkappa,\mathfrak{f},\hbar) = Cos[\varkappa+\mathfrak{f}]$, $X_1(\varkappa,\mathfrak{f},\hbar) = -E^{-1}[\, s^{\theta}\, E[\frac{\partial^3\, X_0(\varkappa,\mathfrak{f},\hbar)}{\partial\, \varkappa^3} + \frac{\partial^3\, X_0(\varkappa,\mathfrak{f},\hbar)}{\partial\, \mathfrak{f}^3}]]$

$X1(\varkappa,\mathfrak{f},\hbar) = \frac{-2Sin[\varkappa+\mathfrak{f}]\,\hbar^{\,\theta}}{\sqrt{\theta+1}}$, $X2(\varkappa,\mathfrak{f},\hbar) = \frac{-4\,Cos(\varkappa+\mathfrak{f})\,\hbar^{\,2\theta}}{\sqrt{2\theta+1}}$

$X_3(\varkappa,\mathfrak{f},\hbar) = \frac{8\,Sin(\varkappa+\mathfrak{f})\,\hbar^{\,3\theta}}{\sqrt{3\,\theta+1}}$, And by the same procedure we can get

$X_4(\varkappa,\mathfrak{f},\hbar) = \frac{16\,Sin[\varkappa+\mathfrak{f}]\,\hbar^{\,4\,\theta}}{\sqrt{4\,\theta+1}}$, $X_5(\varkappa,\mathfrak{f},\hbar) = \frac{-32\,Sin[\varkappa+\mathfrak{f}]\,\hbar^{\,5\,\theta}}{\sqrt{5\,\theta+1}}$,

$X_6(\varkappa,\mathfrak{f},\hbar) = \frac{-64\,Cos[\varkappa+\mathfrak{f}]\,\hbar^{6\,\theta}}{\sqrt{6\,\theta+1}}$, $X_7(\varkappa,\mathfrak{f},\hbar) = \frac{128\,Sin[\varkappa+\mathfrak{f}]\,\hbar^{\,7\,\theta}}{\sqrt{7\,\theta+1}}$

The following figures represent the approximate solutions and consecutive error for X_7.

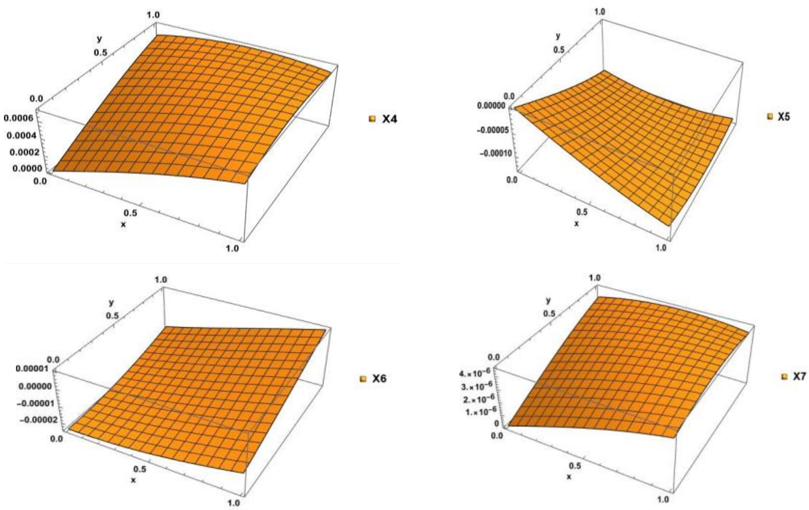

Figure 1
Shows the approximate solutions of X_4, X_5, X_6 and X_7.

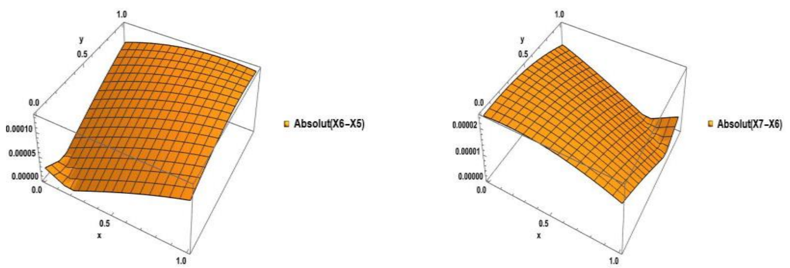

Figure 2
Shows the consecutive error of X_7.

4. Results and Discussion

For Figure 1 presents the approximate solutions for FPDEs using ETADM. Here, we use several terms to get the true solution; we obtain the proposed method ETADM that has a high convergence order and accuracy. For Figure 2 presents the consecutive error between the current solutions X_7 and the solution before it. Here, the proposed method ATADM has a high convergence order and accuracy we get.

5. Conclusion

The fractional model containing Caputo fractional derivatives has been solved using the modified computational technique presented in this paper. This technique is known as ETADM. This paper's numerical example demonstrates the ease of use, accuracy, and effectiveness of the suggested approach. Furthermore, the solutions obtained by this method take a forms for the infinite series which convergences quickly to an exact answers. We have successfully applied the combination of the two methods ET and ADM, namely ETADM, to solve certain types of PDEs, which is a major achievement. The test examples confirm that this new approach is effective, powerful, and yields accurate, reliable results. Also, the results show that ETADM converges to the exact solution.

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